IL NUOVO CIMENTO **47 C** (2024) 31
DOI 10.1393/ncc/i2024-24031-2

Colloquia: COMEX7

Collective wavefunction of Yrast states in ^{50}Cr

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received 31 October 2023

Summary. — In the generator coordinate method the wavefunctions are defined with respect to reference states that can indicate different shapes and deformations, which are examples of collective generator coordinates. In this work, we study the collective wavefunctions of Yrast states up to the terminating state of ^{50}Cr using a recently introduced framework to calculate projected states of spins up to $I = 14$ based on effective Hamiltonians and a 5 dimensional collective coordinate space.

1. – Introduction

In multireference models the wavefunctions are defined with respect to different overlapping reference states. These methods have the advantage of considering strong correlations non–perturbatively, hence provide a natural framework to describe phenomena like deformations. Therefore, multireference methods are used both in nuclear physics and in other fields. The generator coordinate method (GCM) is a multireference model which generates references from collective generator coordinates, like axial deformation and triaxiality β, γ, pairing strengths g_p, g_n and cranking frequency ω in the case of this work [1]. GCM has been used to model nuclei for a variety of observables and use cases, describing both light and heavy deformed nuclei with great precision [2, 3].

Recently, we have developed a method based on generator coordinates to calculate the states of different nuclei from superheavy isotopes like ^{292}Lv to the light ^{24}Mg [1,4,5], with extensions to odd particle transfer spectroscopy [6] and reaction models [7]. This proceeding testifies the progress in representing the states related to this model using collective wavefunctions and presents the ground state of $^{48-52}$Cr isotopes and the Yrast states of ^{50}Cr.

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2. – Method

The method used in this work based on GCM has been developed to compute states of nuclei, including rotational states of deformed nuclei at high spin [1]. In order to use the multireference GCM introduced above to calculate the states of heavy and superheavy nuclei [4], is beneficial to consider an Hamiltonian which is based on operators that depend only on two indexes (rank 2 tensors), constructing the two–body interaction terms as separable in the product of these simpler operators. This operation is without loss of generality, as in [8].

For the present work, the Hamiltonian is chosen as,

$$\hat{H} = \hat{H}_0 + \hat{H}_Q + \hat{H}_P, \tag{1}$$

that is, H includes a spherical mean field $H_0 = \sum_i e_i a_i^\dagger a_i + E_0$, and two separable terms for pairing and a quadrupole interaction inducing deformation. The pairing is the seniority pairing between time reversal states $H_P = -G \sum_{ijkl} P_{ij} P_{kl} a_i^\dagger a_j^\dagger a_k a_l$, the constant strength G is calculated according to the uniform spectra method [9]. The H_Q term is a quadrupole–quadrupole interaction defined as $-\frac{1}{4}\chi \sum_{ijkl} \sum_\mu [Q_{ij}^{2\mu} Q_{kl}^{2\mu*} - Q_{ik}^{2\mu} Q_{jl}^{2\mu*}] a_i^\dagger a_l^\dagger a_k a_j$. The operators are taken from the modified quadrupole force of [10]. The strength parameter χ is adjusted to reproduce axially constrained calculations of a Skyrme functional, in this case SLy4, defining the Sly4-H effective Hamiltonian for a specific nucleus. This effective Hamiltonian reproduces binding energy and deformation properties of energy density functional, without the shortcoming in terms of inconsistencies when the functional is used as an interaction in beyond mean field calculations. For more information cf. [1], and the comparison of Skyrme functionals in [5].

The reference states that are used as overcomplete basis to solve the effective Hamiltonian (1) are generated as the HFB vacua $|\Phi\rangle$ with a variation constrained over a set of generator coordinates $\{|\Phi(\beta, \gamma, g_n, g_p, \omega)\rangle\} \equiv \{|\Phi(i)\rangle\}$. This choice accounts for the most relevant collective degrees of freedom in the form of vibrations, rotations and pairing.

The GCM solution is finally obtained solving the Hill Wheeler equation, which takes into account the non–zero overlap between the reference states, $Hh = EOh$, with h the eigenvector of coefficients of the solutions, and the overlap matrix $O_{ij} = \langle\Phi(i)| P^N P^Z P^I_{MK} |\Phi(j)\rangle$, where P^N, P^Z, and P^I_{MK} are the projection operators. The projection is necessary to enforce the final states are eigenstates of neutrons N, protons Z numbers and total angular momentum I, despite that the reference states might not.

The solutions are then written as a sum of different projected reference states, $|\Psi_I^A(a)\rangle = \sum_{iK} h^{A,a}_{IMK}(i) P^N P^Z P^I_{MK} |\Phi(i)\rangle$, where the state a of angular momentum I and particle number A is obtained summing over the reference states $|\Phi(i)\rangle$. However, the Hill-Wheeler coefficients $h^{A,a}_{IMK}(i)$ cannot be directly interpreted as amplitudes relative to a given collective coordinate i since the reference states are non–orthogonal. The weights representing probabilities of the state being in a certain collective coordinate (i) can be derived taking into consideration the overlap operator O as,

$$g = O^{1/2} h, \qquad \text{that is,} \qquad g(i) = \sum_j O^{1/2}(i,j) h(j), \tag{2}$$

where $O^{1/2}$ is the square root of the Hermitian positive definite overlap matrix and g are referred to as the collective weights of the collective wavefunction [11].

