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In search of the keys of deep understanding: Physics education research in Pavia

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Summary. — In this paper we present recent work of the physics education group at the University of Pavia, some of which in collaboration with other researchers. The unifying trait of research in Pavia, which the present paper aims to clarify, is its focus on conceptual understanding from a wide perspective, including the design of teaching-learning sequences, studies on teacher professional development, conceptual change research. We discuss three different examples of research directions, all concerning work performed in the past two years.

1. – Introduction

Research at the physics education group of the University of Pavia, which is over 40 years old, has been strongly characterized by a focus on conceptual understanding. This includes a long term commitment to prosecute the work done by the 'founding fathers' of the discipline, in recognizing student difficulties and alternate models in the different areas of physics; to devise effective educational strategies, experiments, and teaching-learning sequences, to overcome such difficulties and stimulate fruitful learning trajectories; and a continual interest for conceptual change research. Often, history of physics, an area in which research in Pavia has also been prominent, has served as a source of inspiration and guide. It is impossible not to mention the studies by Bonera and Giudice on Galileo [1, 2] in which, based on a refinement of Koyré's [3] historical analysis of the development of the law of free fall by Galileo, an educational approach to the concepts of instantaneous velocity and acceleration was developed. Or the works, started by Bevilacqua and coworkers [4, 5] and prosecuted in time [6, 7] on using the insights gained by Galileo and Huygens while studying pendulums as a guide for favouring a progressive, historically based foundation of the concepts of potential energy, conservation of mechanical energy, center of mass in educational paths. It may be useful to recall that at the end of the twentieth century, the dominant model of

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conceptual change was the one by Posner and coworkers [8], based on a parallel between the historical development of physics and the cognitive development of students.

Another important recurring theme, started by the works of Borghi, Mascheretti and coworkers [9, 10], was the use of mesoscopic models and toy models in education as a support for conceptual understanding of microscopic phenomena. This direction was pursued over the years with educational models for friction [9] fluid pressure [10] heat exchange [11, 12] phosphorescence [13].

1˙1. *Why focusing on conceptual understanding.* – Physics education nowadays has taken several different routes, and has expanded its horizons, in such a way that a focus on conceptual structuring of content for education may even seem outdated. The advent of lines of research based on 'warm' conceptual change [14] *i.e.*, taking in consideration the learners' *intention* to perform change and importance of affective, situational, motivational, meta-cognitive, epistemological aspects [15] of the learning process, seems at time to have left the problem of conceptual understanding of the science content in the background. While we recognize the importance and relevance of these research lines, and totally, agree, for example, with Pintrich's well known critique of the idea of a "learning ecosystem" [15], we remain strongly convinced that conceptual understanding should continue to occupy a central position in physics education research (PER). In fact, research on student difficulties and alternate mental models, an area which occupies a prominent position in the foundations of physics education like perhaps happens for no other science, has long since proven that while physics is a powerful and coherent interpretive language for the Natural world, such language can, and often will, be misunderstood and deformed, both globally and locally. It demonstrated that key conceptual nodes, links between sub-areas of the discipline, disambiguation of seemingly similar physical concepts and constructs, need to be carefully dealt with, lest leaving even some of the best students with the belief that physics is an inconsistent and fragmented discipline. Finally, research has shown that success in obtaining correct results in standard exercises and problems, which can certainly be achieved by a student with sufficient determination to learn, even with little aid in terms of research based teaching-learning strategies, is a very poor indicator of true conceptual understanding.

On the other hand, in the last 30 years the focus of political decisors responsible for educational policies has been steadily moving towards a greater and greater focus on the development of *competencies*, again putting in the background, albeit from a different perspective, the problem of conceptual understanding. The emphasis on competencies is sometimes accompanied by in principle statements about the development of critical thinking [16], but the link between the two stated objectives is left undeveloped, and appears as vague at best. In this respects, we cannot do better than quoting literally the words by Laurence Viennot [17]: "*Given this emphasis on competencies in combination with the desire to simplify physics, there is a risk that conceptual structuring may be disregarded, potentially resulting in serious inconsistencies in pedagogical resources. This in turn demands increased critical vigilance among students and teachers, inviting the research question What links can be identified between the development of conceptual understanding and critical attitude in physics students —or, in operational terms: Can we help students to develop their critical thinking without a conceptual basis?*"

1˙2. *Broad theoretical framework.* – Research on conceptual understanding is a vast area, and a variety of different methods and approaches are used. It may be worthwhile to reaffirm here the centrality of investigations on student difficulties, which are the most important source of insight in this line of research. Of course, just compiling lists of individual difficulties on a given topic may not be of great help: often they have to be interpreted, and combined with one another, in order to identify alternate mental models, either global or local, either coherent or context-dependent. In some, fortunate cases, a single item may be extremely illuminating, and shed new light on the whole problem of student understanding of a given topic. We make two examples, which are not related to the prime years of the discipline, when such discoveries were abundant, and are both, for different reasons, very important for the research tradition in Pavia. The "*two fishes*" item, coming from the research of Ugo Besson on the understanding of pressure in fluids [18], provided entirely new understanding of the development of student mental models. The statement of the question is rather simple: there are two fishes swimming nearby in the sea at the same depth under the water surface; one is in the open sea, while the other swims within an underwater cave, enclosed in an elevation of the sea bed, the entrance of which is also at the same depth. Is the pressure exerted on the fishes equal, or greater for one of them? It was found that young students, up to middle school, tend to believe that pressure is greater for the fish in the cave, while for older students, the prevailing difficulty is thinking that the pressure is lower for the fish in the cave. So this item pinpointed at the same time an ingenuous model, that is that pressure must be greater in enclosed spaces, and an educated difficulty, presumably coming from mis-application of the formula $P = \rho g h$ with h limited to the distance from the fish to the cave ceiling. A failure of traditional teaching to provide useful conceptual understanding of the presented situation was uncovered.

The second example, perhaps even better known, is the item of the Rainier and Hood volcanoes, coming from the research work of Scherr and coworkers [19] on students' understanding of special relativity. The item, which is today one of the most important diagnostic tools in the area, allows to pinpoint a number of difficulties in understanding simultaneity in different reference frames. But most importantly, it allows to clearly identify the most common, even in university students, *global* misunderstanding of special relativity, appearing especially after initial instruction, *i.e.*, that the theory, and in particular the relativity of simultaneity, concerns the different times of arrival of signals to different individual observers. Discussion of this item in view of the Taylor-Wheeler [20] geometric approach to special relativity has been a cornerstone of teacher professional development in Pavia for about 20 years now.

In many cases, however, the transition between the enumeration of individual difficulties, and the identification of mental models, and strategies to overcome conceptual nodes, is rather complex, although several important contributions have been brought by research over the years. The main pathway is indicated by *models of conceptual change*, among which we have cited already the classical one by Posner et al. [8]. In Pavia, due also to a certain traditional preference for understatement in PER, and taking seriously Cobb *et al*'s famous characterization of *humble theories* [21], we have generally avoided committing exclusively to a particular model, but according to the context and topic at hand, we have tentatively adopted one or the other framework to interpret de learning process. In recent times, the choice has fallen more often on DiSessa's "knowledge in pieces" approach based on the theory of coordination classes [22], or on Vosniadou's "coherence" proposal founded on the idea of framework theories [23, 24]. In research on conceptual change towards understanding of quantum mechanics, as we will see in sect. **4**, the insights brought by Chi [25] about ontological and categorical shifts in conceptual change have been especially useful.

