IL NUOVO CIMENTO **47 C** (2024) 23
DOI 10.1393/ncc/i2024-24023-2

Colloquia: COMEX7

Giant Pairing Vibration in the continuum beyond RPA

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received 31 October 2023

Summary. — The possibility of the existence of collective pairing modes ($J^\pi = 0^+$) lying at high excitation energy (10-15 MeV) resulting from the correlated two-particle or two-hole excitation across major shells, was formulated in the 70's by Bes and Broglia, and today these modes are known as the Giant Pairing Vibrations (GPV). Since then there have been numerous unsuccessful attempts to detect them, and recently a bump has been reported in the excitation spectrum of ^{14}C produced by the reaction ^{12}C(^{18}O, ^{16}O)^{14}C, that was proposed to be a signature of the GPV. In this contribution we present a microscopic calculation of the response function of ^{12}C to the creation operator of two neutrons coupled to 0^+, including the coupling to the continuum. We carry out a pp-RPA calculation, obtaining a concentration of strength close to threshold ($E^* \approx 13$ MeV) based on s and d orbitals, which displays the general features expected for a GPV. We then take into account the coupling between the neutron degrees of freedom and the low-lying collective quadrupole vibrations of the core, that shifts and fragments the *sd* strength, producing two states below threshold in reasonable agreement with the two observed discrete excited 0^+ states, as well as a bump in the continuum ($E^* \approx 18$ MeV).

1. – Introduction

Low-energy collective pairing modes around closed-shell nuclei, have been well determined experimentally [1] and thoroughly studied theoretically (see, *e.g.*, [2-4]), and the parallel with collective particle-hole excitations has been well established [1]. On the other hand, the experimental search for giant pairing vibrations (GPV), the equivalent of the well-known Giant Resonances (GR) in the particle-hole channel, has not been successful despite many attempts (see, *e.g.*, [5] and references therein), until recent experiments between heavy ions populating ^{14}C and ^{15}C with the two-neutron transfer reactions ^{12,13}C(^{18}O, ^{16}O)^{14,15}C [6, 7] have identified bumps in the excitation spectrum, that have been proposed as a signature of the GPV.

The existence of the GPV, correlated 2p and 2h excitations across major shells, was proposed theoretically a long time ago, on the basis of schematic models within the particle-particle Random Phase Approximation (RPA) [8, 9] and has been studied more recently with continuum RPA calculations [10-13]. We intend to take a step forward,

calculating the strength function associated with pairing modes going beyond the RPA, including continuum effects but also the coupling to more complex configurations. This is a prerequisite for the detailed calculation of absolute two-nucleon transfer cross sections, needed for a conclusive comparison between theory and experimental findings.

There are several motivations to go beyond RPA and include the effects of Particle Vibration Coupling (PVC) in the description of pairing modes. The effect of PVC on particle-hole and charge-exchange GR in medium-mass and heavy nuclei has been studied quite extensively, and has been shown to lead to considerable improvement in the calculation of the strength function and, in particular, of the width of the main peaks, in comparison with experimental findings [14, 15]. The effects of PVC on correlated particle-particle states in light nuclei is even more crucial [16]. It is well known that in light nuclei, potentials dependent on parity and angular momentum are needed for an accurate description of single particle states [17]. Furthermore, such potentials must depend on energy in order to reproduce the experimental energies 0^+ excited states [18]. In our opinion, the most physically satisfactory way to account for these dependencies is to include self-energy processes associated with the coupling to surface vibrational states, that renormalize the energy of single-particle states and produce the fragmentation of their strength, improving the agreement of spectroscopic factors with the data [19, 20]. In addition, the coupling to the surface vibrations in the two-neutron problem leads to an effective interaction induced by the exchange of a vibration quantum between the two neutrons. The important role played by the induced interaction has been recognized long ago in neutron and nuclear matter (see, *e.g.*, [21] and refs. therein), and was investigated more recently in finite nuclei, showing that it strongly renormalizes the bare pairing interaction, as found in superfluid nuclei such as ^{120}Sn [22-24], and in light systems such as ^{11}Li [25].

A crucial and challenging feature of the problem we are going to consider lies in the coupling to the continuum. Particle-hole GR are based on excitations that connect a bound state with a state lying in the continuum. Instead, GPVs are built on pairs of particle states both lying in the continuum. Our extended RPA pair addition mode formalism opens the possibility of a robust extrapolation from the low-energy region, where 0^+ addition modes are experimentally identified, to the higher energy region where the experimental situation is still being debated, as in the case of the GPV in ^{14}C.

