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On the transverse coherence of X-ray radiation from micron-sized electron beams

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Summary. — In this work, we investigate the transverse coherence properties of X-ray radiation from micron-sized electron beams in undulator sources. We evidence deviations from the well-known Van Cittert and Zernike theorem when the beam emittance becomes comparable to, or smaller than, the radiation wavelength. In these conditions, we show that the coherence properties of the emitted X-rays strongly depend on the absolute position across the observation plane and exhibit unexpected oscillations caused by the peculiar features of the single-electron radiation pattern. Relevance to current facilities and future fourth-generation light sources approaching the diffraction limit is also highlighted.

1. – Introduction

Light sources with finite physical size emit radiation endowed with limited transverse coherence. For ordinary light sources composed of many independent point-like emitters like atoms generating randomly-phased spherical waves (the so-called incoherent thermal sources), this is expressed by the well-known Van Cittert and Zernike theorem (VCZ) [1]. It directly relates the transverse coherence properties of the emitted light to the Fourier transform of the intensity distribution of the source.

Nowadays, the VCZ theorem lays the foundations of interferometric electron beam size measurements at large-scale accelerator facilities [2-4]. It also underlies coherence studies of the emitted wavefronts in synchrotron light sources [5-7], a fundamental aspect for the full exploitation of coherent methods in the X-ray sciences [8, 9].

In these contexts, undulator sources pose a fundamental challenge [10]. As shown in fig. 1, the far-zone radiation from a single electron cannot be described by a simple spherical wave due to a peculiar sinc modulation resulting from the resonant character of

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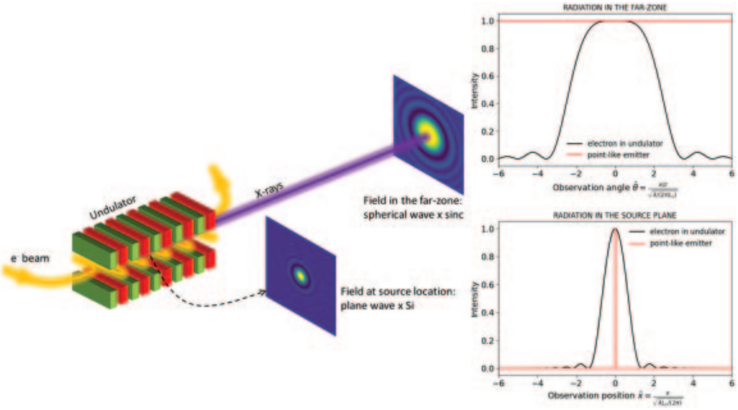

Fig. 1. – Single-electron radiation pattern in undulators (black) and comparison with point-like emitters in incoherent thermal sources (red).

the device. In addition, the far-zone field is generated by an equivalent (virtual) source with finite transverse extent, rather than by a point-like one, described by a plane wave modulated in amplitude by a Si (sine integral) function [10]. Therefore, for what concerns radiation emission processes, electron beams in undulators cannot in general be described as incoherent thermal sources, and deviations from the VCZ theorem might arise.

In this work, we investigate the transverse coherence properties of X-rays from undulator sources for different electron beam configurations, from current state-of-the-art facilities to future fourth-generation light sources approaching the diffraction limit.

2. – Materials and methods

We perform detailed numerical simulations with *FOCUS*, a GPU-based code to compute the transverse coherence properties of X-rays from undulator sources as a function of the electron beam parameters [11]. Transverse coherence is described by the so-called Spectral Degree of Coherence (SDC) [1]:

$$\mu(\mathbf{x}_1, \mathbf{x}_2, z) = \frac{\langle E(\mathbf{x}_1, z) E^*(\mathbf{x}_2, z)\rangle}{\sqrt{\langle |E(\mathbf{x}_1, z)|^2\rangle \langle |E(\mathbf{x}_2, z)|^2\rangle}}, \tag{1}$$

where $\mathbf{x} = (x, y)$ denotes transverse coordinates on an observation plane at a distance z from the undulator center, $E(\mathbf{x}, z)$ is the electric field at the observation plane and brackets denote ensemble averages.

We perform simulations for three different undulator sources with progressively decreasing beam sizes $\sigma_{x,y}$ and divergences $\sigma'_{x,y}$ along the horizontal and vertical plane, respectively. More specifically, we encompass the cases of typical third-generation synchrotrons [4], state-of-the-art facilities [12] and future light sources close to the diffraction limit [13]. For a direct and proper comparison, we also describe the undulator sources in terms of the following reduced parameters [10]:

$$\hat{N}_{x,y} = \frac{2\pi\sigma^2_{x,y}}{\lambda L_u}, \qquad \hat{D}_{x,y} = \frac{2\pi\sigma'^2_{x,y}}{\lambda / L_u}, \tag{2}$$

Table I. – *Main parameters of the three undulator sources. Values in parenthesis represent reduced quantities according to eq. (2).*

	Source 1: ALBA	Source 2: ESRF-EBS	Source 3: PETRA IV
L_u [m]	2.0	2.5	5.0
λ [nm]	0.1	0.17	2.5
σ_x [μm]	130 (562)	29.7 (13.2)	4.47 (0.01)
σ'_x [μrad]	48 (303)	4.37 (1.8)	2.24 (0.06)
σ_y [μm]	6 (1.2)	5.29 (0.42)	4.47 (0.01)
σ'_y [μrad]	5 (3.3)	1.89 (0.33)	2.24 (0.06)

where L_u is the undulator length and λ is the radiation wavelength. The main parameters of the three sources are listed in table I. For all three cases, the SDC is computed at the reduced distance $\hat{z} = z/L_u = 15$.