A different type of contribution altogether, is what we may call the study of the *modes of reproduction* of student difficulties in the teaching-learning process. This kind of analysis is centered not on the student and his/her initial knowledge in relationship to the new one, but on the practices and discourse of teachers, and the structure of learning materials. Great importance is attached to the attitude to critical thinking of teachers, as it allows to a) critically evaluate *ritual* explanations provided in textbooks or learned through instruction, identifying contradictions, missing assumptions and details which might implicitly encourage students to rote learning rather than true conceptual understanding; and b) devise suitable educational strategies to stimulate students to develop their critical thinking skills, and take responsibility for the construction of their own knowledge. Epitomizing this kind of research is the invaluable work of Laurence Viennot, whose influence has been strong on the Pavia group, especially through the presence of Ugo Besson. In sect. **2** we will report on an example of research on teachers' reactions to the evidence of clear contradictions between authoritable learning materials, entirely inspired by the work and thought of Laurence Viennot.

The third major leg of research on conceptual understanding is represented by frameworks for designing effective teaching-learning sequences, didactic interventions, learning materials. Here, a privileged place, both in international research and for the PER tradition in Pavia, is represented by the Model of Educational Reconstruction (MER) [26]. The MER is based on the idea that three elements of educational research are strictly related; the first two, namely

i) analysis of science content (analysis from the perspective of theory),

ii) analysis of students' difficulties and assessment of the crucial features of students' learning processes in this topic (analysis from the perspective of learners),

contribute to the identification of the fundamental structure of the subject matter for instruction (*i.e.*, the issue of *elementarization*) of the possibly fruitful strategies to improve learning outcomes and overcome student difficulties. From such outcomes, the third component of the MER stems:

iii) construction and evaluation of learning environments and activities.

In the design of teaching learning sequences (TLS) according to the MER, the above three elements are combined iteratively [26], in the sense that the educational outcomes, classroom observation and teacher reflection typical of action research contribute as new inputs to a revision of the processes of structuring the material for instruction and practically designing the learning environment and activities.

Although there have been competing proposals in the literature, such as the idea of *didactical transposition* introduced by Chavallard [27] it is probably fair to say that a majority of the PER researchers engaged in educational design sees the MER as the main methodological support for combining empirical and theoretical education research in the design of teaching-learning interventions and educational materials. However, it has to be mentioned that a peculiar trait of research in Pavia has been to refrain from the production of excessively 'closed' and refined TLS's, drawing a line of separation with some Design-Based Research type approaches [28]. Although of course the role of the cycles of testing and revision is acknowledged, the educational proposals and materials designed by the Pavia group have typically been at least partially *open-source* [29]: they have consisted of a *core* of the proposal, made of contents, conceptual correlations and methodological choices considered as indispensable, and a *cloud* of elements that could be re-designed by teachers, according to their own preferences, interests, and needs. In our view, this is very important in order to preserve the proactive role of the teacher, his/her disposition to action research, reflective practices and critical thinking, goals which we believe are globally more important for the real teaching learning process that refinement of a closed-structure TLS to the last minute details.

1˙3. *The choice of research presented in this paper*. – In sect. **1˙2** we have identified three major sub-directions in our research on conceptual understanding: a) *models of conceptual change* to interpret successes and failures in the learning process; b) *teaching rituals* in physics, and the connection between critical thinking and conceptual understanding; and c) *educational design* based on the MER. To each of these directions (in a different order) is primarily related one of the works presented in the rest of this paper, precisely sect. **2** to direction b), sect. **3** to direction c), and sect. **4** to direction a). While sects. **2** and **3** concern works which are so far unpublished, in sect. **4** the reader will find a summarized account of the research in ref. [30], to which we direct him for further information, if interested.

2. – Teaching rituals and critical details in thermodynamics: teachers confront contradictory definitions of efficiency of a thermal engine

This work started from the consideration of a persisting ambiguity in the definition of efficiency of a thermal engine, which had been discussed in the Italian educational literature since the 1990s [31, 32]. Such ambiguity is interesting because it leads to contradictory solutions of similar problems in different textbooks, whereas from the theoretical side, the manuals might seem almost identical, or with negligible differences. In fact, when computing the efficiency for a reversible cycle made of two isotherms and two polytropic, other than adiabatic, transformations, such as the Ericsson or Stirling cycles, some textbooks include the heat absorbed by the system during the polytropic transformation in the computation of efficiency, resulting in a lower efficiency than the Carnot cycle [35] while some don't, resulting in equal efficiency [36]. The basic issue is that reversible cycles operating between two temperatures were historically considered to have the same efficiency as a Carnot cycle, under the additional hypothesis of a perfect regenerator, a device which absorbs heat from the engine in one branch of the cycle, and restitutes it to the engine, ideally with no losses, in the return branch. However, the discussion of a regenerator has disappeared from secondary school and university textbooks, but most of them continue to formulate Carnot's theorem as if a regenerator was implicitly assumed, *i.e.*, stating that all reversible cycles "working between two temperatures" have the same efficiency as the Carnot cycle. Based on these premises, we performed an analysis of secondary school and university textbooks on this topic, and investigated the intellectual dynamics of in-service and perspective teachers presented with excerpts from textbooks, highlighting such contradiction. The study is inspired by similar research works by Viennot (*e.g.*, refs. [37,38,42]) centered on the observation and categorization of the reactions of teachers, especially in terms of activation of critical thinking, when asked to analyze, and reflect upon, excerpts from textbooks containing contradictions, missing assumptions or other defects. We will discuss our results for this interview study based on the theoretical framework and methods devised by Viennot herself.

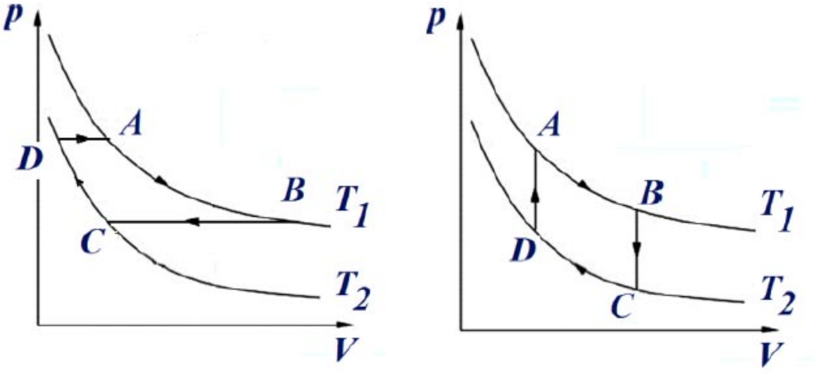


Fig. 1. – The Ericsson cycle (on the left) and the Stirling cycle (on the right) are the most common examples of reversible thermodynamic cycles appearing in textbooks, apart from the Carnot cycle. In some texts, the efficiency of all reversible cycles is computed to be equal [43]. In others, the efficiency of different reversible cycles is computed to be different. For instance, In ref. [35] the efficiency of the Ericsson cycle results to be lower thant the one of the Carnot cycle.

2˙1. *Theoretical framework*. – The above formulation of the problem led us to see almost immediately an instance of *disappearance of critical details* from the teaching-learning practice and materials. In time, the regenerator has been probably considered a device too difficult conceptually to be introduced at the level of secondary school, and its description, or even mention, has been progressively abandoned. However, this would have required either modifications to the formulation of the Carnot theorem and the solution of exercises and problems concerning this issue, or to the definition of absorbed heat [32]. In most cases, neither of these changes have been performed, and the textbooks appear to attempt hiding contradictions under ambiguities of language. The research work of Viennot provided us with the theoretical instruments to perform both the analysis of the contradictions and other shortcomings in textbooks [44] and the intellectual dynamics of teachers when facing such contradictions [37, 38].