2. – Extension of particle-particle RPA including coupling to surface modes

We have calculated the $J = 0^+$ pair modes resulting from the addition of a correlated pair of nucleons to a closed-shell nucleus, using a model space that, besides accommodating ground state correlations associated with the backward amplitudes of the pp-RPA solution, includes polarization effects of the surface through PVC to low energy collective vibrational states, leading to the renormalization of single-particle states and giving rise to a phonon-exchange induced pairing interaction.

2˙1. *The A+1 system* . – Our model is the natural extension of a formalism previously adopted in the description of the A+1 system [19]. In that case we obtained an accurate description of the low-energy experimental spectrum dealing with the Hamiltonian

$$H_{1v} = K + V + H_{vib} + H_{PVC}(1) \tag{1}$$

In eq. (1) K and V denote the kinetic energy and the mean field potential,

$$H_{vib} = \sum_{\nu\lambda\mu} \hbar\omega_{\nu\lambda}[\Gamma^{\dagger}_{\nu\lambda\mu}\Gamma_{\nu\lambda\mu} + 1/2], \tag{2}$$

is the harmonic Hamiltonian that describes the surface vibrations of multipolarity $\lambda\mu$ and deformation parameter $\beta_{\nu\lambda}$,

$$H_{PVC} = -\sum_{\nu\lambda\mu} r_1 \frac{dV_1}{dr_1}\alpha_{\nu\lambda\mu}Y_{\lambda\mu}, \tag{3}$$

represents the coupling between the valence particle and the surface vibrations, where the deformation operators $\alpha_{\nu\lambda\mu}$ can be written as [26]:

$$\alpha_{\nu\lambda\mu} = \frac{\beta_{\nu\lambda}}{\sqrt{2\lambda+1}}[\Gamma_{\nu\lambda\mu} + (-1)^{\mu}\Gamma^{\dagger}_{\nu\lambda\mu}]. \tag{4}$$

The index ν runs over vibrational states of the same multipolarity but with different energy and strength and will be dropped in the following, because we will consider only the coupling to the lowest quadrupole vibrational mode ($\lambda = 2, \nu = 1$).

In principle, the mean field and vibrational states should be obtained from a Hartree-Fock + RPA calculation performed with the same interaction. We follow a more empirical route, adopting a "bare" mean field with the usual Saxon-Woods shape (including spin orbit) and using the experimental deformation parameter of low quadrupole vibration states to determine the strength of the PVC. We have shown that the parameters of the A+1 Hamiltonian system for the N=7 isotones ^{10}Li, ^{11}Be, ^{12}B and ^{13}C can be determined requesting that the calculation including PVC leads to an accurate description of the observed levels and spectroscopic factors [20].

2˙2. *The A+2 system.* – The Hamiltonian of the system A+2 is the sum of the Hamiltonians of the two neutrons plus an interaction term that acts between the two valence neutrons:

$$H_{2v} = [K(1) + V(1) + K(2) + V(2)] + H_{PVC} + H_{vib} + V_{int}(1,2) \tag{5}$$

We have used the finite-range Gogny force for the V_{int} pairing interaction between valence neutrons, with cutoffs E_{cut}= 30 MeV and l_{cut}=10 in the single-particle energy e_{nlj} and orbital angular momentum.

The parameters that appear in H_{PVC} and H_{vib} are those already used in the A+1 system, so that the Hamiltonian H_{2v} is completely determined.

In a first natural approach, the Hamiltonian above could be diagonalized based on $J = 0^+$ states, obtained from the tensor product of the basic states of the system A+1. Limiting ourselves to a first-order calculation involving at most one phonon configurations, the basis states for the A+2 system are of four different types:

i) fermion pp $|[nljn'lj]_{0^+} > (e_{nlj},\, e_{n'lj} > E_F)$

ii) fermion pp $\otimes$ boson $|[nljn'l'j']_{2^+} \otimes \Phi^{\nu}_{2^+}]_{0^+}$ $(e_{nlj}, e_{n'l'j'} > E_F)$

iii) fermion ph $\otimes$ boson $|[[nljn'l'j']_{2^+} \otimes \Phi^{\nu}_{2^+}]_{0^+}$ $(e_{nlj} < E_F,\, e_{n'l'j'} > E_F$, or viceversa)

iv) fermion hh $|[nljn'lj]_{0^+} > (e_{nlj}, e_{n'lj} < E_F)$,

where E_F denotes the Fermi energy.

