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Colloquia: COMEX7

Axial quadrupole and octupole dynamics in heavy even-even nuclei

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Summary. — A phenomenological geometrical model is proposed to describe collective states in even-even nuclei with quadrupole and octupole deformation. The model is applied for a systematic description of the rotation-vibration dynamics associated with the alternate parity bands observed in the isotopic chains of Ra, Th, U, and Pu nuclei. As a result, the $A = 224$–228 mass region of the Ra and Th nuclei was identified as critical for the transition between dynamic and static octupole deformation, which commences at different spins. The model also predicts a specific spin dependence for the transition probabilities in these critical nuclei.

1. – Introduction

A negative-parity band with levels $L^{\pi} = 1^{-}, 3^{-}, 5^{-}, ...$, lying close to the ground-state band, is considered as an evidence of octupole correlations in even-even nuclei. If the intrinsic states with opposite values of octupole deformation are isolated from each other, then the observed bands are simply constructed on symmetric and antisymmetric superpositions of them. This is the case of static octupole deformation, where the two opposite parity bands merge into a single regular rotational band with alternating parity [1]. More often, the two intrinsic states interact, or are considered as turning points for the so-called octupole vibration [2]. The octupole degrees of freedom are usually superposed with the quadrupole ones. The Bohr geometrical approach is one of the few models capable of treating consistently this situation [3]. Here we review briefly the collective model proposed in ref. [4], which treats in an exact manner the interaction between quadrupole and octupole degrees of freedom arising from the geometry of the deformation space. The main focus however is on the model's description of the yrast alternate parity bands in even-even Ra nuclei and light actinides.

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2. – Theoretical formalism

For quadrupole (β_2) and octupole (β_3) deformation variables constrained to axial symmetry, the collective Hamiltonian can be put into the following form [5]:

$$H=-\sum_{\lambda=2,3}\frac{\hbar^2}{2B_\lambda}\frac{1}{\beta_\lambda^3}\frac{\partial}{\partial\beta_\lambda}\beta_\lambda^3\frac{\partial}{\partial\beta_\lambda}+\frac{\hbar^2\hat{L}^2}{6(B_2\beta_2^2+2B_3\beta_3^2)}+U(\beta_2,\beta_3), \tag{1}$$

where B_2 and B_3 are mass parameters. Its solutions will have $\pm$ parity in accordance to their symmetry or antisymmetry with respect to reflection in the plane perpendicular to the intrinsic symmetry axis. The above Hamiltonian is solved by integrating over the Euler angles and making the change of variables

$$\tilde{\beta}_2=\beta_2\sqrt{\frac{B_2}{B_2+B_3}}=\tilde{\beta}\cos\phi,\ \ \tilde{\beta}_3=\beta_3\sqrt{\frac{B_3}{B_2+B_3}}=\tilde{\beta}\sin\phi. \tag{2}$$

The $\tilde{\beta}>0$ variable is a generalized deformation, while the angle $-\pi/2<\phi<\pi/2$ defines the mixture of quadrupole and octupole deformations [5-7]. If the potential is chosen of the form $\frac{2B}{\hbar^2}U(\beta_2,\beta_3)=u(\tilde{\beta})+w_0/\tilde{\beta}^2$ and the total wave function is factorized as $\Psi_L^{\pm}(\tilde{\beta},\phi)=\psi_L^{\pm}(\tilde{\beta})\chi_L^{\pm}(\phi)$, the problem will be separated into two differential equations. The $\tilde{\beta}$ equation is

$$\left[-\frac{\partial^2}{\partial\tilde{\beta}^2}-\frac{1}{\tilde{\beta}}\frac{\partial}{\partial\tilde{\beta}}+\frac{W_L^{\pm}+w_0}{\tilde{\beta}^2}+u(\tilde{\beta})\right]\psi_L^{\pm}(\tilde{\beta})=\epsilon_L\psi_L^{\pm}(\tilde{\beta}). \tag{3}$$

Two choices are considered for the $\tilde{\beta}$ potential: harmonic oscillator (HO) and infinite square well (ISW) potential. Both equations are then exactly solvable and give parameter-free energies $\epsilon=\frac{2B}{\hbar^2}E$. The separation constant $W_L^{\pm}$ is determined from

$$\left[-\frac{\partial^2}{\partial\phi^2}+u_L(\phi)\right]\chi_L^{\pm}(\phi)=W_L^{\pm}\chi_L^{\pm}(\phi), \tag{4}$$

where

$$u_L(\phi)=\frac{3}{\sin^2 2\phi}+\frac{L(L+1)}{3(1+\sin^2\phi)}, \tag{5}$$

is a potential which naturally occurs solely from the geometry of the deformation space. Its infinite inner barrier is modified as:

$$\tilde{u}_L(\phi)=\begin{cases} v_L(\phi)=-a\phi^2+b,\ |\phi|<|\phi_c|,\\ u_L(\phi),\ |\phi|\geq|\phi_c|,\end{cases} \tag{6}$$

in order to allow an interaction between $\pm\phi$ configurations. The parameters a and b are determined from the continuity conditions at the conjunction point $|\phi_c|$. The $\chi_L^{\pm}$ solution is obtained through the diagonalization in a basis of particle in the box wave functions. Finally, the model is determined only by two parameters, w_0 and the conjunction angle ϕ_c.