2˙2. *Research questions*. – In this work we considered the following two research questions:

- RQ1: Which issues and shortcomings can be observed in the discussion of thermal engines and their efficiency in textbooks?
- RQ2: What are the reactions and reflections of teachers in front of two contradictory definitions of efficiency?

2˙3. *Methods*. – The study uses typical methods of qualitative research [33], and as such it does not attempt to provide an answer to the research questions which is valid in a statistical sense, but only at the level of case study. Concerning RQ1, we analyzed 10 current secondary school physics textbooks, and 3 university level ones. Notwithstanding the above statement of limitation on the significance of our results in a statistical sense, the list still includes the five most widely used physics textbooks in Italian secondary school, according to open data provided by the Italian ministry of Education [34]. For RQ2, structured interviews were used, based on a protocol initially designed by two researchers and refined after a test interview. The sample was composed by 11 respondents in total, comprising 8 in-service teachers, generally with significant teaching experience, and 3 student teachers with limited experience.

2˙4. *Results*. – In the following we report on our main findings concerning the two previous research questions.

2˙4.1. Textbook analysis. The following elements summarize the results we gathered from the analysis of textbooks (RQ1):

1) No secondary school textbook discusses an (ideal or real) regenerator in any form. However, a majority of them when formulating Carnot's theorem state that all reversible cycles "operating between two temperatures" (a ritual formula which is most often left underspecified, and used interchangeably with "absorbing heat from only two sources") have the same efficiency as Carnot's cycle. Such statement, at this point, is *intrinsically incomprehensible* [39]. In particular, in the subtle linguistic shift between "operating between two temperatures" and "absorbing heat from only two sources" it is never explained whether whatever the system exchanges heat with in the transformations connecting the two isothermal ones should be considered as a "source" or not.

2) Similarly, the concept of "absorbed heat" is not sufficiently specified, and sometimes the ambiguity appears intentional, and a consequence of the omission of the regenerator hypothesis. Often textbooks define "absorbed heat" in general, but then continue only considering two possible heat exchanges, with the hot and cold sources, as if no heat exchange would be possible in the transformation connecting the two isotherms. Sometimes [40] the absorbed heat is explicitly defined as "heat transferred from the hot source", but then the definition is not general, and applicable only to a restricted class of thermodynamic cycles. No textbook adopts the definition suggested in ref. [32], *i.e.*, defining absorbed heat as the *net* heat exchanged with a source at a given temperature. Such definition, however, would be problematic for other reasons which have been discussed in ref. [41].

3) In general, very few textbooks contain exercises in which reversible cycles different from the Carnot one are considered, which would allow to critically evaluate the content of their formulation of Carnot's theorem. Other sources do, and typically find the efficiency to be the same as the one of the Carnot cycle, by canceling heat exchanges from polytropics connecting two isothermal transformations, with no justification. These textbooks, in the language of ref. [38], depending on their definition of absorbed heat suffer either from *internal contradictions* (absorbed heat is computed in a way which is incompatible with its definition) or *logical incompleteness* (the definition of absorbed heat does not cover all the cases considered in problems).

4) Only one [45] of three University level textbooks we took in consideration provides a consistent treatment, by stating that the efficiency of any thermal engine, reversible or not, is lesser than or equal to the one of the Carnot engine, and consistently following such formulation in exercises.

The analysis of the content of textbooks, carried out under the light of educational research, confirms the importance for teachers to always activate critical thinking in in front of proposals and formulations of textbooks. In fact, it is essential that teachers identify unresolved or poorly specified conceptual issues in the textbook, and take charge of the problem of how to clarify them, in order to promote deep and satisfactory learning of an undoubtedly difficult discipline like thermodynamics.

2˙4.2. Results from the interview study. The following elements summarize our results from the teacher interviews (RQ2). We will use the terminology introduced in refs. [37, 38, 42] to categorize the intellectual dynamics of teachers when confronted with an ambiguity in textbooks.

1) Out of eleven teachers interviewed, seven, when confronted with two solved exercises from different textbooks, reaching opposite conclusions about the efficiency of a reversible thermal engine different from the Carnot one (Ericsson and Stirling cycles), followed the problems to the end finding no contradiction. Two of them refused to admit a contradiction even after it was explained to them (*absence of critique*). One of the two was in such discomfort that decided to end the interview early, stating that he was not prepared for the type of questions that were asked. Three teachers conceded that there was a contradiction after being stimulated to reflection (*retarded critique*).

2) In the latter group, teachers finally seeing the contradiction either declared incompetence to judge it (again, *delayed critique*), or justified the textbooks by stating that the topic is too difficult for students to hope achieving a consistent treatment, and that “one has to make compromises” (*anesthesia by substitution*) thus, in fact, accepting that students should be given sometimes inconsistent treatments.

3) Three teachers, out of the eleven interviewed, found that “something was wrong” after reading the texts, but seemed reluctant to openly express a critique to either text, and often changed opinion (*unstable critique*).

4) Only one teacher immediately recognized the contradiction (*prompt critique*) and was able, after some reflection, to frame it in the context of an ambiguity in the definition of the efficiency of thermal engines. It can be helpful to remark that this teacher profile was quite peculiar, being president of AIF (Association for Physics Teaching) in his county, and frequent organizer of, and participant in, physics teacher courses for professional development.

Teachers were also asked to make explicit their views about the teaching of critical thinking skills in the classroom, and its connection to conceptual understanding. In general, the interviewees struggle to define research-based practices in physics education that favor the development of critical thinking in the classroom (*e.g.*, inquiry based and problem based learning, assignment of non standard problems and tasks). Several teachers identify a basic issue in the relationship that often arises in a learning/teaching process between student and teacher, a relationship that can be summarized in two sentences from one interviewee “*the main difficulty with critical thinking is that the narrative about school is of a context where you go for the purpose of learning, and this definition entitles those involved in this process (both students and teachers) to have a passive attitude*”, and, “*school makes you forget that you* [teacher] *are also learning, not only transferring knowledge.*” Some interviewees complain that they have not had sufficient instruction in strategies to promote critical thinking and that they themselves are not trained to put into practice what they should then ask of the students (*e.g.*, to look at textbooks and problems with a critical attitude, rather than accepting them without scrutiny as automatically authoritative sources). In general, we collected ample consensus that these topics should be dealt with also at the level of initial teacher formation.

