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Colloquia: COMEX7

A consistent description of the monopole resonance in spherical nuclei

G. Colò([1]), Z. Z. Li([2]) and Y. F. Niu([2])

([1]) *Dipartimento di Fisica, Università degli Studi di Milano and INFN, Sezione di Milano via Celoria 16, 20133 Milano, Italy*

([2]) *School of Nuclear Science and Technology, Lanzhou University and Frontiers Science Center for Rare isotope, Lanzhou University - Lanzhou 730000, China*

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Summary. — We have recently implemented a fully self-consistent model based on Quasiparticle-Vibration Coupling (QPVC model). This can be applied to Giant Resonances of any kind, and can account for the position of the resonance main peak (or centroid) and for the resonance width. In this contribution, we show how this model can solve the problem of the different incompressibilities (K_∞) that spherical nuclei display. In other words, we discuss here that the values of K_∞ extracted from Sn isotopes and ^{208}Pb turn out to be compatible, so that the famous issue of the "fluffiness" of Sn is set. Ca isotopes and ^{90}Zr are also compatible with the same values of K_∞, that are around 225–230 MeV. This conclusion relies on the use of the so-called subtraction method.

1. – Introduction

The nuclear structure community has striven to infer the value of the nuclear incompressibility for several decades. In the 1980s, measurements of the Isoscalar Giant Monopole Resonance (ISGMR) have been carried out, and the first estimate of the incompressibility has been attempted in the pioneering works by J.P. Blaizot [1]. The nuclear incompressibility K_∞ is defined by the equation

$$\frac{E}{A}(\rho) = \frac{E}{A}(\rho_0) + \frac{1}{2}K_\infty\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \dots, \tag{1}$$

where E/A is the energy per particle of symmetric nuclear matter, ρ is the density and ρ_0 is the saturation density. As such, it is a property of nuclear matter that characterises its Equation of State (EoS). Seeking an accurate value of this quantity is not only of interest for validating our understanding of nuclear structure. In fact, the incompressibility of nuclear matter also affects the dynamics of heavy-ion collisions and the physics of astrophysical compact objects [2,3]. Core-collapse supernova, and the process of formation of

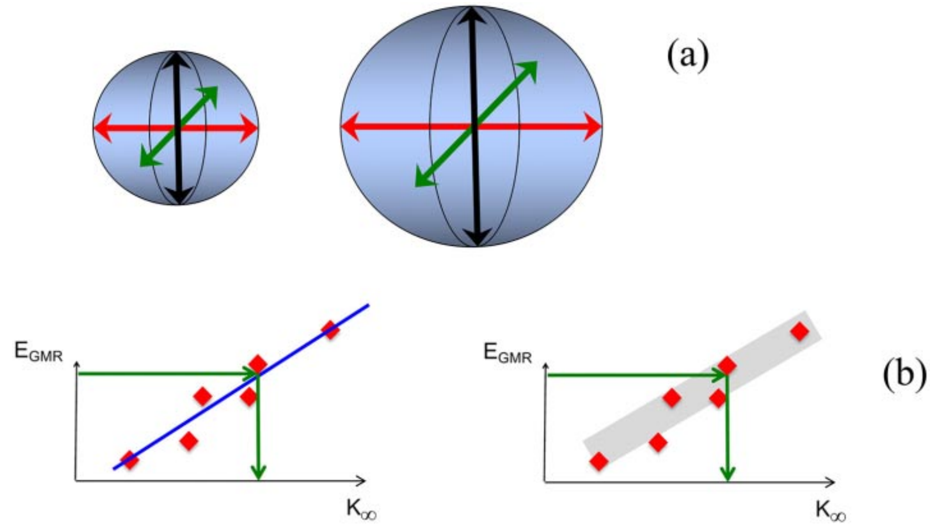

Fig. 1. – In panel (a), a schematic picture of the nuclear breathing mode, or ISGMR, is shown. In (b), two examples of the same correlation, between the energy of the ISGMR calculated in a self-consistent model and the value of K_∞, are displayed. The shadowed area in the right figure is meant to illustrate the model dependence of the procedure to extract K_∞ from the experimental data.

a proto-neutron star, are sensitive to the nuclear EoS and recently it has been shown that the same holds for the merging of compact objects and the gravitational-wave emission [4]. In this respect, one can claim that the study of the nuclear incompressibility is an interdisciplinary topic.

