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The nuclear Josephson effect and γ-ray emission

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Summary. — We discuss the analogy between the transfer of nucleon pairs in reactions involving superfluid nuclei and the transfer of electron pairs through Josephson junctions, based on measured and theoretical probabilities for one- and two-neutron transfer in the reactions ^{116}Sn(^{60}Ni, ^{61}Ni)^{115}Sn and ^{116}Sn(^{60}Ni, ^{62}Ni)^{114}Sn. We consider the possibility of the emission of γ-radiation, analogous to the emission of microwave radiation in the Josephson effect. We implement the quantum mechanical description of the coupling of the electric dipole associated with the (2n)-transfer reaction process, establishing the connection between the dynamics of the collision process and the number and energy dependence of the emitted photons. We present predictions of the angular distributions, analyzing powers and strength functions of the emitted γ-radiation.

1. – The nuclear Josephson effect

Nucleon pair transfer processes between superfluid nuclei in heavy ion reactions have been previously considered [1-6] as possible analogues of the transfer of electron Cooper pairs through Josephson junctions [7, 8]. We have recently presented a new viewpoint on the subject [9-11], predicting the emission of γ-radiation, analogous to the emission of microwave radiation occurring in the Josephson effect (see, *e.g.*, [12] and references therein; see also [13]). We summarise here the main results obtained from the analysis of the reaction ^{116}Sn(^{60}Ni,^{61} Ni)^{115}Sn and ^{116}Sn(^{60}Ni,^{62} Ni)^{114}Sn. The absolute differential cross sections were measured at various bombarding energies at the Laboratori Nazionali di Legnaro [14, 15], where a dedicated measurement aimed at the detection of the predicted radiation has been recently carried out [16].

Let us consider a two-nucleon transfer process occurring in a heavy ion collision between superfluid nuclei

$$a(= b + 2) + A \rightarrow b + B(= A + 2), \tag{1}$$

characterised by a Q-value Q_{2n}. Treating each nucleus as the intrinsic state of a pairing rotational band, one can write

$$\mu_a = \hbar\dot{\phi}_a \ ; \ \mu_B = \hbar\dot{\phi}_B \tag{2}$$

where μ stands for the Fermi energy (chemical potential) and $\dot{\phi}$ denotes the rotational frequency in gauge space. The matrix elements of the transfer Hamiltonian between entrance and exit channels α, β will be multiplied by the phase $i\frac{E_\beta - E_\alpha}{\hbar}t$, where

$$\frac{E_\beta - E_\alpha}{\hbar} = \frac{Q_{2n}}{\hbar} \approx 2\frac{\mu_B - \mu_a}{\hbar} = 2(\dot{\phi}_B - \dot{\phi}_a). \tag{3}$$

One can make a parallel between the nucleon transfer and the transfer of electrons through a weak link between two identical metallic superconductors, placed at the left (L) and at the right (R) of an insulating barrier. In the latter case, the angular frequency ω_J associated with the alternating current (ac) Josephson effect obeys the relation [8]

$$2\frac{\mu_R - \mu_L}{\hbar} = \frac{2eV}{\hbar} = \omega_J, \tag{4}$$

where V is the direct current bias applied to the barrier. The tunnelling of electron pairs is accompanied by the emission of microwave radiation of the frequency ω_J. We will consider the possibility of the emission of γ-radiation across the Josephson-like junction transiently established in heavy ion collisions (collision time $\tau_{coll} \approx 10^{-21}$ s) between two superfluid nuclei at energies below the Coulomb barrier.

2. – The inspiring experiment

A recent advance in the subject of nuclear superfluidity was accomplished through the experimental study and the theoretical analysis of one- and two-neutron transfer reactions between superfluid nuclei, enabled by the use of magnetic and gamma-ray spectrometers, and carried out in inverse [5] and direct [6] kinematics:

$$^{116}\text{Sn}(^{60}\text{Ni},^{61}\text{Ni})^{115}\text{Sn} \ (Q_{1n} = -1.74 \text{ MeV}) \tag{5}$$

$$^{116}\text{Sn}(^{60}\text{Ni},^{62}\text{Ni})^{114}\text{Sn} \ (Q_{2n} = 1.31 \text{ MeV}) \tag{6}$$

