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Delta-T noise in an inhomogeneous quantum Hall junction

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Summary. — The current fluctuations due to a temperature bias, *i.e.*, the delta-T noise, allow accessing properties of strongly interacting systems which cannot be addressed by the usual voltage-induced noise. In this work, we theoretically study the delta-T noise between two different fractional quantum Hall edge states, with filling factors (ν_L, ν_R) in the Laughlin sequence, coupled through a quantum point contact and connected to two reservoirs at different temperatures.

1. – Introduction

Noise is a fundamentally inescapable ingredient of any electronic device, that has now been broadly accepted as a key tool to improve our understanding of nanoscale conductors. Electronic noise is typically separated into two contributions: thermal (or Johnson-Nyquist) noise [1,2] and shot noise [3]. Using atomic-scale metallic junctions [4], it was recently showed that under a temperature rather than a voltage bias, a new non-equilibrium noise signal could be measured, which the authors dubbed as delta-T noise. This previously undocumented source of noise is actually a form of temperature-activated shot noise.

Here, we propose to investigate the fate of delta-T noise in a prototypical strongly correlated state, namely the edge states of the fractional quantum Hall (FQH) effect [5] between different FQH states instead of equal ones as done in ref. [6]. We consider an inhomogeneous junction involving two coupled edge states belonging to Hall fluids with different filling factors (ν_L, ν_R). In the specific case of a hybrid junction (1/3, 1), the problem is exactly solvable for all couplings and for any set of temperatures, showing that contributions linear in the temperature gradient dominate [7]. This motivated us to derive a universal analytical expression connecting the delta-T noise to the equilibrium one up to the lowest order in the temperature mismatch, for any junction involving two

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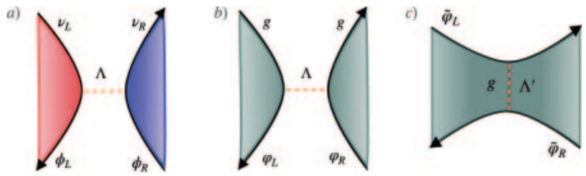

Fig. 1. – (a) Schematic view of the inhomogeneous FQH system in a QPC geometry. (b) Mapping of the original junction to one between FQH with the same effective filling factor g. (c) Through duality relation, we map the problem of two FQH liquids in the weak coupling regime to the strong coupling one described by a single FQH liquid. Figure adapted from ref. [7].

fluids belonging to the Laughlin sequence [8]. Remarkably enough, we can take into account all orders in the tunneling amplitudes. Starting from the weak coupling regime, where the two edges are almost decoupled, we turn our attention to the more complex and interesting strong coupling regime, which corresponds to a non-trivial situation, where the perfect transmission of current is reduced by scattering of fractional quasiparticles.

2. – Modeling the problem

We consider two FQH bars at different filling factors ν_α $(\alpha = L, R)$ belonging to the Laughlin sequence, *i.e.*, $\nu_\alpha = 1/(2n+1)$ $(n \in \mathbb{N})$ [5, 8]. They are kept at two different temperatures $T_L = T_R + \Delta T$ and $T_R = T$, where parametrization has been chosen in view of experimental implementations, and coupled through a point-like tunneling region as depicted in fig. 1(a). The edge states of such a system are described in terms of a hydrodynamical model [9] by a chiral Luttinger liquid free Hamiltonian of the form $(\hbar = k_B = 1)$

$$H^{(0)} = H_L^{(0)} + H_R^{(0)} = \sum_{\alpha=L,R} \frac{v_\alpha}{4\pi} \int \mathrm{d}x\, [\partial_x \phi_\alpha(x)]^2, \tag{1}$$

where ϕ_α are the bosonic fields describing the counterpropagating modes traveling along the edge of the left and right QH bars. They satisfy the usual commutation relation $[\phi_\alpha(x), \phi_\beta(y)] = i\pi\delta_{\alpha\beta}\,\mathrm{sgn}(x-y)$, with $\alpha, \beta = L, R$ [9]. We assume that the two QH systems are coupled via a QPC, placed in $x = 0$, which allows local tunneling between the two counter-propagating edges. The tunneling Hamiltonian is

$$H_\Lambda = \frac{\Lambda}{2\pi a} e^{i\frac{1}{\sqrt{\nu_R}}\phi_R(0)} e^{-i\frac{1}{\sqrt{\nu_L}}\phi_L(0)} + \mathrm{H.c.} \tag{2}$$

By considering a suitable rotation in the field space [10]

$$\begin{pmatrix} \varphi_L(x) \\ \varphi_R(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_L(x) \\ \phi_R(x) \end{pmatrix}, \tag{3}$$

with angle satisfying $\sin 2\theta = (\nu_R - \nu_L)/(\nu_R + \nu_L)$, we are able to map the problem of electron tunneling between two different FQH edges to the problem of electron tunneling between two identical chiral Luttinger liquids (see fig. 1(b)) with the same effective filling factor $g^{-1} = \frac{1}{2}(\frac{1}{\nu_L} + \frac{1}{\nu_R})$. The total Hamiltonian then becomes

$$H = \sum_{\alpha=L,R} \frac{v}{4\pi} \int \mathrm{d}x \big[\partial_x \varphi_\alpha(x)\big]^2 + \frac{\Lambda}{2\pi a} e^{i\frac{1}{\sqrt{g}}\big[\varphi_R(0)-\varphi_L(0)\big]} + \text{H.c.} \tag{4}$$