3. – Results and discussion

In fig. 2 we report the horizontal (first row) and vertical (second row) SDC maps computed at $y_1 = y_2 = 0$ and $x_1 = x_2 = 0$, respectively. We also show the on-axis 2D coherence maps at $(\mathbf{x}_1 + \mathbf{x}_2)/2 = 0$ (third row). Notice that this represents the expected outcome of a Young's double-pinhole experiment.

According to the VCZ theorem, the SDC should be described by Gaussian functions depending only on the relative position $\Delta\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$ between the two observation points. Results show that this is indeed the case for relatively large electron beam sizes ($\hat{N}_{x,y} \gg 1$), as for example for the ALBA and ESRF-EBS sources along the horizontal plane. Opposite to this case, however, deviations from the VCZ theorem arise for small beam sizes ($\hat{N}_{x,y} \leq 1$). For the ALBA source along the vertical direction, a mild dependence of the SDC on the absolute position of the observation point can be noticed at large y_1 and y_2. These variations of the coherence properties across the observation plane become even more pronounced for smaller and smaller $\hat{N}_{x,y}$, and ultimately result in the squared shape of the PETRA IV SDC. Furthermore, in these cases, the SDC also exhibits a markedly non-Gaussian behavior with unexpected oscillations. They can be ascribed to the peculiar sinc modulation of the single-electron radiation pattern, since the changes in sign of the radiation pattern induce anti-correlations in the electric field which manifest themselves as deep oscillations in the SDC maps [11]. Therefore, results suggest that coherence properties of undulator sources are also affected by the single-electron emission processes in the presence of a finite, albeit small beam emittance (parametrically, for $(\hat{N}_{x,y}\hat{D}_{x,y})^{1/2} \leq 1$).

4. – Conclusions

We have performed a thorough study of the transverse coherence properties of X-ray radiation from micron-sized electron beams in undulator sources. Results evidence deviations from the well-known Van Cittert and Zernike theorem, thereby proving that undulator sources cannot in general be described by the fully-incoherent thermal model. In particular, in the presence of a finite, albeit small beam emittance, coherence properties strongly depend on the absolute position across the observation plane and exhibit

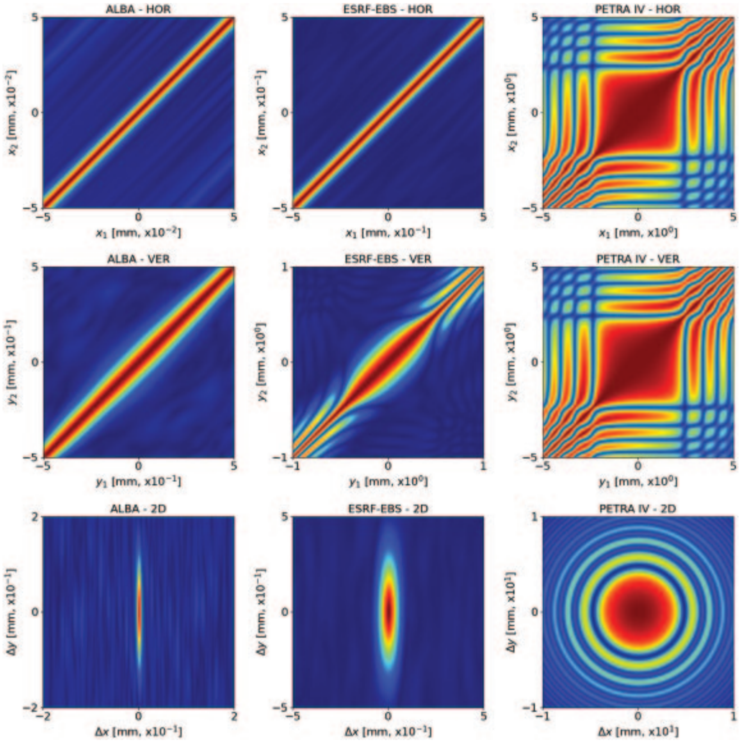

Fig. 2. – 2D maps of the SDC for the three undulator sources of table I. See text for details.

peculiar oscillations caused by the single-electron radiation pattern. Such features are expected to impact the actual quality of the X-ray wavefronts when the electron beam emittance becomes comparable to, or smaller than, the radiation wavelength, and will therefore represent a fundamental aspect for coherence diagnostics and coherent methods in future fourth-generation light sources approaching the diffraction limit.

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