3. – Disentangling the core of quantum computation for instruction

We now turn to provide an example of research on educational design, switching to the field of quantum physics education, and more precisely to the emerging area of education to quantum technologies (QT). The advent of the second quantum revolution [46] is changing our perspective in looking at physical systems, and QT are one of the most relevant outputs of such an epochal change. However, awareness of the importance and potential of such revolution is still low beyond the restricted circle of professional physicists. The "*creation of the learning ecosystem necessary to inform and educate society about QT*", guided by the vision that "*a quantum-ready society, with knowledge about and positive attitudes towards QT, will enable the emergence of a quantum-ready workforce*" are main objectives of the QTEdu Coordination and Support Action [47] in which the Pavia group is actively involved. In the pursuit of these objectives, secondary school plays a pivotal role, as the high school curriculum is the common background for many key roles of the future, such as scientists (physicists, mathematicians, information scientists, chemists, biologists, physicians, engineers...); technicians (programmers, developers, maintainers...); political decisors and industry leaders (political scientists, lawyers, economists...). Of course, in order to reach any significant result, intervention in the secondary school cannot be limited to the design of episodic events and seminaries, but must include a massive action for teacher professional development, in the perspective of a synergic drive towards curriculum change, from above and from below (from political decisors to teachers, and vice versa). Furthermore, learning about QT, and gaining conceptual understanding of its basic principles, can help secondary school students imagine the future, and take action to play an active role in shaping it [48]. Finally, teaching QT at school may have a bearing on the emerging problem of quantum misinformation. In fact, recent research shows [49] that vague or inaccurate statements about quantum mechanics (QM) and QT which are reported in media, or sometimes implicitly passed even in formal instruction by means of improper formulations and analogies [50], may lead students and the general public to view the above topics as more akin to magic than science, as astray deviation from the rational scientific thought, and ultimately as the supposed base for pseudo-scientific claims, alternative medicines, syncretic philosophical doctrines, and conspiracy theories [51]. Our hypothesis is that adopting the context of QT may help students perceive quantum mechanics as an integral part of the modern scientific tradition, whose successes lead directly to crucial technological advancements for humanity, rather than as a backdoor from true science to a number of pseudo-scientific ideas. However, in order to reach these objectives, conceptual understanding plays a crucial role. In fact, research has convincingly shown [52] that no profound personal re-elaboration of science content, in terms of developing a drive towards activism, or promoting a positive change in epistemological beliefs, can happen without deep conceptual understanding.

3·1. *Research in education to quantum technologies as an emerging field.* – While research on the teaching and the learning of quantum physics is a well-developed field within physics education, the teaching-learning of QT and represents still a largely uncharted territory, which however is very recently receiving increasing attention. For example, Pospiech [53] proposes a course on QM for the German high school, in which quantum computing and quantum cryptography are introduced as rich technological contexts in which the fundamental concepts of quantum theory (*e.g.*, superposition, entanglement, incompatibility, measurement) find their full development and application. Walsh and coauthors [54] designed and tested a one year high school course on quantum computing based on classical wave optics, with a focus on hands-on experiments and simulation activities adopting an inquiry-based approach. Satanassi *et al.* [55] developed a quantum computing course for high school students based on the general idea of leading students to follow the evolution of computational thinking in human history, from the most primitive computing machines, and ending with quantum computers and algorithms. Indeed, as we will see in the following, the underlying philosophy of our educational design closely aligns with the one of the Bologna group. Angara and coauthors [56] proposed a coding approach, revolving around the introduction to basic quantum computing concepts using the Qiskit [57] and Jupyter [58] notebooks. In the context of the QTEdu coordination and support action (2020-22, [47]) individual pilot projects, some of which included members of the Pavia group, laid important foundations for current and future research on the teaching-learning of QT [59] such as preliminary work for the development of a Quantum Concept Inventory for secondary school [60] and the construction of a map of Pedagogical Content Knowledge [62] about QT for high school teachers. Besides research on the introduction of QT in secondary school, other recent works target for example undergraduates with little or no physics background [63]; and a comprehensive review of educational efforts aimed at expanding and improving education on QT has been published in 2022 [64].

3˙2. *Theoretical framework.* – Motivated by the considerations stated in the introduction to this section, the Physics Eduction group and the Quantum Science and Technology group at the Department of Physics of the University of Pavia have undertaken a joint effort teacher professional development on QT in 2019 [65] based on a strongly interdisciplinary approach whose unifying theme was the gradual convergence and progressive overlap, in science, between the discourse on physics and the discourse on computation.

To design a TLS according to the MER [26] as described in generality in sect. **1**˙2 we initially performed a content analysis focusing on the theoretical perspective, the students perspective and teachers perspective.

From the theoretical side we considered two aspects as most significant for our educational reconstruction: the history of physical information theory starting with the work of C. Bennett [66] and the diagrammatic approach linked to category theory [67]. The history of quantum computation and information provided us with the first conceptually relevant element: the extension of the semantic field of the word computation from the area of logic-mathematics to that of physics. More precisely, what is brought to light is the need to consistently problematize, when talking about computation, whether we are referring to hardware or software, to physics or logic. Therefore, we need a language capable of exploring such dichotomy as deeply as possible, and at the same time allowing to express both classical or quantum computation. The second aspect, the diagrammatic language, serves precisely these purposes. The use of category theory and its possible diagrammatic circuit representations is deeply embedded in some of the more recent axiomatic formulations of the quantum theory, and we find it adopted e in several more application-oriented works, such as in computer sciences, and the physics of computation [68]. The literature in this field, much of which is currently of great interest for artificial intelligence and machine learning, points at the possibility of providing categorical descriptions of any kind of processes, be they physical, chemical, linguistic (texts), musical (compositions), or otherwise. The unifying effort of these works translates in our research into the use of a language able to create a unified model for logic, the physical theory of computation, and the corresponding experimental realizations in the quantum case using optical devices. Therefore, we have introduced useful categorical tools, the diagrams, appropriate for defining a unifying language for computational theory, physical theory and implementation using optical devices.

As for the students' and teachers' perspective, in addition to the recent literature on teaching quantum technologies discussed in sect. **3**˙1, we used data from previous explorative tests carried out by our research group, from online courses realised in collaboration with a group of Italian universities [69] and the previously mentioned course for teacher professional development realized in 2019–20 [65]. Finally, the model describing the educational transition from classical to quantum mechanics, accounted for in sect. **4**, although not specifically centered on QT, was always kept in consideration for interpreting difficulties and conceptual nodes within a broad theoretical framework.

There are some general features that seem to characterize the analyzed data on previous educational experiments: on average, the educational path about quantum technologies was useful to familiarize students with fundamental aspects of quantum mechanics, and student understanding of the basic quantum mechanics formalism benefits from the use of multiple representations [70] (formal, graphical, diagrammatic); in particular the diagrammatic language is appreciated as a tool for conveying different meanings. However, a strong need also emerged to make the concepts introduced more concrete and physically grounded. Therefore, starting from the embryonic idea of representing graphically the action of quantum gates in the state space of polarization states, which worked very well in the trial here presented (subsect. **3**˙6.1) we then proceeded to the development of an educational strategy in which all logic gates, circuits, algorithms and protocols introduced are constantly juxtaposed and compared to their possible experimental realization with optical elements. This idea has evolved significantly from the experimentation presented here, as will be discussed in sect. **3**˙6.

A basic issue, in the perspective of the Model of Educational Reconstruction concerns *elementarization*, *i.e.*, identifying the entities within a complex content domain which may be viewed as elementary features, the composition of which explains the relevant scientific content. Steps in this direction were taken by the Bologna group [55], with the parallel between quantum computation and the more general description of a quantum system, consisting of the preparation of the qubits (input information), the manipulation of their state (processing), and measurement (output information). However, information processing represents the engine of the algorithm and, as such, it is the central and most variable element of such structure. An elementarization of its internal structure was still missing. Without a clear picture of the elements that bestow quantum information processing its peculiar form and operational advantages, and of the composition of these elements in order to perform a specific task, students risk to see quantum computing as a set of magic formulas producing wondrous effects, that need to be memorized without engaging in their design. Based on an analysis of educationally significant algorithms (*e.g.*, Deutsch's and Grover's algorithms), we suggest to decompose the structure of the information processing phase into three sequential processes: (1) the initial enabling of parallelism by means of Hadamard gates on the previously prepared registers involved in the computation (2) the transfer of information encoded in the oracle function to the register(s), exploiting the multiplicative structure of compound quantum systems; (3) the enabling of interference by means of a network of logic gates to produce the desired state on which measurement can be performed. A primary goal of the first experimentation with secondary students hereby presented was to determine whether such structure would prove efficient in providing significant scaffolding to support students' understanding of quantum algorithms.