The reactions were carried out at twelve c.m. energies in the range 140.60 MeV - 167.95 MeV. That is, from energies above the Coulomb barrier (E_B = 157.60 MeV) to well below it. Absolute differential cross sections were measured at $\theta_{cm} = 140^o$ for one- and two-particle transfer reactions. These reactions can be reliably studied within a semi-classical approximation, that accurately reproduces the measured cross sections. The spectroscopic amplitudes are calculated making use of a monopole pairing interaction with a pairing constant $G = 25/A$ MeV. While the two-nucleon transfer reaction σ_2 is calculated for the transition between the ground states of the two superfluid nuclei, the cross section for one-nucleon transfer σ_1 is obtained summing the incoherent contributions from a number of excited states below 2 MeV, in keeping with experimental evidence about the inclusive character of the reaction in this channel. In the semiclassical approach, it is possible to plot the cross sections as a function of the distance of closest approach $D_0(E)$, associated with the angle $\theta_{cm} = 140^o$, for energies below the Coulomb

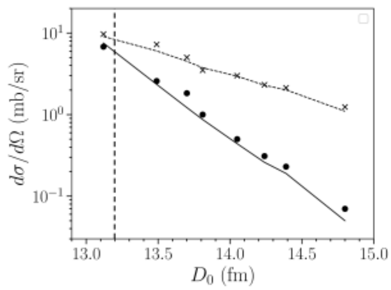

Fig. 1. – Absolute differential cross sections at $\theta_{cm} = 140^o$ associated with the reaction ^{116}Sn+^{60}Ni$\rightarrow$^{115}Sn+^{61}Ni, $d\sigma_{1n}/d\Omega|_{\theta_{cm}=140^\circ}$ (experimental values are shown by x's, theoretical values by the dashed line), and ^{116}Sn+^{60}Ni$\rightarrow$^{114}Sn+^{62}Ni, $d\sigma_{2n}/d\Omega|_{\theta_{cm}=140^\circ}$ (exp., solid dots, th., continuous line), displayed as a function of the distance of closest approach D_0, for eight of the twelve bombarding energies for which the experiments were carried out. The vertical dotted line at $D_0 = 13.2$ fm indicates the distance of closest approach at the Coulomb barrier, $E_B = 157.60$ MeV.

barrier, at which the imaginary part of the potential plays a minor role. The theoretical and experimental cross sections are shown in this way in fig. 1, in which we display the results of quantal DWBA calculations, which confirm the semiclassical calculations reported in [14]. Remarkably, σ_2 increases exponentially, until it becomes of the same order as σ_1 ($\sigma_2/\sigma_1 \approx 0.5$) for $E_{cm} \approx 154$ MeV, corresponding to $D_0 \approx 13.5$ fm.

3. – Depairing critical distance/velocity

The fact that the theoretical calculations reproduce the experimental data is certainly satisfactory, but nevertheless one would like to have a simple qualitative explanation of the dependence of the ratio σ_2/σ_1 on the bombarding energy. The velocity of the Cooper pair with respect to the lattice plays a crucial role in determining the strength of pairing correlations in metals. In fact, pairing is quenched when such a velocity exceeds a critical velocity, given by $mv_{cr} = \hbar/\xi_P$, where ξ_P denotes the Pippard coherence length, namely the mean square radius of the Cooper pair, usually estimated by

$$\xi_P = \frac{\hbar v_F}{\pi \Delta}, \tag{7}$$

in terms of the Fermi velocity and of the paring gap. It can be argued that the velocity of the transferred neutrons plays an analogous role in the nuclear case. Taking into account that fact that during the collision two neutrons have to be transferred, such velocity can be estimated by the ratio $v_{tr} = 2D_{sep}/\tau_{coll}$, where $D_{sep} = D_0 - R_a - R_A$ is the distance between the two nuclear surfaces, while the collision time τ_{coll} can be estimated by $\tau_{coll} \approx (R_a + R_A)/v_{rel}$ [18], where v_{rel} is the relative velocity. The quantity v_{tr} decreases as the bombarding energy increases, because $D_0 \propto 1/E$ while $\tau_{coll} \propto 1/\sqrt{E}$. One can then define the critical velocity v_{cr} as the value of the transfer velocity associated with the bombarding energy at which the probabilities for one- and two-neutron transfer become equal. One can also introduce an effective coherence length ξ_{cr}, satisfying the relation $mv_{cr} = \hbar/\xi_{cr}$. According to the Heisenberg principle, ξ_{cr} provides an estimate of the width of the relative velocity distribution of the Cooper pair. If v_{tr} is lower than the

