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Colloquia: COMEX7

Interplay between strong, weak and electromagnetic interactions revealed by nuclear decay processes

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Summary. — We discuss a beyond-the-mean-field theory of α-decay, namely a solution to the α-decay problem through the use of the Hartree-Fock-Bogoliubov (HFB) method together with a residual nucleon-nucleon Surface Gaussian Interaction (SGI), a generalization of the well-known Surface Delta Interaction. We present a systematic description of the alpha-particle formation amplitude in superfluid nuclei in terms of microscopic degrees of freedom, namely protons and neutrons. This opens a path towards the understanding of a less-known interplay between the fundamental interactions explored through decay processes, namely the interplay between the α-particle formation amplitude and electromagnetic observables or electron capture transition strengths.

1. – Introduction

There has been almost a century between the very first theories of α-emission [1, 2] and the experimental observation of α-particles on the surface of atomic nuclei [3]. At present, describing the formation of α-particles on the surface of nuclei in terms of proton and neutron degrees of freedom remains a considerable theoretical challenge. Historically, absolute decay widths were calculated within one major shell [4,5] which led to a discrepancy of several orders of magnitude with respect to experimental values. The value of the calculated decay width increased substantially by increasing the number of single-particle (sp) configurations [6, 7]. Adding a very large number of shells simulates the continuum part of the spectrum [8,9], the high-lying configurations being included in the formation process in the form of nucleonic clustering. However, even when taking all of these features into account, the calculated decay widths still differ from experimental observations by at least one order of magnitude [10-13].

Our goal here is to extend the description of nuclear interactions beyond the mean field picture in terms of the HFB approach. We want to show that a phenomenological potential that describes α-decay accurately follows from this method, provided that the usual nucleon-nucleon interaction is enhanced in regions of low nuclear density, *i.e.*, on the nuclear surface. This follows the theoretical developments from [10, 14], with full details given in [15]. The connection between α-decay, electromagnetic transitions and electron capture (EC) is discussed in more detail in [16].

2. – HFB Theory and α-clustering

We wish to describe the α-decay process

$$P\,(\text{parent}) \to D\,(\text{daughter}) + \alpha \tag{1}$$

in terms of a mean field generated from the HFB equations [17]

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + \Gamma^{(\text{dir})}\left(\mathbf{r}\right)\right]\psi_{am}\left(\mathbf{r}\right) + \int \mathrm{d}\mathbf{r}'\Gamma^{(\text{exc})}\left(\mathbf{r},\mathbf{r}'\right)\psi_{am}\left(\mathbf{r}'\right) = \epsilon_a\psi_{am}\left(\mathbf{r}\right) \tag{2}$$

together with a standard nucleon-nucleon interaction having a SGI term

$$v\left(r_\tau, R_\tau\right) = -v_0 \exp\left(-\frac{r_\tau^2}{b_{\text{rel}}^2}\right)\left[1 + x_c \exp\left(-\frac{\left(R_\tau - R_0\right)^2}{b_{\text{c.m.}}^2}\right)\right] \tag{3}$$

where $\tau = p, n$ for protons/neutrons, r and R are the relative and center of mass (c.m.) coordinates for a given pair of nucleons and x_c is the residual interaction strength centered at the radius R_0. The b values are the length parameters of the relative and c.m. Gaussians. The end result is a clustered mean field that describes the dynamics of pp and nn quasiparticle collective states coupled by the residual interaction enhanced on the nuclear surface. The procedure predicts a potential of the form

$$V_{\text{MF}}\left(r_\tau\right) = V_0\left(r_\tau\right) + V_{\text{cl}}\left(r_\tau\right) \tag{4}$$

where the first term is a standard mean field close to the Woods-Saxon shape and the second term is a Gaussian surface correction. The many-body description up to this point is valid for spherical nuclei, but many α-emitters exhibit a deformation which plays a significant role in the decay process [18]. This can be taken into account through Fröman's approximation [14] where the decay width factorizes in a spherical and deformed component

$$\Gamma = \Gamma_0 D\left(\beta_2\right) \tag{5}$$

the latter factor containing the effects of the Coulomb field characterized by the quadrupole deformation β_2. This framework allows one to calculate the α-particle formation amplitude by expanding in proton and neutron degrees of freedom the following overlap integral [14]