For the initial introduction to quantum physics, comprising the first 8 hours of the TLS, we adopted the two state approach based on polarization, as developed by the University of Udine [71].

3˙3. *Structure of the TLS*. – the TLS is summarised in table I. Instruction proceeded through a variety of activities, including lectures based on slides, but also inquiry-based and modelling tasks described in two-three page worksheets. Worksheets are to be completed by students step by step in suitable short pauses of the lesson flow, and are designed to emphasize written explanations of student reasoning. Therefore, the worksheets that are used in our work have multiple uses. First and foremost, they are designed to get students to work independently to become personally active in constructing knowledge. Second, they allow instructors to understand the difficulties their students may be having, through a bird's eye inspection after each lesson. In particular, the micro-steps in which the worksheets are structured allow teachers to grasp specifically where the significant difficulties lie. Finally, thanks to the collection of the worksheets and their analysis after the experimentation, it was possible to monitor students' learning, identify possible changes to the worksheets themselves and modify some parts of the TLS.

TABLE I. – *Structure of the teaching-learning sequence.*

Content	Learning goals
Introduction to quantum physics	Introducing quantum physical quantity, state, vector, superposition, interference, measurement
Computational approach to problems: physics and logic in classical computation	Interpreting a problem and its solution from a logic-computational point of view. Linking logical to physical aspects (software to hardware).
From bit to qubit: one qubit computation	Introducing and developing quantum computation: Dirac's vector formalism and its geometric interpretation for new single-qubit computation.
Polarization model for computation.	Describing the transition from the known wave model of polarization via Jones vectors and use it to build the polarization qubit
Spatial model for computation	Building the single-photon qubit model based on the spatial mode in a Mach-Zehnder interferometer.
The Deutsch algorithm	Understanding the application of the quantum Deutsch algorithm to the case of determining whether a coin is genuine or counterfeit. Compare to the classical algorithm and understand the origins of quantum advantage.
Entanglement, Bell inequalities, quantum protocols	Understand the basic characteristics of entangled states and their application.

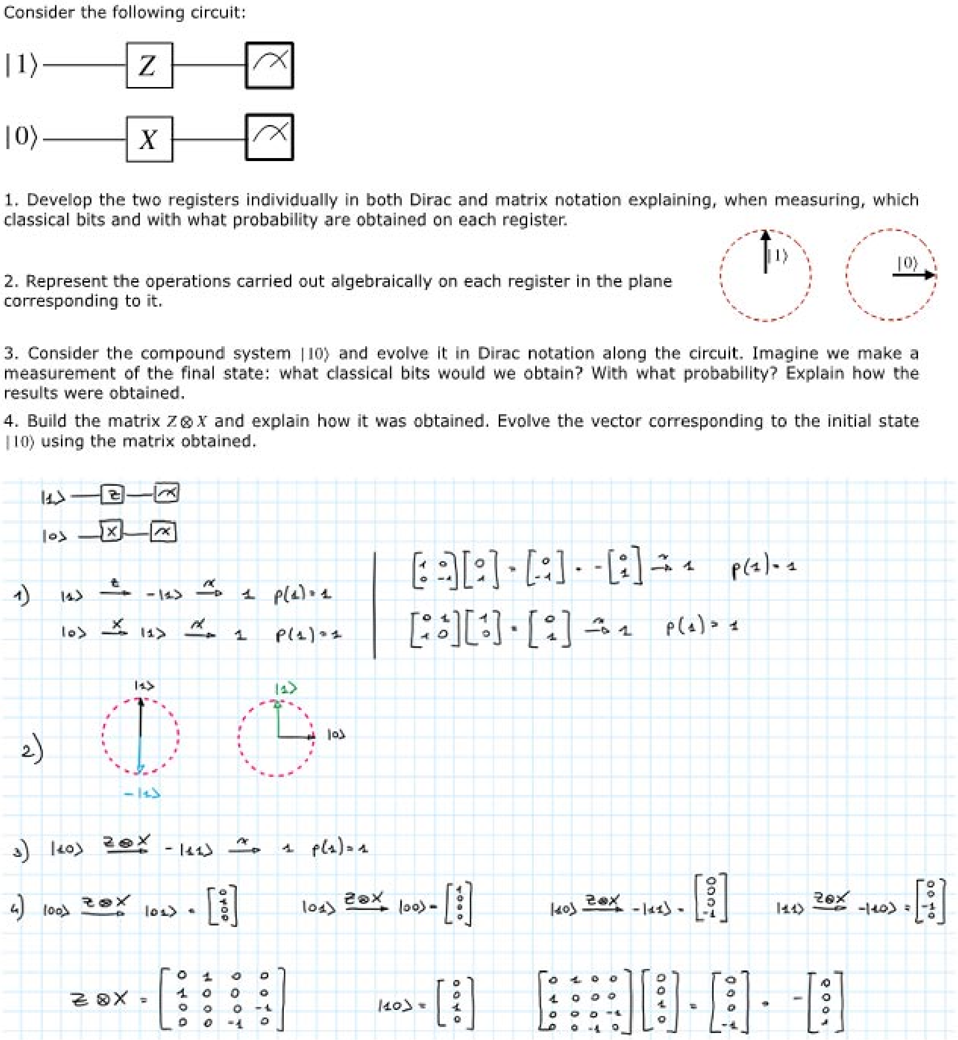
3˙4. *Context.* – The intervention was performed with 8 self-selected 18 and 19-year-old students from the *liceo classico* (2) and *liceo scientifico* (6), both belonging to the institute Galilei-Grattoni of Voghera, Italy. Both *liceo classico* and *liceo scientifico* in the Italian system are types of schools typically attended by students who intend to continue their studies in university; *liceo classico* is more oriented towards literacy and human sciences while *liceo scientifico* is more focused on STEM disciplines. The instructors of the course were two researchers in physics education. The experiment lasted from November 2020 to May 2021 for a total of about 25 hours in distance learning. Data were collected from both intermediate work sheets assigned to students and a final test.

3˙5. *Methods.* – Data collected in written answers to exercises and worksheet questions were analyzed for correctness and for student reasoning paths, since both aspects were instrumental to the revision of the activities. The second type of analysis was conducted according to qualitative research methods [33]. Since research data on secondary school student difficulties were not available, due to the novelty of the topic, a-priori categories were structured by theoretical identification of crucial aspects of the content. Then, based on conceptual elements introduced by student answers, the categories were revised. This process led to the identification of conceptual clusters in student reasoning.

3˙6. *Results.* – In general, the trial provided support for our design hypotheses, and encouraged us to proceed further on the directions undertaken. In the following, we discuss more thoroughly some aspects of the analysis of worksheets (subsect. **3**˙6.1) and final test (subsect. **3**˙6.2).

3˙6.1. Worksheet analysis: understanding of different representations of quantum logical circuits. Concerning the intermediate worksheets, we limit ourselves to the discussion of one of them, which consisted in a series of items probing the capabilities of students in a) translating circuital diagrams into Dirac and matrix notations, and using such notations to perform actual calculations; b) connecting the operations performed by logical gates to the graphical representation of quantum states; c) connecting the formal expression of a state to the probabilities of measurement outcomes. The text of the worksheet and an example answer of one of the students are reported in fig. 2. The work made for the construction of the qubit concept starting from polarization, and the subsequent interpretation of logic gates as axial symmetries in the state space seems to have been well understood by almost all students. Furthermore, the 6 students from liceo scientifico also solved, for the most part correctly, more demanding questions in terms of mathematical manipulations (mostly sub-item 4, asking to determine the action of an operator on basis vectors and compute the corresponding 4×4 matrix), while students of liceo classico displayed difficulties in this sense, presumably due to their more limited mathematical background.

