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New developments in modeling Heavy Ion Double Charge Exchange reactions

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Summary. — Heavy Ion Double Charge Exchange (HIDCE) nuclear reactions are described in terms of sequential meson-exchange, *i.e.*, as a sequence of two independent single charge exchange reactions (Double Single Charge Exchange, DSCE). HIDCE reactions may represent a powerful tool to get information on the Nuclear Matrix Element (NME) describing double beta decays, thanks to the analogies found between the structures of DSCE nuclear response tensors and the nuclear matrix elements of double beta decays. Heavy Ion DSCE reactions are treated within second order DWBA framework; this allows to express DSCE reaction amplitude as a superposition of distortion factors, accounting for initial and final state ion-ion elastic interactions, and nuclear matrix elements. Explicit expressions for the latter are derived using QRPA theory. Reduction schemes for treating DSCE transition form factors are discussed, allowing to get disentangled projectile and target NMEs within DSCE cross section expression. Calculations are performed for some of the reactions studied within the NUMEN collaboration, such as ^{40}Ca (^{18}O, ^{18}Ne) ^{40}Ar, ^{76}Se (^{18}O, ^{18}Ne) ^{76}Ge and ^{76}Ge(^{20}Ne,^{20}O)^{76}Se, at a beam energy of 15.3 AMeV.

1. – Introduction

Heavy Ion induced Double Charge Exchange (HIDCE) reactions are attracting increasing interest during the last decades, because they allow to probe a wide range of frontiers physical phoenomena, such as the drip-line nuclei and the theorized Double Gamow-Teller Giant Resonance (DGTGR) and neutrinoless double beta decay ($0\nu\beta\beta$). In particular, the latter process would represent a telltale of physics beyond the Standard Model and it turns out to share some peculiar aspects with HIDCE reactions, such as the

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same kind of spin-isospin operator, accounting for charge changing transition [1]. Relying on these analogies, the NUMEN collaboration, at LNS in Catania, aims at looking for constraints on the nuclear matrix elements (NMEs), involved in $0\nu\beta\beta$ decay observables, from measurements of HIDCE cross section [1].

HIDCE reactions can proceed via two main reaction mechanisms: the exchange of charged mesons ("direct" DCE) or sequential multi-nucleon transfers feeding DCE ("transfer-DCE") [1, 2]. This proceeding focuses on the former reaction mechanism, which is the one allowing to recover analogies between HIDCE reactions and $0\nu\beta\beta$ decay. The feasibility of this kind of studies is supported by the existence of a linear correlation between the NMEs of these two processes [3, 4]. Direct HIDCE reactions are described in terms of a sequence of two uncorrelated single charge exchange (SCE) reactions (each one induced by charged-meson exchange), *i.e.*, as a two-step process (Double Single Charge Exchange, DSCE), but correlations between the two SCE transitions can also be accounted for, thus leading to an effective-one-step transition [5]. Here, the DSCE reaction mechanism is described, which allows to recover close analogies between DSCE and $2\nu\beta\beta$ decay NMEs [6]. DSCE reaction mechanism could also allow to look for possible connections to $0\nu\beta\beta$ NME, being the nuclear states involved in the off-shell intermediate channel the same in both processes.

2. – Formalism and results

DSCE reaction cross section is described within the 2nd order DWBA, where the DSCE transition matrix element (TME) can be expressed as the convolution of the two SCE TMEs, and the propagator G_γ.

A first step toward disentangling projectile and target DSCE NME, within DSCE cross section expression, is to switch from the standard description of two consecutive SCE reactions (t-channel representation) [6] to a representation scheme that highlights the two-step evolution of each interacting nucleus (s-channel representation) [7]; this can be achieved simply by means of a rotation in angular momentum space. The propagator and the nuclear states populated within the intermediated reaction channel are treated within the Closure approximation and the bi-orthogonality property of the distorted waves, involved in the intermediate channel, is exploited. The unitary transformation in angular momentum space and the above treatment of the intermediate reaction channel allow to write the two-step TME as