In this effective picture, as the tunneling amplitude increases ($\Lambda \to \infty$) we switch from the two identical, but separate, FQH liquids to a unique one (see fig. 1(c)). This process is embodied by a powerful electron-quasiparticle duality [11] which reflects the duality relation between the weak- and strong-coupling limits. The strong coupling limit is accessible through a weak-strong duality transformation [12]

$$\begin{aligned} \varphi_L(x) &= \widetilde{\varphi}_L(x)\Theta(-x) + \widetilde{\varphi}_R(x)\Theta(x), \\ \varphi_R(x) &= \widetilde{\varphi}_L(x)\Theta(x) + \widetilde{\varphi}_R(x)\Theta(-x). \end{aligned} \tag{5}$$

Then, the dual Hamiltonian of the one in eq. (4) is

$$\widetilde{H} = \sum_{\alpha=L,R} \frac{v}{4\pi} \int \mathrm{d}x\, [\partial_x \widetilde{\varphi}_\alpha(x)]^2 + \frac{\Lambda'}{2\pi a} e^{i\sqrt{g}\big[\widetilde{\varphi}_R(0)-\widetilde{\varphi}_L(0)\big]} + \text{H.c.}, \tag{6}$$

where we have considered substitution $g \to 1/g$ and the two tunneling strengths are connected by relation [13]

$$\left(\frac{\Lambda'}{\omega_c a}\right) = \left[2^{-2g+1}\,\Gamma^g\left(1+\frac{1}{g}\right)\Gamma(1+g)\right]\left(\frac{\Lambda}{\omega_c a}\right)^{-g}, \tag{7}$$

where $\omega_c = v/a$ is a high-energy cut-off and $\Gamma(x)$ is the Euler Gamma function of a given argument x.

3. – Universal expression for the delta-T noise

The current operator describing the tunneling current and its expectation value, respectively, read

$$\begin{aligned} I(t) &= ie\frac{\Lambda}{2\pi a} e^{i\frac{1}{\sqrt{\nu_R}}\phi_R(t)} e^{-i\frac{1}{\sqrt{\nu_L}}\phi_L(t)} + \text{H.c.}, \\ \mathcal{I} &= \frac{1}{Z}\,\text{Tr}\Big\{\exp\Big[-\textstyle\sum_{\alpha=L,R}\dfrac{H_\alpha^{(0)}}{T_\alpha}\Big] I(t)\Big\}, \end{aligned} \tag{8}$$

with $Z = \text{Tr}\big\{\exp\big[-\sum_{\alpha=L,R}\frac{H_\alpha^{(0)}}{T_\alpha}\big]\big\}$ being the partition function. Then, the zero-frequency current noise is written as

$$S\left(T_L, T_R\right) = 2\int_{-\infty}^{+\infty} \mathrm{d}\tau \left[\frac{1}{Z}\,\text{Tr}\Big\{\exp\Big[-\sum_{\alpha=L,R}\frac{H_\alpha^{(0)}}{T_\alpha}\Big]\Delta I(\tau)\Delta I(0)\Big\}\right], \tag{9}$$

where $\Delta I(t) = I(t) - \mathcal{I}$.

From this expression, we can derive a universal formula for the first order expansion in temperature gradient ΔT of the noise that applies to all orders in the tunneling amplitude Λ and for any set of filling factors (ν_L, ν_R). For the whole calculation we refer to ref. [7]. Here, we report only the final results, which read

$$S(T_L, T_R) = S_0(T) + \Sigma(\nu_L, \nu_R, T)\,\Delta T + O(\Delta T^2) \tag{10}$$

with

$$\Sigma(\nu_L, \nu_R, T) = -\left(\frac{\nu_R}{\nu_R + \nu_L}\right)\frac{1}{T^2}\frac{\partial S_0}{\partial \beta}, \tag{11}$$

where $\beta = 1/T$.

We underline the relevance of this result, which enables calculating the first-order correction to the noise in the temperature gradient only by knowing the expression for equilibrium noise $S_0(T)$. In particular, our derivation does not require any assumption concerning the strength of the tunneling between the two QH bars. This allows us to obtain the out-of-equilibrium delta-T noise in various tunneling regimes, provided that one is able to compute the corresponding thermal noise at equilibrium. Since eqs. (10) and (11) are valid for all values of Λ, it is worth noticing that they can be exploited for describing both the weak-coupling regime and the dual strong-coupling model through the duality procedure presented in sect. **2**.

In this work, we have reported on a universal expression, in terms of the tunneling parameter for a completely generic junction, for the linear correction to the full delta-T noise in the temperature gradient starting from the knowledge of the equilibrium noise. Moreover, since the delta-T noise depends on both filling factors separately rather than the sole factor describing junction, it could give us access to a more detailed analysis of strongly correlated systems.

Then, as shown in ref. [7] it is possible to cross from the weak-coupling regime to the strong-coupling one by applying a duality transformation.

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