3˙6.2. Analysis of the final test: productivity of the three-step decomposition of quantum information processing. In the final test we preferred to design questions that would allow us to evaluate whether the students had understood the conceptual aspects of the proposed topics, leaving the elements of pure algebraic calculation as optional. Here we limit ourselves to discussion of an item concerning the decomposition of quantum algorithms into three fundamental processes. To understand whether the proposed approach had supported the conceptual understanding of the Deutsch algorithm, we proposed the following open response item:



Consider the following circuit:

1. Develop the two registers individually in both Dirac and matrix notation explaining, when measuring, which classical bits and with what probability are obtained on each register.

2. Represent the operations carried out algebraically on each register in the plane corresponding to it.

3. Consider the compound system $|10\rangle$ and evolve it in Dirac notation along the circuit. Imagine we make a measurement of the final state: what classical bits would we obtain? With what probability? Explain how the results were obtained.

4. Build the matrix $Z \otimes X$ and explain how it was obtained. Evolve the vector corresponding to the initial state $|10\rangle$ using the matrix obtained.

1) $|1\rangle \xrightarrow{Z} -|1\rangle \xrightarrow{\nearrow} 1 \quad p(1)=1$

$|0\rangle \xrightarrow{X} |1\rangle \xrightarrow{\nearrow} 1 \quad p(1)=1$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\nearrow} 1 \quad p(1)=1$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\nearrow} 1 \quad p(1)=1$

2)

3) $|10\rangle \xrightarrow{Z\otimes X} -|11\rangle \xrightarrow{\nearrow} 1 \quad p(1)=1$

4) $|00\rangle \xrightarrow{Z\otimes X} |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad |01\rangle \xrightarrow{Z\otimes X} |00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad |10\rangle \xrightarrow{Z\otimes X} -|11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \qquad |11\rangle \xrightarrow{Z\otimes X} -|10\rangle = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

$Z \otimes X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Fig. 2. – The student correctly analyses (figure below) the circuit proposed (figure above) in both Dirac notation and using matrices and vectors. Each operation performed on the individual registers (separable states) is visualized as a relative geometric transformation in state space. The student performs a consistent transition to considering a compound system rather than two separate subsystems (apart from an inaccuracy in sub-item 3). The answer attests the student's ability to both formally manipulate algebraic tools, and connect them to other relevant forms of representation.

Use Deutsch's algorithm to introduce the main elements of quantum algorithms: quantum parallelism, the role of the operator on target and ancilla, interference and measurement. For each of these elements, identify the parts of the circuit that represent them and identify which aspects of quantum physics are involved. (If you think it is necessary, carry out some calculations)

We conducted a qualitative analysis of the answers to the final test aimed at determin-

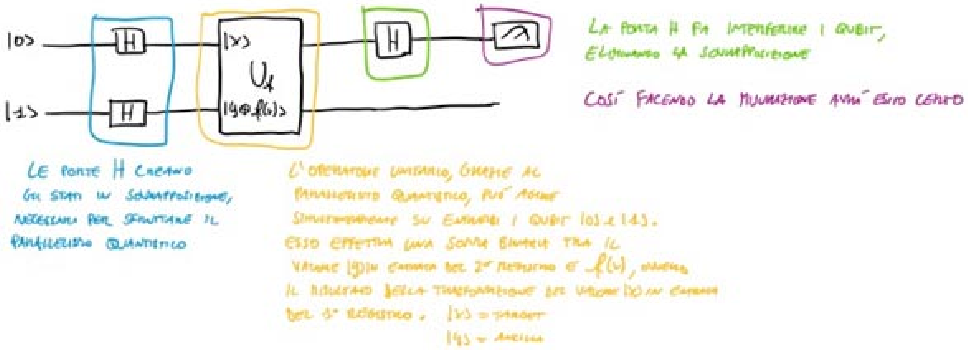


Fig. 3. – Part of the answer of one of the students to the item discussed in subection **3**˙6.2. Student's writing in different colours (in the original) can be translated as follows: "H gates produce the superposition state necessary to exploit quantum parallelism" (cyan); "The unitary operator, thanks to quantum parallelism, can act simultaneously on both qubits $|0\rangle$ and $|1\rangle$. It performs a binary sum between the value $|y\rangle$ at the input of the second register and $f(x)$, that is the value $|x\rangle$ at the input of the first register. $|x\rangle$ = target; $|y\rangle$ ancilla" (yellow); "The H gate produces interference on the qubits, eliminating the superposition" (green) "Thus, measurement will have a certain outcome" (magenta).

ing whether the division in subprocesses we provided students with had been productive for their learning. An example of these answers is reported in fig. 3. All students were able to correctly highlight, in the circuital representation of Deutsch's algorithm, the portions of circuit corresponding to each of the three significant subprocesses (the enabling of quantum parallelism, the transfer of information from the oracle to the ancilla and target, the final selection of the result). All students also identified, at least partially, the links between each subprocess and individual features of quantum physical behavior and description, such as quantum superposition, the multiplicative structure of quantum compound systems which allows a phase factor to be considered related indifferently to the target or ancilla, quantum interference. Some students supported their reasoning with explicit calculations, but only as a complement to the considerations made earlier, so that they do not seem to rely solely on mathematics for sense-making. By examining students' answers, we can conclude that the elementarization we performed of the internal transformations from input to output of a quantum algorithm was successful, within the privileged setting of this first experimentation, in scaffolding students' learning process and providing a general framework to imbue such transformations with conceptual meaning.

3˙7. *Discussion.* – The numerous limitations of this preliminary trial (very small sample, composed of self-selected students, in distance learning) did not allow to draw conclusions on the effectiveness of the approach for curricular teaching. Nevertheless, some of the results obtained appeared very encouraging. Some critical elements to take into account for future implementations were also identified: for example, in the intermediate worksheets, for a number of items which were meant to be solved through calculations, but also encouraged to provide explanations and arguments, students either ignored the request for a comment, or answered with extremely concise remarks. This phenomenon was very general, and among the factors which prompted us to insert in the final test items in which, instead, discursive analysis was the center of the question, and mathematics was optional. In subsequent experimentations started in 2022 we introduced corrections aimed at improving the quality of students' argumentation about quantum computing, among which:

- Structured worksheets for the most significant activities, including passages in which students are required to produce extended verbal analyses and explanations, in such a way to activate and stimulate their abilities of verbal expression in different moments during the course;
- Groupwork activities, based on laboratory experiences on the optical bench aimed at providing an experimental analogue of the behaviour of quantum gates with classical coherent polarized light.

As stated in sect. **3**˙2, the transposition of quantum logic gates into optical devices in the version of the sequence tested in 2021 was only sketched, and provided at the level of significative examples. The element which we mostly emphasized was the possibility to interpret several quantum gates as axial symmetries in the state space for polarization encoded qubits. We did not try to translate each and every quantum circuit we discussed into an optical device, as was instead done in the later version of the sequence, whose experimentation started in 2022. In fact, building from the experience of the trial hereby discussed, we subsequently devised a semi-general translation strategy from two qubit quantum circuits to optical devices, where one qubit is coded as the polarization state, and the other as the spatial state (which way information) which we believe can further exploit the educational value of multiple representation, allowing students to switch between logical and physical representations of the same algorithm or protocol. A complete analysis of the data from the three trials performed in 2022 (two of which in curricular teaching, one in the context of a vocational summer school) is now in progress, and the results will soon be presented to the scientific community.