From the nuclear structure viewpoint, the compression modes of nuclei are the main source of information on K_∞. The ISGMR is the so-called nuclear "breathing mode", in which the nucleus contracts and expands. This mode is sketched, in a sort of macroscopic view, in panel (a) of fig. 1. A series of experimental measurements exist, and both the experimental and theoretical status have been reviewed a few years ago in [5]. This latter review paper covers also another nuclear compressional mode, *viz.* the Isocalar Giant Dipole Resonance (ISGDR), which is not discussed in this contribution. Panel (b) highlights the "standard" view of how we can extract K_∞ from experimental data. Assuming that a correlation exists, in calculations, between the energy of the ISGMR [E(ISGMR)] and the value of K_∞ associated with the model that has been employed, the left figure of panel (b) highlights how the value of K_∞ can be deduced. The right figure of panel (b) is meant to give a pictorial impression of the fact that there is not a "universal" correlation between E(ISGMR) and K_∞, as implied by the shadowed area. A large amount of research has been devoted to quantify this spread, or model dependence. As we now argue, there is also a nucleus dependence — that manifests itself if using the procedure in panel (b) leads to different results when different nuclei are considered.

Some model dependence is probably unavoidable. At present, only nuclear Density Functional Theory (DFT) models can be used to calculate the ISGMR in medium-heavy nuclei. Such calculations are not doable, so far, using *ab initio* nuclear theory. Comparisons between various implementations of DFT have been carried out in the last two decades, in particular focusing on confronting nonrelativistic Energy Density Functionals

(EDFs) *vs.* relativistic ones. To cut this story short, in a given paragdigmatic nucleus like ^{208}Pb, the model dependence of K_∞ has been quantified as $K_\infty = 240\pm20$ MeV in [5] and references therein. However, and this is a key point for what follows, the calculations of E(ISGMR) that have been used to this aim are (Quasiparticle) Random Phase Approximation [(Q)RPA] calculations. QRPA is a self-consistent theory but it is only the simplest approximation one can employ to calculate Giant Resonances (GRs). A systematic analysis of how the correlation depicted in fig. 1 changes when going beyond QRPA, is called for.

The purpose of this contribution is to discuss precisely this point: we show in what follows that going beyond QRPA we can solve, or mitigate, the model dependence and nucleus dependence of K_∞ at the same time. In particular, we emphasize and focus on the following problem. A while ago it was pointed out that the models that reproduce well the energy of the ISGMR in ^{208}Pb overestimate the energy of the ISGMR in the Sn isotopes [6, 7]; in other words, an inconsistency between the values of K_∞ deduced from ^{208}Pb and Sn has been pointed out - with the Sn value being lower than the Pb value. This fact that led to the question "why is Tin so soft?" [8-10]. If we use the "paradigm shift" that we just mentioned, that is, if we abandon the idea that the correlation between the energy of the ISGMR and K_∞ must be explored at the QRPA level, we can solve this "softness" puzzle.

More specifically, we employ a theoretical framework based on the (Quasi-)Particle-Vibration Coupling (QPVC) that goes beyond the QRPA. In it, the ISGMR energies in spherical nuclei appear to be essentially consistent with the result provided by the same EDFs and with similar values of K_∞. A few caveats are in order, though. First of all, our conclusions rely on the so-called "subtraction method" as we describe below. Secondly, we have so far to leave aside the case of deformed nuclei. We come back to this latter point in the conclusion of this manuscript.

