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Colloquia: COMEX7

Electric dipole strength functions of Lambda hypernuclei obtained by the time-dependent mean-field calculation

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Summary. — We investigate the electric dipole ($E1$) resonance states of Λ hypernucleus using a time-dependent mean-field model. The model is the canonical-basis time-dependent Hartree-Fock-Bogoliubov theory (Cb-TDHFB) with Skyrme-type effective interaction and smoothed constant pairing interaction. The linear response calculation with Cb-TDHFB is performed in the three-dimensional Cartesian coordinate space to treat deformed nuclei. We report the $E1$ strength functions of carbon isotopes and discuss the influences of a Λ particle on the ground states and the excited states of the hypernuclei.

1. – Introduction

The hyperon-nucleon (YN) interaction in the nuclear matter system is necessary to understand the equation of state (EOS) for neutron stars. A two-solar-mass neutron star was measured in 2010 [1]. The density in the neutron star's core is expected to be several times the normal nuclear density ($\rho_0 \sim 0.16\text{fm}^{-3}$). Under the environment, a hyperon will appear in the matter, and its degree of freedom should be considered in the EOS. The EOS is softened by simply introducing the attractive YN interaction and cannot describe the massive neutron star. This is called a "Hyperon puzzle". We should know the correct YN interaction to solve the hyperon puzzle, but this is difficult due to the very short half-lives of hyperon. Therefore, we need to investigate the structures of the hypernuclei while assuming the YN interaction, and we need accurate data for hypernuclei to verify them.

A high-intensity high-resolution (HIHR) project in Japan Proton Accelerator Research Complex (J-PARC) has been planned to study a nuclear system with hyperon. The accuracy of nuclear structure on the Λ hypernuclei will be expected to be significantly improved by the progress. Furthermore, the targeted hypernucleus might expand to a broader nuclear mass number of regions.

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In this work, we develop a theoretical method to investigate the structure and excitation states of the hypernuclei, including heavy ones. We focus on single Λ hypernuclei, introduce a mean-field model with the Skyrme-type effective interactions to describe it, and then report the obtained results.

2. – Method

To prepare the ground state of a hypernucleus, we employ a Hartree-Fock plus Bardeen-Cooper-Schrieffer (HF+BCS) and HF model for the nucleons and a Λ-particle, respectively. The intrinsic nuclear density $\rho(\boldsymbol{r})$ in HF+BCS is represented as

$$\rho(\boldsymbol{r}) \equiv \sum_{l,\sigma} v_l^2 |\phi_l(\boldsymbol{r},\sigma)|^2, \tag{1}$$

where v_l^2 means the occupation probability of the l-th single-particle state $\phi_l(\boldsymbol{r})$. The σ means the spin degree of freedom. The density is fully self-consistently obtained according to HF and BCS gap equations [2],

$$[h,\rho] = 0, \tag{2}$$

$$2\bar{\epsilon}_l u_l v_l + \Delta_l (2v_l^2 - 1) = 0, \tag{3}$$

where h and Δ_l are a single-particle Hamiltonian and a gap parameter. The u_l is one of the BCS parameters obtained from the gap equation with the norm and number conservation conditions. The $\bar{\epsilon}_l$ is the averaged single-particle energies of l-th state pairs: $\epsilon_l \equiv \langle\phi_l|h|\phi_l\rangle$. In the even-even nucleus, the l-th pair state has a time-reversal relation with ϕ_l. The label of l-th pair state is often noted as $\bar{l}$, which means $\bar{l} = -l$. The gap parameter Δ_l is defined as $\Delta_l = \sum_{m>0} G_{lm} u_m v_m$, where G_{lm} is the smoothed constant pairing strength which has the energy cutoff. The Λ-particle states are treated in the HF equation eq. (2). Here, we treat the single Λ hypernucleus, therefore, the occupation probabilities of $s_{1/2}$ Λ-particle states with $\sigma =\uparrow,\downarrow$ are the same value: $|v^{\Lambda}_{s_{1/2}}|^2 = 0.5$, which is the equal filling approximation.