4. – Characterizing conceptual change in the transition between scientific theories

Most research on conceptual change has initially focused on the transition from naïve to scientific knowledge, giving rise to ongoing debates on the nature and features of the former, *e.g.*, as concerns the extent of coherence or fragmentation of naïve ideas. However, in more recent years, there has been a steady increase of research on the learning of successive scientific models or theories in the context of formal instruction [72,73]. Based on the knowledge provided by the literature on specific difficulties, conceptual resources and effective educational strategies, we have proceeded to characterize the processes involved in learning a successor of a scientific paradigm already familiar to students. The tool of dynamic frames [74], originally proposed to describe the evolution of system of thought, was revised and used to analyze the educational transition between classical and quantum mechanics, focusing on ontological and representational continuity and change, respectively in concepts and mathematical constructs.

4˙1. *Theoretical background.* – Empirical research on challenges in learning QM spans the last thirty years and has been recently summarized in comprehensive reviews at both the level of secondary school [75] and university [76]. The authors of both reviews identify the paradigm shift as a central source of difficulties: "*Because quantum mechanics led to fundamental changes in the way the physical world is understood and how physical* *reality is perceived, quantum mechanics education is faced with several challenges*" [75] or more explicitly "*Because the quantum mechanics paradigm is radically different from the classical paradigm, students must build a knowledge structure for quantum mechanics essentially from scratch, even if they have built a robust knowledge structure of classical mechanics*" [76].

Theory change generally involves a global restructuring of a conceptual system and a new conceptual construction. According to Vosniadou and Skopeliti [77], *categorization is the most fundamental learning mechanism*, as once it is categorized by the learner an entity inherits all the properties common to all entities that belong to the domain. Important contributions were given by Chi [25] who explained the robustness and universality of some misconceptions by arguing that they were caused by concepts assigned to wrong ontological categories, for example "force" may be categorized by a child as a "property of a physical object", and such mis-categorization may lead to erroneous inferences when dealing with physics problems. Although the works by Chi did not deal with the educational transition between successive theories, they provided significant insight for its interpretation, as argued already, for example, in ref. [72].

Quantum mechanics is a multiform theory, which comprises different "pictures", which differ mostly in the mathematical formalism used, and different "interpretations", which differ in ontology, epistemological tenets, and in some cases mathematical formalism also. In order to construct a well defined map of the educational transition from CM to QM we have to preliminary perform a choice of the quantum ontology and of mechanisms of causality we intend to deal with. We chose the Schrödinger picture of QM since most educational literature and curricular materials currently in use operate within this framework, and the so-called *standard* interpretation (*e.g.*, [78]). The issue of different interpretations is however educationally relevant and needs further investigation.

4·1.1. Dynamic frames. As already stated, dynamic frames are a tool originally introduced by Andersen, Barker, and Chen [74] who traced conceptual change in the history of science by comparing different frames corresponding to subsequent or competing views of the structure of a scientific concept. Dynamic frames represent concepts by means of layers of nodes. the single node on the left in fig. 4 represents the superordinate (S) concept (BIRD) all the other nodes represent specific subsidiary concepts. The second layer represents attributes (A) of the concept (*e.g.*, NECK, COLOR); the third one, values (V) of those attributes (*e.g.*, NECK: LONG or SHORT). Each triplet S, A, V represents a proposition: "V is the A of S". When a particular subset of values is chosen to represent a specific subordinate concept, values are said to be "activated".

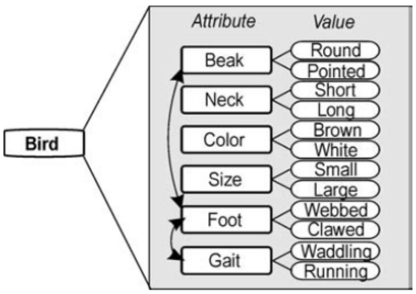


Fig. 4. – Frame representation of the categorical structure of a 'natural kind', bird in this case.

4·2. *Research questions.* – Although the paradigm shift has been identified by several researchers as a fundamental source of student difficulties, a global analysis of the connection between theory change and learning challenges was lacking. Therefore, in this work we proposed to address the following research questions:

- RQ1: How is it possible to develop a clear and comprehensive picture of what dynamics of theory change from classical to quantum mechanics are educationally significant and of the corresponding learning demands?
- RQ2: Can an exploration of conceptual continuity and change in the transition from classical to quantum mechanics help us put prior intuition in the service of learning quantum mechanics?

4·3. *Methods.* – Addressing the research questions required, first, the development of methods of analysis suitable to identify the connections between theory change in individual aspects of a concept/construct and student difficulties elicited by research; second, the design of a customized representational tool aimed to visualize with a bird's eye view all those changes of the same concept/construct related to learning difficulties, in the search of educationally meaningful regularities. For the first goal (RQ1) we started by building a detailed taxonomy of specific changes that may affect scientific concepts (ontological and metaphorical changes) and mathematical/visual constructs (different types of representational changes). Then we applied this taxonomy to the classical/quantum transition, selecting basic entities of both theories, and classifying the dynamics of these entities and of their properties according to the taxonomy. Finally, we scanned the literature on student understanding of the new theory, associating common difficulties to the specific conceptual trajectory involved. For the second goal (RQ2) we adopted and accordingly restructured the dynamic frame construct. In the type of frames we used, the superordinate concept is a basic term, mathematical object or procedure common to classical and quantum mechanics. The frame describes an educationally significant portion of the categorical structure of the concept, differentiating the classical and the quantum version through the patterns of activated values (fig. 5).

The relations between values activated in the old and the new version of a concept represent its specific pattern of change. We make an explicit choice of attempting to preserve continuity as far as possible by choosing common superordinate concepts between the two theories; the choice is also functional to the identification of a productive role

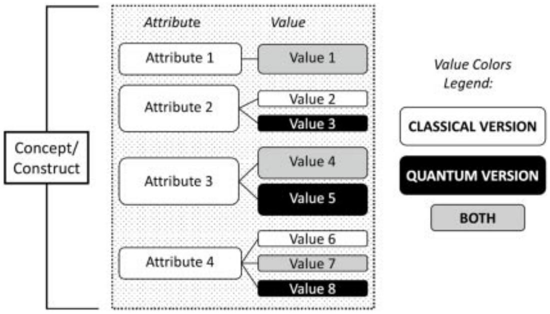


Fig. 5. – An abstract example of the iconic structure of our frames.

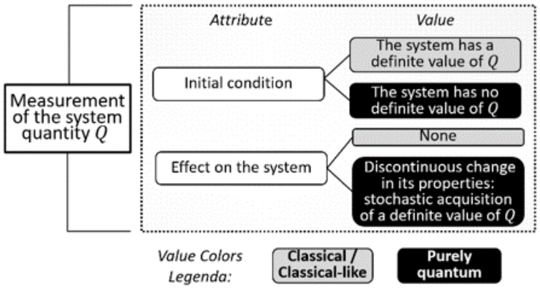


Fig. 6. – Frame representation of the superordinate concept "measurement of a system quantity". Values in gray are the only ones possible for classical systems, but can also belong to attributes of selected quantum system which have a classical-like behavior in certain respects.

of prior intuition based on the specific pattern of change at hand. Notwithstanding this choice, as we will see, radical change still emerges.

4˙4. *Three (plus one) patterns of change.* – The process of producing the frame representation of corresponding superordinate concepts in classical and quantum mechanics involved both a thorough analysis of student difficulties from all, or most, available sources, and theoretical reflection on the conceptual structure of the theories themselves. Such process is accounted for in [30]. In this paper we limit ourselves to a classification of the different patterns of change which occur in the frame structures.