$$\begin{aligned}\mathcal{M}^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) =&\overline{G}_\gamma \int d^3\frac{\xi}{2}\int d^3\eta\, \tilde{\rho}_{1P}\left(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2}\right)\tilde{\rho}^*_{2P}\left(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}\right)\tilde{\rho}_{1T}\left(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2}\right)\tilde{\rho}^*_{2T}\left(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}\right)\\ &\tilde{V}^{(SCE)}_{NN}\left(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2}\right)\tilde{V}^{(SCE)}_{NN}\left(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}\right)N_{\alpha\beta}(\boldsymbol{\eta})\end{aligned} \tag{1}$$

where η and ξ are the sum and the difference of the linear momenta transfer involved in the two sequential SCE reactions, respectively, $\overline{G}_\gamma$ is the propagator (treated as a constant), $N_{\alpha\beta}(\boldsymbol{\eta})$ is the DSCE distortion factor, $\tilde{V}^{(SCE)}_{NN}$ is the Fourier-Bessel transform of NN interaction potential and $\tilde{\rho}_{ij}$, $(i = 1, 2\,, \quad j = P, T)$ are the Fourier-Bessel transforms of projectile (P pedice) and target (T pedice) one-body transition densities, accounting for first (i=1) and second (i=2) SCE transition. One can note that eq. (1) shows still entangled projectile and target transition densities. In order to gain separate information on these transition densities, *i.e.*, DSCE NMEs of the two interacting nuclei, two approximations are used:

- average-ρ approximation

$$\tilde{\rho}_{1P}\left(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2}\right)\tilde{\rho}^{*}_{2P}\left(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}\right) \rightarrow \frac{(2\pi)^3}{V_\xi}\int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}}\rho_{1P}(\mathbf{r})\rho^{*}_{2P}(\mathbf{r}) \equiv \tilde{\rho}^{2BTD}_{P_{av}}(\boldsymbol{\eta}) \tag{2}$$

where the product of the Fourier-Bessel transform of first and second step SCE one-body transition densities (OBTDs) is replaced by their average over ξ; V_ξ is a normalization volume allowing to recover the correct dimensions of these 2BTDs.

- collinear approximation

$$\tilde{\rho}_{1P}\left(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2}\right)\tilde{\rho}^{*}_{2P}\left(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}\right) \rightarrow \tilde{\rho}_{1P}\left(\frac{\boldsymbol{\eta}}{2}\right)\tilde{\rho}^{*}_{2P}\left(\frac{\boldsymbol{\eta}}{2}\right) \equiv \tilde{\rho}^{2BTD}_{P_{coll}}(\boldsymbol{\eta}) \tag{3}$$

where only the contribution from $\xi = 0$, is retained both in first- and second-step SCE OBTDs, which means to assume equal momenta transfers in the two SCE reactions.

In both cases, the remaining integral over ξ, in eq. (1) allows to find a quite simple expression of the DSCE NN interaction potential, $V^{DSCE}_{NN}(\boldsymbol{\eta}) \equiv (2\pi)^3 \int d^3r\, |V^{(SCE)}_{NN}(\mathbf{r})|^2 e^{i\boldsymbol{\eta}\cdot\mathbf{r}}$.

Eventually, a single-step like expression for the DSCE TME emerges,

$$\mathcal{M}^{(2)}_{\alpha\beta} = \overline{G}_\gamma \int d^3\eta\, \tilde{\rho}^{2BTD}_{P_x}(\boldsymbol{\eta})\tilde{\rho}^{2BTD}_{T_x}(\boldsymbol{\eta})V^{DSCE}_{NN}(\boldsymbol{\eta})N_{\alpha\beta}(\boldsymbol{\eta}) \tag{4}$$

where $x = av$ or *coll* according to average-ρ or collinear approximation choice, respectively. TME expression of eq. (4) can be factorised for small momentum transfer values, thus leading in turn to a factorized DSCE cross section expression (see ref. [8] for more details), representing a first step toward the possibility of carrying out data-driven information on DCE NMEs.