We obtain the strength function of the electric dipole ($E1$) mode by the linear response calculation with time-dependent mean-field models. The time-dependent models are the canonical-basis time-dependent Hartree-Fock-Bogoliubov theory (Cb-TDHFB) [3] and TDHF for nucleons and Λ-particle states. The initial states of these time-dependent calculations can be prepared by the HF+BCS and HF, although the instantaneous external field with the $E1$ operator $\hat{F}_{E1}$ added to the states. The initial state ϕ_+ is written as,

$$|\phi_+\rangle = e^{ik\hat{F}_{E1}}|\phi_{\text{g.s.}}\rangle, \tag{4}$$

where k is the strength of the external field and is small enough to induce the linear responses. The $\hat{F}_{E1}$ is

$$\hat{F}_{E1} = \frac{ZM_N}{AM_N + M_\Lambda}\left(\frac{M_\Lambda}{M_N}\hat{z}_\Lambda + \sum_n \hat{z}_n\right) - \frac{NM_N + M_\Lambda}{AM_N + M_\Lambda}\sum_p \hat{z}_p, \tag{5}$$

where M_N and M_Λ are the masses of a nucleon and a Λ particle, and $\hat{z}_\tau$ with $\tau = n, p, \Lambda$ means a position operator for z-axis which is a rotational-symmetry axis of the nucleus here. This $\hat{F}_{E1}$ is defined to keep the center of mass in the time-evolution calculation.

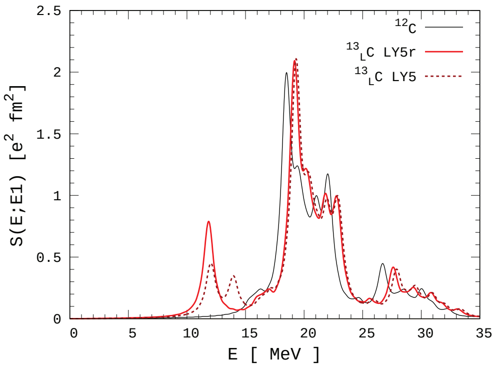

Fig. 1. – $E1$ strength functions for ^{12}C and $^{13}_{\Lambda}$C. See text for details.

For the nucleons, the SLy4 Skyrme effective interaction is employed, and for Λ-nucleon (ΛN) effective interaction, we use a parameter set suggested in ref. [4]. The ΛN interaction is adjusted to reproduce the split between $p_{3/2}$ and $p_{1/2}$ Λ-particle states on the $^{13}_{\Lambda}$C, and is labeled in "LY5r". The difference between the original ΛN interaction (LY5) [5] and LY5r is the strength of the spin-orbit (LS) term. The LS term of LY5r is about 15 times smaller than LY5. The LY5 interaction does not include the Σ-nucleon coupling, which might be repulsive in a nuclear medium.

The wave functions are represented in the three-dimensional Cartesian coordinate space, which can describe any nuclear deformation. As summarized, our model describes the single Λ hypernucleus considering its deformation and pairing correlation for nucleons. The targets are carbon (C) isotopes with a Λ-particle occupying the lowest state.

3. – Results

Figure 1 shows $E1$ strength functions for ^{12}C (thin solid line) and $^{13}_{\Lambda}$C with LY5r (thick solid line) and LY5 (dotted line) interactions. We can see the giant dipole resonance (GDR) peak around 20 MeV for each result. The both GDRs of $^{13}_{\Lambda}$C with LY5r and LY5 are about 1 MeV higher than ^{12}C. Furthermore, we can also find the characteristic low-energy dipole (LED) strengths lower than 15 MeV, which cannot be found in ^{12}C. The LED shows apparent differences between results with LY5r and LY5. The strong LS term induces the split LED of the LY5 result and corresponds to the split of single-particle energies for Λ-particle $p_{3/2}$ and $p_{1/2}$ states.

To evaluate the peak position of their GDR, we introduce the mean energy $\bar{E}_{\rm GDR}$,

$$\bar{E}_{\rm GDR} \equiv \frac{\int_{E_{\rm LED}} dE\ ES(E;E1)}{\int dE\ S(E;E1)}, \tag{6}$$

where $S(E;E1)$ is the strength function of $E1$ mode, and the $E_{\rm LED}$ is the threshold energy for LED strength to subtract their strength from the higher strengths. Here, we use $E_{\rm LED}$=14 MeV. The evaluated $\bar{E}_{\rm GDR}$s of C isotopes are shown in fig. 2 concerning the number of nucleons, excluding Λ-particle. The GDR shifts less than 1 MeV evenly appear in the C isotopes.