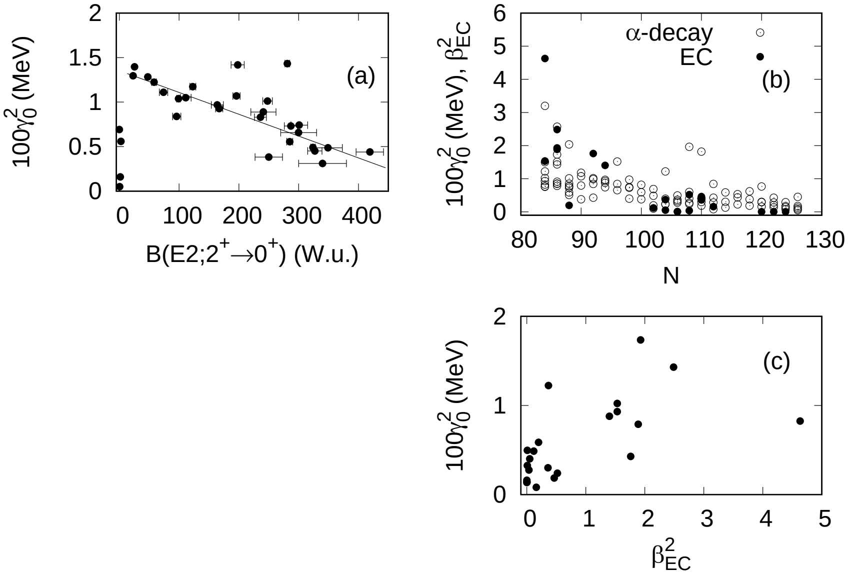

Fig. 1. – Reduced α-decay widths *versus* B(E2) values above ^{208}Pb (a). Reduced α-decay widths and experimental EC transition strengths *versus* the neutron number (b). Reduced α-decay widths *versus* experimental EC transition strengths (c).

$$\mathcal{F}_0\left(R\right) = \langle\Psi_P|\Psi_D\Psi_\alpha\rangle. \tag{6}$$

The above is a good approximation beyond the geometrical touching radius, where antisymmetrization becomes less important.

3. – α-clustering, γ-decay and electron capture

The theory outlined in the previous section opens the way to understanding correlations between decay processes mediated by three distinct interactions: the electromagnetic interaction, the strong nuclear force and the weak nuclear interaction. A summary of these basic observations is represented in fig. 1. Panel (a) shows the linear correlation between the reduced α-decay width of the standard R-matrix theory [19] and the experimental B(E2) values above ^{208}Pb. What is observed is that large collectivity involving many nucleons leads to a dissolution of four-body correlations in the nuclear matter, while above magic nuclei, where only few nucleons are involved, α-clustering is favored. Panel (b) shows the α-reduced widths together with the experimental Fermi EC transition strengths *versus* the neutron number. The transition strengths are calculated according to

$$g_A\beta_{\mathrm{EC}} = \sqrt{\frac{6147\left(2J_i+1\right)}{10^{\log ft}}} \tag{7}$$

where the overall value of the axial-vector coupling constant is $g_A = 1$, J_i is the total angular momentum of the parent state and the $\log ft$ values are taken from experiment [20]. Corresponding shell effects are clearly visible for both quantities, leading to the linear correlation depicted in panel (c). A proper understanding of these observations requires a simultaneous description in terms of sp degrees of freedom for all the decay processes involved. In this regard, the main challenge has been with respect to the description of α-decay. Progress was made through the use of the HFB theory with surface residual interactions previously presented.

4. – Conclusions

In this brief report we have outlined a theory of α-decay based on the HFB equations together with a residual surface gaussian interaction acting between quasiparticle pairs. The effects of nuclear deformation have been taken into consideration through Fröman's approximation. We have shown several correlations involving α-clustering, shell effects and transitions mediated by electromagnetic and weak interactions and we expect that a new theory of α-decay based on the developments presented here will provide an adequate quantitative description of these observations.

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