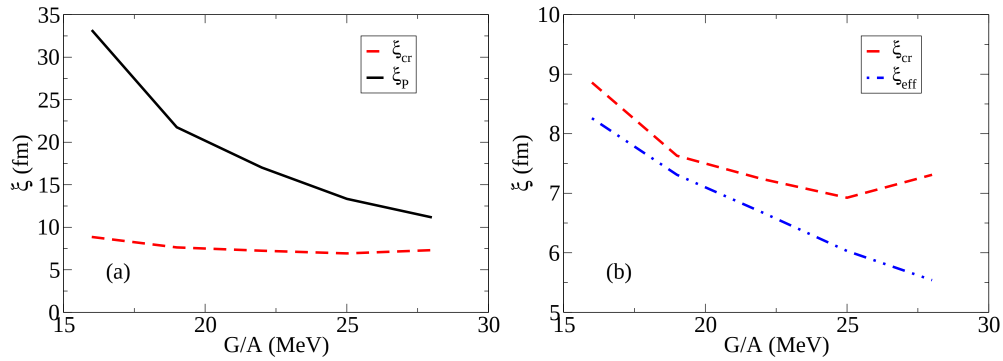

Fig. 2. – The coherence length (dashed line) $\xi_{cr} = \hbar/mv_{cr}$, calculated from the critical velocity for different values of the pairing constant G, is compared with Pippard's estimate ξ_P (7) (a) and with the effective coherence length given by eq. (8), using $L_{mf} \approx R_a + R_A \approx 11$ fm (b). Note the different scale in the two figures.

critical value the structure of the Cooper pair will be remain unaffected in the sequential transfer process. When v_{tr} is larger than $\hbar/m\xi_{cr}$, the correlated Cooper pair is broken and the two-nucleon transfer process is hindered.

An independent estimate of the coherence length in the present situation is a delicate issue. On the one hand, the extension of Cooper pairs in infinite nuclear matter is well described by eq. (7) [19,20]. On the other hand, the situation is quite different in atomic nuclei, where the extension of the Cooper pair is mostly determined by the confining mean field [20], rather than by the intensity of pairing correlations, and the coherence length is of the order of the nuclear radius. We can then estimate the coherence length by introducing an effective coherence length ξ_{eff}, obtained by interpolating between the two extreme situations (isolated nucleus and infinite matter):

$$\frac{1}{\xi_{eff}} \approx \frac{1}{\xi_P} + \frac{1}{L_{mf}}, \tag{8}$$

where L_{mf} represents the size of the mean field, that we can take equal to $R_A + R_a$ in the present case of two colliding nuclei. In the case of the isolated nucleus, $L_{mf} = R < \xi_P$ and $\xi_{eff} \approx R$. In the case of infinite matter, $L_{mf} \to \infty$ and ξ_{eff} reduces to ξ_P.

In order to assess the coherence of this picture we have carried out a set of DWBA calculations of the transfer cross sections σ_1 and σ_2 for the reaction ^{116}Sn(^{60}Ni,^{62} Ni)^{114}Sn as a function of the strength of the pairing interaction G in the two superfluid colliding nuclei. For each value of G, we have determined the distance of closest approach D_{cr} at which $\sigma_1 = \sigma_2$ and the corresponding value of the critical transfer velocity $v_{cr} = 2D_{sep}/\tau_{coll}$. In fig. 2 we compare the associated coherence length $\xi_{cr} = \hbar/mv_{cr}$ with the Pippard coherence length ξ_P (eq.(7)) (panel (a)) and with the effective coherence length given by eq. (8) (panel (b)). We observe that the latter provides a good approximation of ξ_{cr}.