4˙4.1. Categorical generalization. In this case the classical version of the concept is a categorical subset of the quantum one. In the quantum version, attributes can assume new values in addition to the classical ones (see fig. 6).

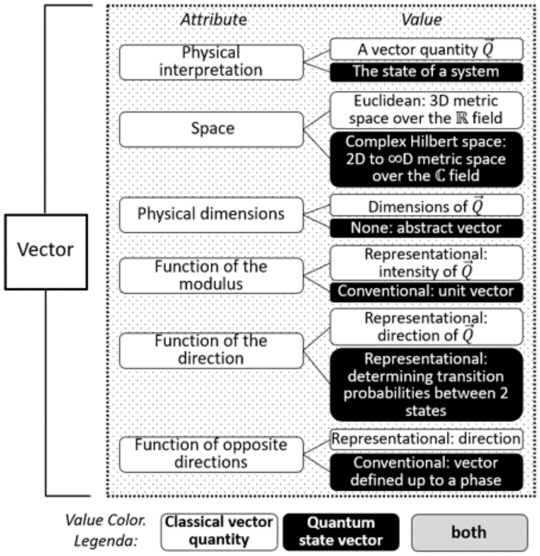


Fig. 7. – Frame representation of the superordinate concept "vector". Values in white are exclusive of attributes of classical systems, values in black are exclusive of attributes of quantum systems. There is no overlap between the two sets of values.

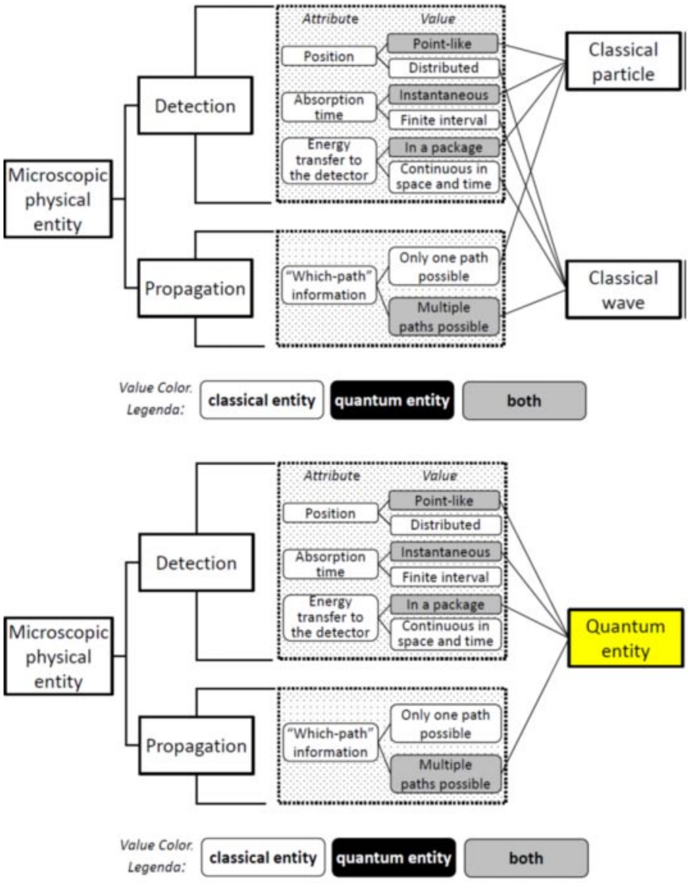


Fig. 8. – Frame representation of the superordinate concept "microscopic quantum entity". In the quantum case, a new subordinate concept appears ("quantum entity") which has values of attributes which, in the classical cases, are proper both of particles and waves.

4˙4.2. Value disjunction. In this case, the classical and the quantum version of the same entity assume necessarily different values. This is usually the case for mathematical constructs (see fig. 7).

4˙4.3. Change of value constraints. In this pattern of change, a new subordinate concept appears whose properties are a reorganization of the properties belonging to old subordinate concepts (see fig. 8).

4˙4.4. Special case: radical change. Although our frame representation, by construction, privileges continuity, "radical change", in the sense intended in ref. [74], that is a mismatch in the frame structure of the two theories, can still be identified. It consists of the case in which for either the older or the new theory (typically, the former is the case) the value "none" appears as the unique value for an attribute. This can happen both in the cases of categorical generalization, and value disjunction. This essentially means that our choice to preserve continuity has a fictitious character here: the structure of concepts in the two theories do not map to each other (see fig. 9).

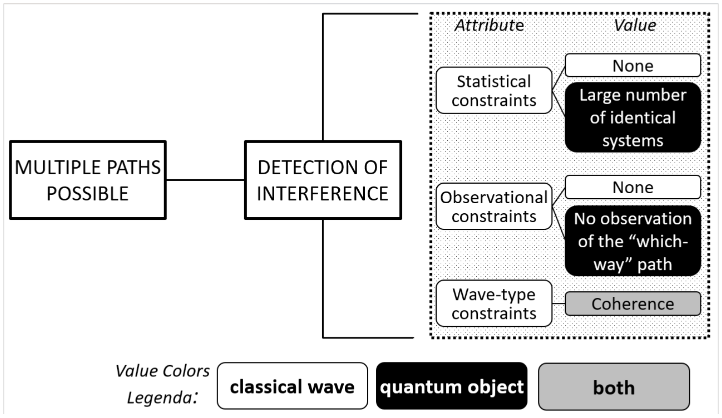


Fig. 9. – Frame representation of the value of Fig. 8 "multiple paths possible". The transition from classical to quantum interference is a clear instance of radical change.

4·5. *Recommendations for instruction.* – In ref. [30] we proposed several recommendations for instruction based on our categorization of the dynamics of change in different concepts from CM to QM. We here provide only two examples.

- For the categorical generalization pattern, a strategy can consist in starting from quantum situations corresponding to values common to both theories, that can be interpreted classically and then extending them to cases corresponding to exclusively quantum values. For example, in the introduction of quantum measurement in the setting of polarization, sending to the measuring device random mixtures of photons whose polarization corresponds either to one or the other eigenvalue of the measured observable (a classical ideal measurement). Introducing this case first, will favor the interpretation of the interaction of a photon with a polarizer as measurement, which is problematic for students. At this point, the concept can be generalized to photons with different polarizations.

- In the Radical change special case there is a need of developing with students the consciousness of implicit assumptions of the older theory which may need to be made explicit before they are challenged; for example, the assumption that measurement produces no effect on the system. As in Foucault's work on the history of evolution theory [79], an implicit assumption cannot be challenged before it is made explicit. Also, it can be hypothesized that the frame representation may give indications on the points at which the discussion of historical or ongoing debates about the foundations of the new theory may be most productive, being connected with genuine elements of radical change from the old theory to the new one of the two theories (*e.g.*, ref. [80]).

5. – Conclusions

In this article we have reported on recent research by the PER group in Pavia, some of which with external collaborators, broadly related to conceptual understanding of physics, especially, though not exclusively, at the level of secondary school. Research in the near future will continue in the directions here discussed, and others, which have only been mentioned in passage in the Introduction. All research work in education, however, risks to be at least partly self-referential and ineffective, without an active involvement of teachers engaged in action–research. Unfortunately, in Italy, highly controversial choices at the political level in the past few years have deprived significantly the resources and spaces for teacher professional development. At the time we are writing, we receive the long-awaited news of the opening of a new cycle of the PLS - plan for scientific degrees project, and thus, the reviving of a researcher community of invaluable importance for the organization of such actions [81]. Our hope is that, in the near future, the reform of teacher initial recruitment will finally be completed, putting an end to the shame of a generation of teachers entering the classroom with no background in disciplinary education research.

* * *

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