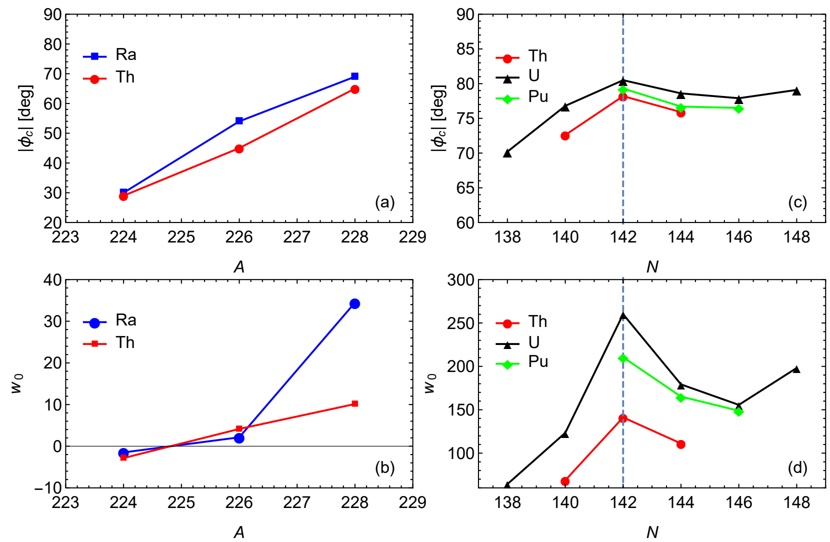

Fig. 1. – Model parameters $|\phi_c|$ (a) and w_0 (b) as a function of mass number A for the critical Ra and Th nuclei. The same $|\phi_c|$ (c) and w_0 (d) parameters are given as a function of neutron number N for the considered octupole vibrational isotopes of Th, U, and Pu.

3. – Results

The model fits showed that ^{224,226,228}Ra(Th) nuclei are better described with an ISW potential. Moreover, these nuclei have lower values for both parameters than the rest of the considered nuclei described with a HO potential. The lower and moderate values of $|\phi_c|$ imply a double well shape of their corresponding $\tilde{u}_L(\phi)$ potential which is realized starting from a certain spin. This can be understood as a dynamical transition from an octupole vibration to an increasingly stabilized octupole deformation. This transition undergoes at different spins and through different quantum phases such as single well vibration, double well vibration with or without tunneling and the stable quadrupole-octupole phase where the two $\pm\phi(\pm\beta_3)$ configurations no longer interact. An example of such a behaviour is represented by the chiral and wobbling bands of triaxial nuclei [11-13]. The obtained values of both parameters shown in fig. 1 have a regular evolution with nucleon numbers. The critical Ra and Th nuclei with $A = 224, 226$ and 228 have similar increasing behaviour for both $|\phi_c|$ and w_0 as a function of mass number A. The heavier Th isotopes and those of U and Pu have higher values of $|\phi_c|$ which are associated with octupole vibration. For these nuclei, the two parameters have an almost scalable dependence on the neutron number with a maximum at $N = 142$.

The transition probabilities are more sensible to structural changes and therefore we found that the dynamical transition identified in the critical nuclei is transposed into a specific spin dependence of the intra-band $E2$ transition probabilities. Indeed, the $E2$ transition rates for the lightest Ra and Th nuclei show a moderate increase for few angular momentum states and then decrease asymptotically towards a saturation value. As can be seen from fig. 2, this behaviour is quite different from the case of the octupole vibrational nuclei, where all states are of the same nature and the $E2$ rates increase as expected from simple rotational excitation.

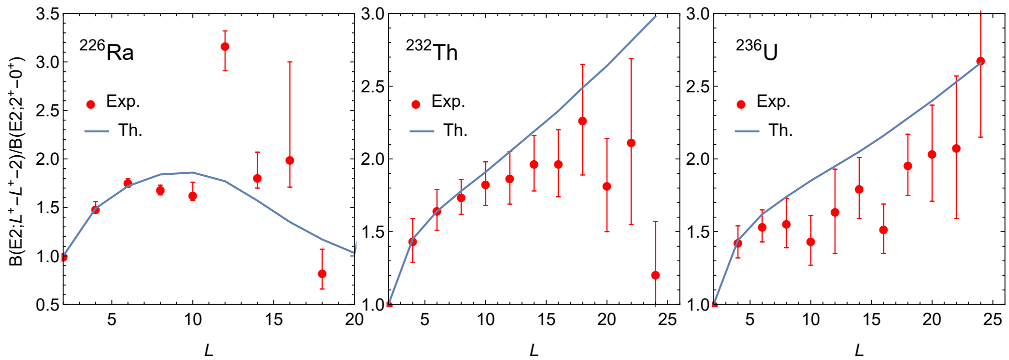

Fig. 2. – Theoretical and experimentally available $E2$ transition probabilities in units of $B(E2; 2^+ \to 0^+)$ for the critical nucleus ^{226}Ra [8] and two octupole vibrational nuclei ^{232}Th [9] and ^{236}U [10].

4. – Conclusions

Using a quadrupole-octupole axially symmetric collective model, we identified a critical region with $A = 224, 226, 228$ in the Ra and Th nuclei, where a shape phase transition commences between stable and dynamic octupole deformation. The model is also successful in reproducing the alternate parity bands of U and Pu nuclei with a strong octupole vibration character. The obtained model parameters have a very smooth evolution with neutron and mass number, offering thus extrapolation opportunities. A specific spin dependence of the intra-band $E2$ transition probabilities is forwarded as a possible signature for the critical nuclei.

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