4. – The prediction

In keeping with the fact that the bombarding energy associated with D_0=13.5 fm is 3.9 MeV/A, that is an order of magnitude smaller than the Fermi energy, one can expect

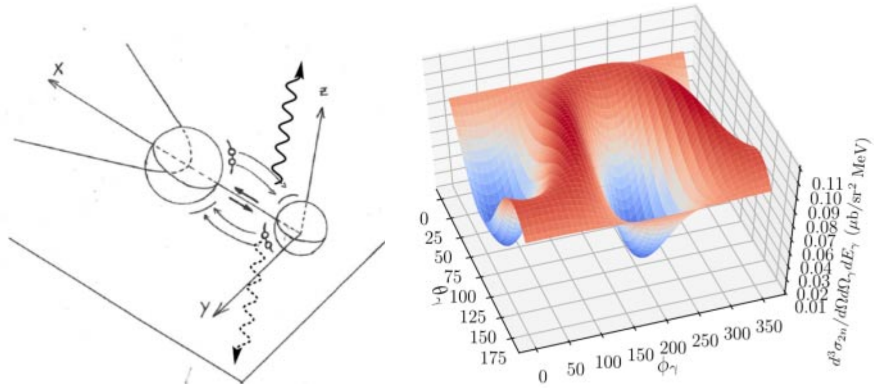

Fig. 3. – (Left) Schematic representation of the quasielastic process in which a Cooper pair is transferred back and forth between the two superfluid nuclei ^{116}Sn and ^{60}Ni. An oscillating dipole with frequency $\nu = Q_{2n}/\hbar$ is established because the transferred neutrons carry an effective charge, leading to the isotropic emission of γ-rays perpendicular to the reaction plane. (Right) Angular radiation pattern for $E_\gamma = 4$ MeV, $E_{cm} = 154.26$ MeV and $\theta_{cm} = 140^\circ$ as a function of θ_γ and ϕ_γ. Colours are a guide for the eye.

that there can be time for the nuclear Cooper pair to be transferred back and forth more than once between target and projectile. That is, for more than one cycle of the quasi-elastic process ^{116}Sn + ^{60}Ni $\rightarrow$ ^{114}Sn+^{62}Ni $\rightarrow$ ^{116}Sn + ^{60}Ni. The neutron Cooper pair carries an effective charge $2e_{eff} = -e \times 2(78/176) = -e \times 0.89$ and consequently, the nuclear junction can be viewed as biased by a potential $V = Q_{2n}/(2e_{eff}) = 1.469$ MeV, and the process can be considered as an alternating nuclear Josephson supercurrent of frequency $\nu_J = 2e_{eff} \times V/\hbar = Q_{2n}/\hbar$. As such, and in keeping with the fact that the collision takes place at a few MeV below the Coulomb barrier, where tunneling proceeds essentially free of dissipation, one expects the process to be a source of ZHz photons emitted perpendicular to the reaction plane (see fig. 3, left panel). In fig. 4, the γ-strength function is shown. This quantity has been calculated using the same DWBA approach adopted to calculate the two-neutron transfer cross section, but introducing the

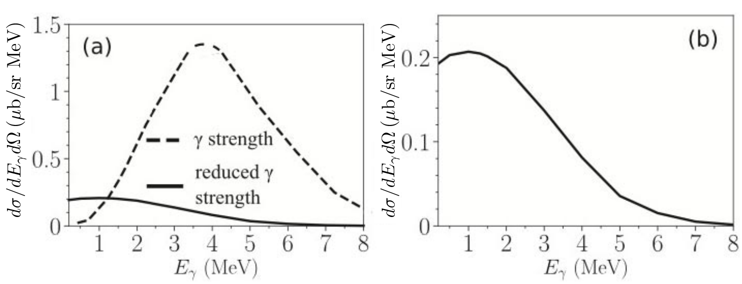

Fig. 4. – (a) Double differential cross section for γ emission at $\theta_{cm} = 140^o$ as a function of the energy of the emitted γ-ray (dashed curve). The reduced strength (continuous curve) has been obtained by dividing out from $d\sigma/dE_\gamma$ the phase-space factor E_γ^2, and multiplying it by the corresponding quantity with $E_\gamma = 1.307$ (MeV). The reduced strength is shown in (b) on a different scale, so that the width and the position of the centroid are more apparent (from [9]).

dipole moment $D_\gamma = 2e_{eff}\sqrt{4\pi/3}Y_{1m}(\hat{r})$ in the T-matrix [9]. Remarkably, the reduced strength function (that is, $d\sigma/dE_\gamma d\Omega$ without the phase space factor E_γ^2) is peaked at 1.2 MeV, which almost coincides with the Q_{2n} value, as expected for the Josephson radiation. An experimental measure of the predicted strength distribution would thus provide a verification of the nuclear analogue of an alternating Josephson current. The predicted angular radiation pattern is shown in fig. 3 (right panel).

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