In sect. **2**, we describe our QPVC implementation. In sect. **3**, we provide our main results. Conclusions and perspectives are drawn in sect. **4**. The main results discussed in this paper have been published in [11].

2. – Theoretical framework

Our QPVC model is based on self-consistent Hartree-Fock-Bogoliubov (HFB) plus QRPA; it includes, on top of this, the coupling of the QRPA two-quasiparticle states with so-called doorway states, made with two-quasiparticles plus one phonon. In formal terms, this means that the excited states $|N\rangle$ of the nucleus are described by the ansatz

$$(2)\qquad |N\rangle = \left(\sum_{ab} X^{(1)}_{ab}\alpha^\dagger_a\alpha^\dagger_b - Y^{(1)}_{ab}\alpha_b\alpha_a + \sum_{ab;n} X^{(2)}_{ab;n}\alpha^\dagger_a\alpha^\dagger_b\Gamma^\dagger_n - Y^{(2)}_{ab;n}\Gamma_n\alpha_b\alpha_a\right)|0\rangle,$$

where $|0\rangle$ is the nuclear ground-state, α ($\alpha^\dagger$) are annihilation (creation) operators for quasi-particles (labelled by $a, b\ldots$), and $\Gamma^\dagger_n$ is the creator of a QRPA state $|n\rangle$, that is,

$$(3)\qquad \Gamma^\dagger_n = \sum_{\alpha\beta} X^{(n)}_{\alpha\beta}\alpha^\dagger_\alpha\alpha^\dagger_\beta - Y^{(n)}_{\alpha\beta}\alpha_\beta\alpha_\alpha.$$

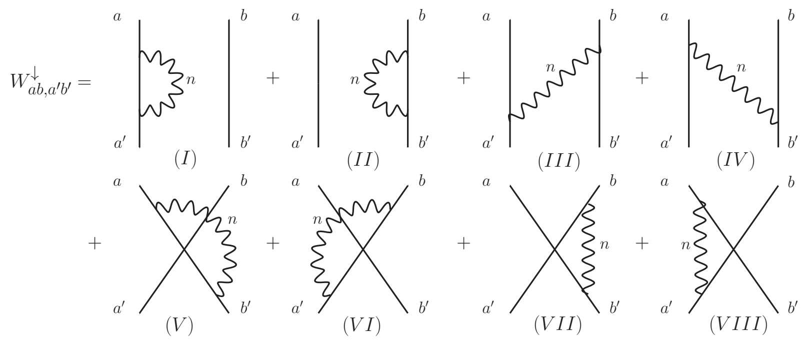

Fig. 2. – Feynman diagrams associated with the coupling between two-quasiparticles and the doorway states. See the main text.

X and Y are amplitudes to be determined. To this aim, the Hamiltonian that we adopt is

$$H = T + V_{\text{Skyrme}}, \tag{4}$$

where the first term is the kinetic energy and V_{Skyrme} is a Skyrme-type interaction. In principle, one could solve this Hamiltonian given the ansatz of Eq. (2) for the wave function; however, this is too demanding from the computational viewpoint, in particular for heavy nuclei. Therefore, we use standard projection techniques in order to obtain an equation expressed in the two-quasiparticles basis only. This reads

$$\begin{pmatrix} \mathcal{A}(\omega) & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^*(-\omega) \end{pmatrix} \begin{pmatrix} \mathcal{X} \\ \mathcal{Y} \end{pmatrix} = \hbar\omega \begin{pmatrix} \mathcal{X} \\ \mathcal{Y} \end{pmatrix}, \tag{5}$$

where

$$\begin{aligned} \mathcal{A}(\omega) &= A + W^{\downarrow}(\omega), \\ \mathcal{B} &= B. \end{aligned} \tag{6}$$

Here, A and B are the standard QRPA matrices while $W^{\downarrow}(\omega)$ is the self-energy associated with the coupling between the two-quasiparticles and the doorway states. In the present case, we take the doorway states as non interacting (this limitation is waived in ref. [12]). The diagrams corresponding to the self-energy are displayed in fig. 2. The model is described in full detail in ref. [13]. The reader can also consult ref. [14] for a discussion about the comparison between this and other models that are similar to QRPA or go beyond it.