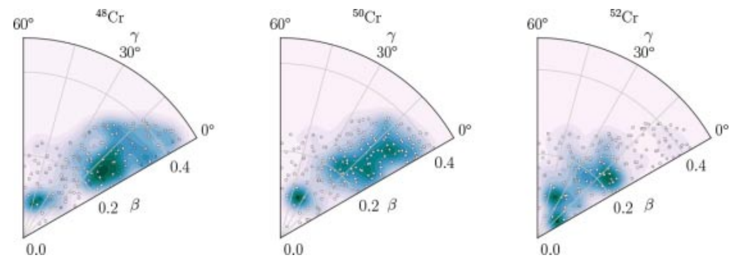

Fig. 1. – Collective wavefunctions (2) of ^{48}Cr, ^{50}Cr, and ^{52}Cr represented in the β, γ plane, where β is the amount of deformation and γ is the axis of deformation with $\gamma = 0$ corresponding to prolate shape. The coordinates of the reference states i are indicated on the landscape as points, with more intense colors corresponding to higher values. The contour are obtained by convoluting these points with Gaussians of $\sigma = 0.02$.

3. – Collective wavefunction of the Yrast line

We can now use eq. (2) to analyse the states of different isotopes and in particular the lowest energy states of different angular momenta, called Yrast states. For this scope, the isotopes of $^{48-52}$Cr are particularly interesting since they represent a transition between the $N = Z$ half-occupied $f_{7/2}$ shell of ^{48}Cr, that is often taken as an example of rotational band structure [12], to neutron closed shell of ^{52}Cr.

The calculations of these isotopes have been executed with the parameters in [1]. The five dimensional collective landscape is constructed with a 192 points uniform sampling of β, γ for those points in which the HFB energy is within 12 MeV of the minimum point, while g_n, g_p and ω collective coordinates are randomly sampled. This sampling is projected on one sixth of the β, γ plane to more clearly analyse the intrinsic shape of the nucleus. In figs. 1 is shown the initial result for the representation of the wavefunctions of the ground states of these nuclei. The corresponding spectra can be found in [1].

In fig. 2 is possible to appreciate how the collective wavefunction of ^{50}Cr evolves along the Yrast states from the ground state to the the terminating state. The terminating state is the highest total spin that is possible to make by aligning the angular momenta of all particles in the valence shell. In the case of ^{50}Cr 4 protons and 6 neutrons can align in the $f_{7/2}$ shell, giving $I = 14$.

The ground state wavefunction of ^{50}Cr shows some a distribution located around the mien field minimum of $\beta \approx 0.2$ and $\gamma \approx 0°$. Nevertheless it is interesting to follow the development of the wavefunction as angular momentum is increased. At spins $2 \leq I \leq 6$ the wavefunction is better represented by a deformed and rotating reference state and therefore the collective wavefunction assumes a more definite prolate character. After $I = 8$ the wavefunction starts reducing its deformation, distributing more widely across references of different shapes. Finally, the terminating state at $I = 14$ that is well described by a spherical reference states confirming the representation of the terminating state as particles aligned in the spherical shell to form the corresponding angular momentum.

4. – Conclusions

The generator coordinate method can enable the detailed study of a variety of different phenomena and provides precise account of several physical degrees of freedom

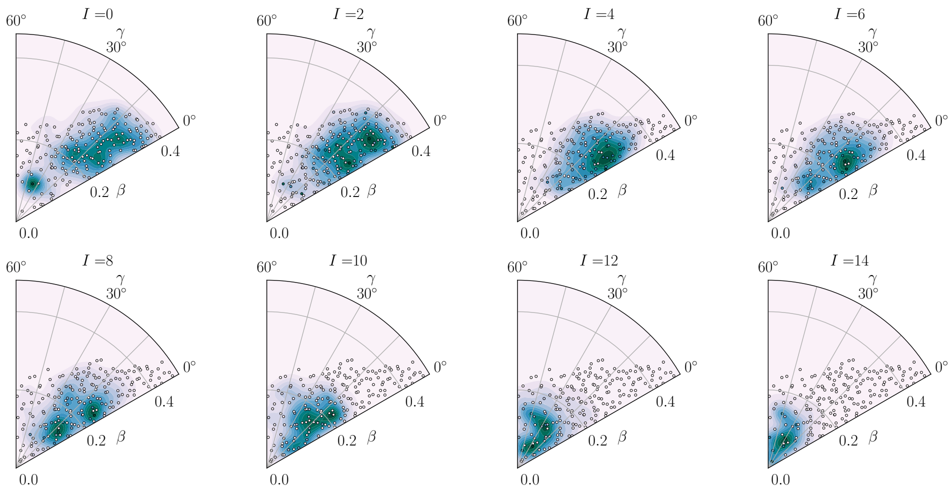

Fig. 2. – Collective wavefunctions (2) for the even angular momentum Yrast states in ^{50}Cr from $I = 0$ to $I = 14$, as in fig. 1.

that are relevant for the nuclear case. Among that, the study of detailed wavefunction of rotational states. Using the collective wavefunction is possible to give an intuitive representation of the nuclear wavefunction without sacrificing rigour in the treatment of the nucleus as a many–body system.

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We were supported by Swedish Research Council 2020-03721, Crafoord fundation, Knut & Alice Wallenberg fundation (KAW 2015.0021), Lunarc computing facility and computational resources provided by the National Academic Infrastructure for Supercomputing in Sweden (NAISS) at Tetralith partially funded by the Swedish Research Council through grant agreement no. 2022-06725

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