TABLE I. – *Schematic representation of the extended pp-RPA matrix. The sub-matrix connecting pp-states among themselves is the conventional pp-TDA matrix. The sub-matrix connecting pp- and hh- states is the conventional pp-RPA matrix.*

H_{2v}	pp	pp$\otimes b$	ph$\otimes b$	hh
pp	$H_{mf}+V_{int}$	H_{PVC}	H_{PVC}	V_{int}
pp$\otimes$b	H_{PVC}	$H_{mf}+\hbar\omega_\lambda+V_{int}$	0	0
ph$\otimes$b	H_{PVC}	0	$H_{mf}+\hbar\omega_\lambda+V_{int}$	0
hh	V_{int}	0	0	$H_{mf}+V_{int}$

Adopting a monopole pairing interaction and neglecting the term H_{PVC} (*i.e.*, the coupling to states ii) and iii)), we recover the well known pp-RPA (or pp-TDA if also states iv) are neglected) for the study of pair addition modes [27].

The diagonalization of the full matrix, schematically shown in table I (in which $H_{mf} \equiv K(1)+V(1)+K(2)+V(2)$) corresponds to an extended pp-RPA approach, by which a number of relevant effects are taken into account, namely the influence of PVC on the single-particle self-energy (with the corresponding fragmentation of single-particle strength), the coherent superpositions of pairs of neutrons coupled to 0^+ (type-i and type-ii) and the pairing interaction induced by exchange of bosons. It should be noted that self-energy effects arise only for type-i configurations. For the renormalization of type-ii and type-iii configuration two-phonon configurations should be included. For these configurations we use instead the single particle energies already obtained for the odd system A+1, which essentially reproduce the empirical values. Ground state pairing correlations of the core are also taken into account by the inclusion of configurations involving two hole states (type-iv), Also in this sector we will use the empirical single-particle energies, this time for the odd system A-1. These states are coupled to type-i configurations through V_{int} according to the standard pp-RPA formalism [27].

It should be noted that we are building a model for an accurate description of $J=0^+$ pair addition modes, which at zero order are based on type-i configurations. If we were interested in an analogous description of pair removal modes states in the A-2 system, also hh $\otimes$ boson configurations should be considered.

2˙3. *The strength function and the treatment of the continuum.* – The results of pp-RPA calculations can be conveniently presented by means of the pair addition strength function $S(E)$, which is constructed from the obtained discrete eigenvalues and transition amplitudes, E_a and $S(a)$ $(a=1,...N)$ calculated in a box of radius R_{box}. The associated strength is computed from the eigenfunction amplitudes as

$$S(a)=\sum_{pp'} X_{pp'}(a) < pp'|f(r)a_p^\dagger a_{p'}^\dagger|0> + \sum_{hh'} Y_{hh'}(a) < hh'|f(r)a_h a_{h'}|0>, \tag{6}$$

$f(r)$ being a suitable radial form factor. The associated strength function is obtained as the convolution with a Lorentzian function $\mathcal{L}(E;\Gamma)$ with a convenient FWHM Γ,

$$S(E,R_{box})=\sum_a S(a) * \mathcal{L}(E-E_a;\Gamma). \tag{7}$$

The position and relative strength of collective or resonant states are quite independent of the specific $f(r)$, at least if they are concentrated in the volume or on the surface of the nucleus, as occurs with the form factors which are usually adopted. However, the details of $S(E, R_{box})$ also depend on the specific value of the radius R_{box} of the spherical reflecting wall surrounding the nucleus. More precisely, the positive energy part of the strength function show a collection of spikes whose position and height change as a function of R_{box}. Similar problems appear when studying the single-particle strength in the A+1 system [19]. In that case it has been shown that the meaningful result concerning eigenstates in the continuum is not the exact position of the spikes for a given box, but rather their underlying distribution, which can be reconstructed by superposing the different $S_f(E, R_{box})$ functions obtained with N different values of R_{box}, that is taking the average $S(E) = \frac{1}{N} \sum_{i=1,N} S(E, R^i_{box})$. In order for the averaging to be meaningful, the set of adopted boxes must satisfy certain conditions. The lowest single-particle states lying in the continuum should be smaller than the energy of the resonant states under study; the energies of the continuum levels of given angular momentum l, j should cover rather uniformly the excitation energy interval up to a maximum cutoff energy (in other terms, the energies of the levels having n nodes in the smallest box and having $n + 1$ nodes in the largest box should be close); the typical energy change of a level going from one box to the next should be smaller than the width Γ used to generate the strength function (which in turn should be smaller than the physical width of the resonance under study). In practice, we have obtained our results averaging over a the strength functions calculated in a set of 17 boxes of radius R_{box} ranging from 20 fm to 28 fm in step of 0.5 fm. We have validated our procedure carrying out a calculation of the strength distribution associated with the pair addition modes in ^{18}O and finding a good agreement with the results obtained in [10] (see also [11]) by means of a pp-RPA continuum calculation, using the same mean field and pairing interaction.