The numerical details of our implementation are given in the Supplemental Material of [11]. The HFB equations are solved in spherical symmetry, with box boundary conditions. The QRPA basis is large enough so to guarantee the respect of the appropriate sum rules. In a second step, the coupling of the given multipole strength (that is monopole in the case of the calculations described in the next section) with doorway states is carried out, namely the self-energy $W^{\downarrow}$ is calculated and inserted into the previous equations. We have carefully checked that the results are stable with respect to the number of phonons, or the number of doorway states, that we include.

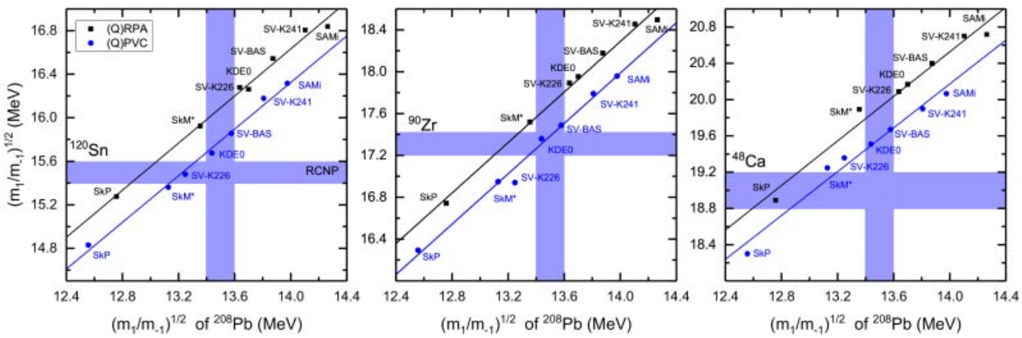

Fig. 3. – Correlation plots of the ISGMR energies in different nuclei. The energies are defined as the constrained energies (see the text). In the figure, we display the energy in ^{208}Pb on the horizontal axis, and the energies in the other spherical nuclei, ^{120}Sn, ^{90}Zr and ^{40}Ca, on the vertical axis of the three panels. The points represent either QRPA or QPVC calculations that have been performed with an extensive set of Skyrme forces. The coloured bands correspond to experimental data.

We conclude this section by mentioning the use of the so-called subtraction method, in which the self-energy $W^{\downarrow}(\omega)$ is replaced by

$$W^{\downarrow}(\omega) \;\to\; W^{\downarrow}(\omega) - W^{\downarrow}(\omega = 0). \tag{7}$$

This procedure has been introduced in [15], by arguing that in this way one avoids the double-counting of static ($\omega = 0$) correlations, that are effectively taken care by when EDFs are fit. The procedure has been justified in more formal terms in ref. [16].

3. – Results

The eigenstates of the matrix (5), that we have already denoted as $|N\rangle$, are associated with complex eigenvalues $E_N \equiv \hbar\omega_n - i\frac{\Gamma_N}{2}$. What we usually calculate, to compare with experimental findings, is the strength function $S(E)$ corresponding to a given operator F, that is

$$S(E) = -\frac{1}{\pi}\Im \sum_N \frac{\langle N|F|0\rangle^2}{E - \hbar\omega_N + i\frac{\Gamma_N}{2}}. \tag{8}$$

In principle, we can calculate the strengths associated with various operators of interest, like, *e.g.*, the dipole operator which is associated with the dipole polarizabiliy. Work along this line is in progress [17]. In this section, we focus on results obtained with the monopole operator. In particular, we compare the theoretical centroid energy of the strength function with recent experimental findings. From the moments of the strength function m_k, defined as

$$m_k \equiv \int_0^\infty dE\; E^k S(E), \tag{9}$$

we can extract the centroid energy in different ways. We focus here on the quantity $(m_1/m_{-1})^{1/2}$, that is often referred to as the constrained energy.