To asses the quality of the approximations made within s-channel representation, a comparison with t-channel calculations is performed, for some of the nuclear reactions studied within the NUMEN collaboration: ^{40}Ca(^{18}O, ^{18}Ne$_{gs}$)^{40}Ar$_{gs}$, ^{76}Se(^{18}O, ^{18}Ne$_{gs}$)^{76}Ge$_{gs}$ and ^{76}Ge(^{20}Ne, ^{20}O$_{gs}$)^{76}Se$_{gs}$ at 15.3 AMeV beam energy. Figure 1 illustrates that for all the analyzed systems, average-ρ approximation allows to reproduce t-channel diffraction pattern at small scattering angles, while collinear approximation results reproduce the diffraction pattern of t-channel calculations over a wider angular range. However, both the approximations do not allow to recover the order of magnitude of t-channel angular distributions for the three nuclear reactions studied. In particular, calculations within collinear approximation need a very small scaling factor (N_C in fig. 1) to be normalized to t-channel result. Moreover, average-ρ approximation turns out to better reproduce both the order of magnitude and the diffraction pattern of t-channel result (for heavier systems) if only transitions with multipolarities smaller than $J_\gamma = 5$ are considered in the sum over intermediate channel states. Hence, the approximations adopted within s-channel framework need further checks to fix the discrepancies with t-channel results. Nevertheless, the similar diffraction patterns of t-channel and s-channel calculations, at least at small scattering angles, reveal the feasibility of the approach here discussed. To make a consistent comparison with DCE data, it is necessary to coherently sum the contribution from all the possible reaction mechanisms, which are the transfer-DSCE, the DSCE mechanism here described and the direct reaction mechanism including correlations between the two SCE reactions (Majorana-like DCE).

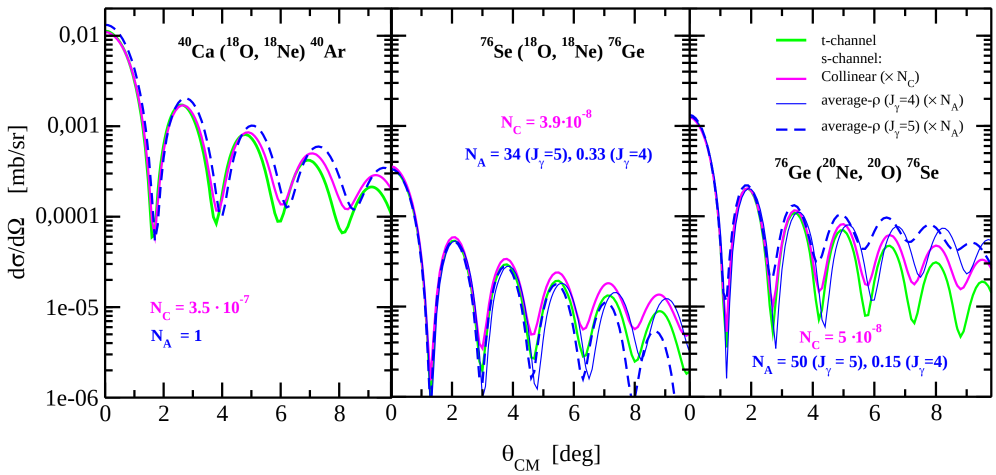

Fig. 1. – Comparison among t-channel and the two kind of s-channel calculations of DSCE cross sections for three nuclear reactions at 15 AMeV beam energy.

3. – Conclusions

HIDCE reactions are treated as two-step process within second order DWBA. By a proper recoupling of all angular momenta, simple expressions of projectile and target 2-body transition densities and DSCE NN interaction potential can be derived. The adopted formalism allows to directly relate DSCE cross section to the disentangled product of projectile and target NMEs. This result turns out to be useful for directly extracting information on double beta decay-like NMEs, once the contributions to the DCE cross section from all the possible reaction mechanisms are known and coherently added to the present calculations. Further improvements on the approximations discussed and on the nuclear structure inputs used (nuclear deformation effects, better description of experimental energy spectra, effects of different nuclear structure models) are in progress.

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