3. – Results

For ^{14}C we then obtain the strength functions shown in fig. 1, where the results of the PVC calculation are compared with those of unperturbed and of pp-RPA calculations. The results have been obtained with a surface peaked form factor (dU/dr) , but they remain essentially the same using a volume form factor. We have included the effects associated with coupling to the lowest collective quadrupole vibration of the ^{12}C core, characterized by the experimental value of the energy, $E_{2^+} = 4.44$ MeV, and by the deformation parameter $\beta_2 = 0.58$, determined in our previous fit of $N = 7$ isotones [20]. In fig. 2(a) the PVC strength shown in fig. 1 is decomposed into the contributions S_{lj} associated with two-neutron states of given angular momentum {l,j}. In fig. 2(b) we present a similar decomposition, but weighting the pp-amplitudes by the square of the dU/dr form factors. The latter should determine which components are more relevant in a pair transfer experiment.

Going from the unperturbed to the pp-RPA calculation the ^{14}C ground state lowers its energy and increases its strength, confirming its nature of collective pair addition mode. It is interesting to observe that the *sd* peak which in the unperturbed calculation is located around 2 MeV above threshold is lowered by about 4 MeV and its strength is increased. The resulting excitation then lies in the interval $E^* = 10 - 15$ MeV and represents a collective excitation in the shell next to the pair vibrational state. It then displays the characteristics expected for the high-lying resonance (GPV) proposed by Bes and Broglia.

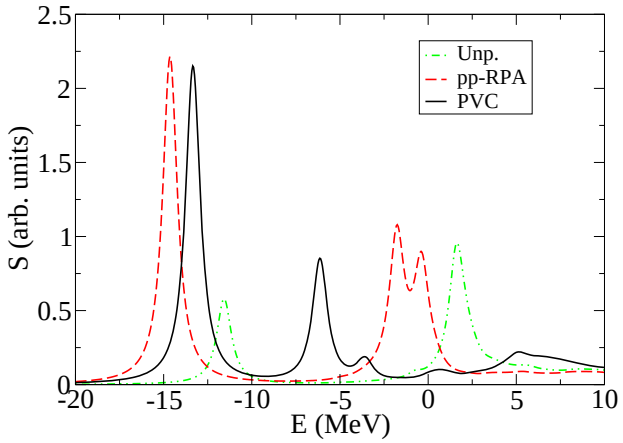

Fig. 1. – Pairing strength functions calculated in ^{14}C with the dU/dr form factor, averaging over the results obtained in 17 boxes having different radii. The unperturbed strength (Unp.) is compared to the results obtained with a pp-RPA calculation with the Gogny force, and with the results obtained including the PVC to the lowest quadrupole vibration in ^{12}C. The strengths have been averaged by a Lorentzian function with FWHM =1 MeV.

Comparing the pp-RPA and the PVC calculation, one can observe that including the PVC the ground state increases its energy slightly, which is due to the Pauli correction self-energy term that acts very specifically on the $1p_{1/2}$ level, which is at the basis of this pair addition mode. The calculated two-neutron separation energy (13.1 MeV) agrees with the experimental result (13.12 MeV), and the ground state wavefunction is dominated by a $(p_{1/2})^2$ component. Furthermore, the PVC shifts and fragments the *sd* bump obtained in the pp-RPA calculation and described above, due to strong polarization effects on the single-particle states $2s_{1/2}$ and $1d_{5/2}$. Most of the strength is shifted at lower energy, producing two peaks well below threshold. A peak is found at $E \approx -6.1$ MeV ($E^* \approx 7$ MeV), with dominant $(d_{5/2})^2$ and $(s_{1/2})^2$ components, and another one at $E \approx -3.5$ MeV ($E^* \approx 9.6$ MeV), also mostly of *sd* character. Experimentally, two excited 0^+ states bound for two-neutron emission are known, that lie at $E^*(0^+_2) = 6.59$ MeV and $E^*(0^+_3) = 9.74$ MeV. The state 0^+_3 lies above the threshold for one-neutron emission ($S_n = 8.18$ MeV, in good agreement with the theoretical calculation) and therefore acquires a width from the coupling to the continuum. This physical width is smaller than the width visible in fig. 1, which rather reflects the value of the width used to smooth the pair strength, equal to 1 MeV. In the recent ^{12}C(^{18}O, ^{16}O)^{14}C transfer experiment, only the 0^+_3 state was weakly populated at $E_{lab} = 84$ MeV, while neither the 0^+_2 nor the 0^+_3 state were identified at $E_{lab} = 275$ MeV [6, 7]. This appears to be in contrast with the present calculation, that shows a rather large strength for the 0^+_2 state. On the other hand, these two excited states have been populated in (t,p) reactions [28] and the ratio of the cross sections relative to the ground state is in fair agreement with our calculated strength function. No conclusion can be drawn before an actual calculation of the two-neutron transfer cross section is carried out.