In fig. 3 these energies, that we may call ISGMR energies for the sake of brevity, are calculated with an extensive set of different Skyrme forces, either at the QRPA level or at the QPVC level. Pairing is actually relevant in Sn while, in the other nuclei whose results are in the three panels, QRPA (QPVC) reduces to RPA (PVC). The lines are fit to the points, and are meant to show that, to a large extent, energies in different nuclei are well correlated when one employs one of these models. The (Q)PVC line is shifted in a systematic manner with respect to the (Q)RPA line: in particular, in three nuclei at hand the shift between the (Q)RPA energy and the (Q)PVC energy is larger than in ^{208}Pb and, thus, the blue line is below the black line.

As seen from the figure, this is a key point when comparing to experiment. Experimental data are taken here from refs. [6, 18-20] and correspond to the colored areas that take the reported 1σ uncertainties into account. (Q)RPA can give a consistent description of Pb and Zr, but not of the other nuclei. However, including (Q)PVC correlations we can have a full consistent description of this set of four nuclei by using some of our interactions. In particular, these effective interactions are SV-K226 and KDE0. They are chracterized by values of the nuclear incompressibility, K_∞, equal to 226 MeV and 229 MeV, respectively,

4. – Conclusions

In this work, we have presented an application of the Quasiparticle-Vibration Coupling (QPVC) model to the study of the ISGMR. QPVC can account for the line shape of the nuclear Giant Resonances, including the conspicuous width that is usually of the order of several MeV. QRPA cannot account for it, but only for a small part of it, that is configuration mixing (the so-called Landau damping). Most of the conspicuous width, at least in medium-heavy nuclei, is spreading width and calls for beyond-QRPA approaches. At the same time, the QPVC energies are usually shifted downward with respect to QRPA. In our recent work [11], we have shown that this is a key element to provide a consistent description of monopole in a series of spherical nuclei.

In fact, in different nuclei the shifts can be, and are, different. The main conclusion of our work is shown in fig. 3 of the current paper. The centroid energies of the ISGMR in ^{48}Ca, ^{90}Zr, ^{120}Sn, and ^{208}Pb appear to be correlated when calculated with different Skyrme effective forces. However, it is hard to match the experimental findings with a given force at the QRPA level. At the QPVC level, instead, there are models like SV-K226 and KDE0 that reproduce the energies in all nuclei, in keeping with the experimental uncertainty.

While this is a novel and very promising conclusion, a few words of caution are in order here. This conclusion relies on the subtraction method, that consists in employing the above Eq. (7) when including the self-energy associated with (Q)PVC correlations. The underlying philosophy is that (Q)PVC should not affect static quantities. Along this line, the correct value of K_∞ is the value associated with either SV-K226 or KDE0 at the mean-field level, namely either 226 or 229 MeV. The subtraction method, and its impact on the extraction of other properties of the nuclear EoS, should be further investigated, though.

Another open question regards deformed nuclei. Some nuclei that are deemed to be "soft", in the sense that their ISGMR energy is consistent with low values of K_∞, may have an intrinsic deformed shape. In the case of, *e.g.*, axial deformation, the total angular momentum J is not a good quantum number any longer but only the projection K on the intrinsic symmetry axis is. This produces a coupling between the ISGMR and the $K = 0$ component of the quadrupole resonance. All this has been discussed in the literature (cf., *e.g.*, [21] and references therein), but its impact on the extraction of K_∞ has still to be carefully assessed. Projected QRPA is still under development in exact form, while only approximate versions have been implemented so far.

Last but not least, a recent paper [22] has confirmed the importance of PVC correlations also when staring from a covariant EDF. The calculations of this latter work point to a larger K_∞ than what we have found ($K_\infty = 251$ MeV). However, the analysis of [22] is based on a single EDF. Extensive QPVC calculations based on a series of functionals with different incompressibility, in a similar way as we have shown here, would be highly welcome.

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