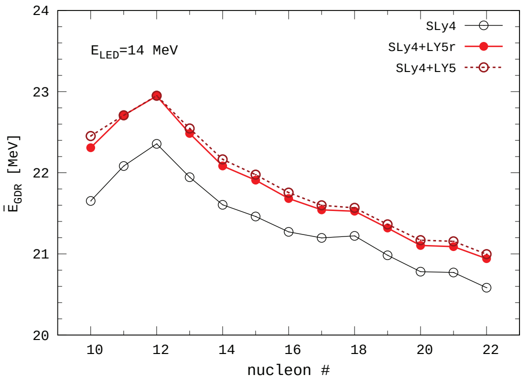

Fig. 2. – Mean energy of the giant dipole resonances for carbon isotopes.

We have checked the differences between C isotopes with and without Λ-particle for their ground state. Figures 3 and 4 show the quadrupole deformation parameter β of the nucleon and the root mean square radius $R_{\rm rms}$ of the nucleon, respectively, for the C isotopes concerning the nucleon number. The βs of Λ hypernuclei get smaller than those of normal C isotopes, consisting of the decrease of $R_{\rm rms}$. The difference between results with LY5r and LY5 is negligibly small on the β and $R_{\rm rms}$. But, it is a remarkable influence of Λ-particle that the $R_{\rm rms}$ shrinks evenly appear in the isotopes. The $R_{\rm rms}$ shrink will be a major reason to shift the GDR peak position.

4. – Summary and perspective

Using the mean-field models, we investigated the single Λ hypernuclei for C isotopes. The models described nuclear deformation and pairing. Furthermore, we discussed the differences between C isotopes with and without a Λ-particle on their ground states and $E1$ strength functions. The Λ-particle reduces the nuclear $R_{\rm rms}$ and shifts the $\bar{E}_{\rm GDR}$ to about 1 MeV higher energy region. For the $E1$ excited states, the LED strength

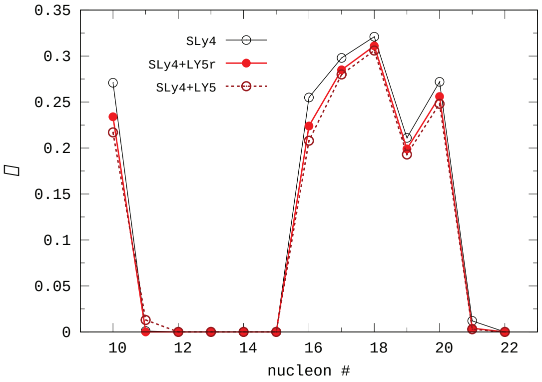

Fig. 3. – Quadrupole deformation parameter of nucleon for C isotopes.

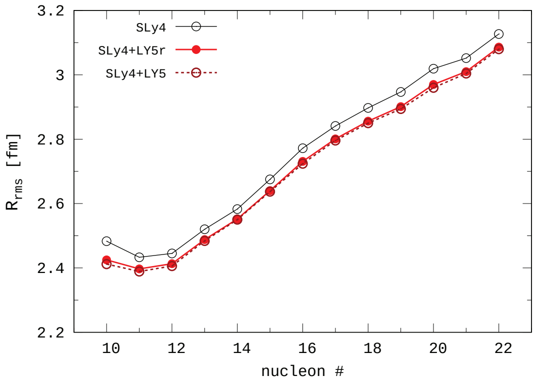

Fig. 4. – Root mean square radius of nucleons for C isotopes.

is found in the Λ hypernuclei, which does not appear in the normal C isotopes. The LED mode is consistent with the modes of ${}^{18}_{\Lambda\Lambda}$O in the previous study [6]. We found the difference in *LS* term appears in the LED mode from comparing results with LY5r and LY5 interactions.

In the future, we will extend our models to heavier and multi-strangeness hypernuclei and contribute to revealing the general influence of Λ-particle on nuclear structure.

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