Considering now the *sd* strength shifted at higher energy, the PVC produces a bump in the continuum, which is located in the excitation region $E^* \approx$ 16-20 MeV, not far from the bump detected in the ^{12}C(^{18}O, ^{16}O)^{14}C experiment. It can be seen from fig. 2(a)

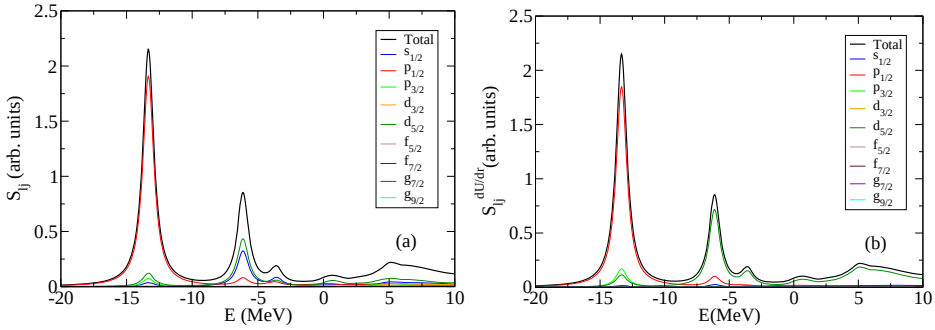

Fig. 2. – (a) The PVC pairing strength function shown in fig. 1 is decomposed into the contributions associated with different angular momenta lj (b) The same, but the various {l,j} contributions are weighted by the square of the corresponding dU/dr form factors.

that several waves contribute to form the bump besides the *sd* orbitals; however, from fig. 2(b) one observes that the $d_{5/2}$ contribution dominates the strength weighted by the form factor squared. This is due to the resonant character of the $d_{5/2}$ orbital. The eigenstate amplitudes of the main peaks associated with this bump show a very strong amplitude (about 60%) in configurations involving the 2^+ surface vibration. Thus, only 40% of the wave function contributes to the monopole pair addition strength function.

4. – Conclusions

We have formulated an extension of the pp-RPA equations to describe 0^+states in the A+2 system, which incorporates polarization effects of the core surface via PVC. The theory has been applied to the pair response in ^{14}C, in which a bump at about 16 MeV excitation energy was recently reported in the experimental study of the ^{12}C(^{18}O, ^{16}O)^{14}C reaction. The parameters of the calculations were determined in a previous analysis of ^{13}C. which has already been well established in the A+1 system. To describe this energy zone, an accurate treatment of the continuum is necessary. We have based our calculations on an average of excitation spectra obtained in spherical boxes of different radii. The adequacy of this procedure has been verified by comparing with continuum pp-RPA calculations in the case of ^{18}O.

The pp-RPA produces a concentration of strength close to threshold, that has the features usually attributed to a GPV. In particular it is based on the *sd* strength, namely it represents a collective excitation in the shell next to the pair vibrational state. The PVC then plays a crucial role, because causes a strong renormalization of the strength and leads to a spectrum which is in reasonable agreement with experiment in the region below threshold. Some structure appears in the continuum in the region where the bump was observed experimentally. Our results show that more than 50% of the wave functions in this region is based on components including the quadrupole excitation of the core. A quantitative comparison between theory and experiment requires a study of the absolute two-nucleon transfer cross section, that can be based on our computed strength function.

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This work stems from a long time collaboration with the late R.A. Broglia. We acknowledge useful discussions with M. Matsuo. F.B. acknowledges the I+D+i project with ref. PID2020-114687GB-I00, funded by MCIN/AEI/10.13039/501100011033.

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