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Colloquia: COMEX7

Pair condensation in the excited states of nuclei

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Summary. — The low-lying excited states of neutrons or protons interacting by pairing forces are usually described by breaking a pair from the ground state pair condensate and replacing it with an "excited" pair. In this study we focus on a particular type of excited states which have the structure of a pair condensate built by identical excited pairs. As an example, we discuss the properties of these states for the case of ^{108}Sn.

1. – Introduction

In the BCS framework the excited states are generated by breaking pairs from the BCS ground state and distributing the corresponding particles on single-particle levels which are blocked in the pairing calculations [1]. The well-known drawback of this approach is that it does not conserve the particle number exactly. Another limitation is related to the blocking procedure, which is not appropriate for the excited states of zero seniority. For these excited states all the particles are distributed on time-reversed single-particle orbits. Since the pairing interaction acts on these two-particle configurations, the correlations between the particles originated from the broken pairs should be taken into account, which is not the case for the blocking approximation.

The two drawbacks mentioned above can be avoided if one works in the framework of the particle-number projected-BCS (PBCS) approach [2-4]. In PBCS the excited states are usually built by breaking a pair from the PBCS condensate, which conserves the particle number exactly, and replacing it with an excited collective pair [5, 6]. This one-broken-pair approximation was discussed recently for the case of zero seniority states [7]. In particular we have studied a special class of zero seniority states obtained by breaking all the pairs from the PBCS condensate and replacing them with identical excited pairs. The main results of this study are summarised below.

2. – Formalism

We consider systems formed by an even number of neutrons or protons distributed on Ω single-particle orbits $|i\rangle$ of energies ϵ_i and interacting by a pairing force. The

Hamiltonian describing such systems is of the form:

$$H = \sum_{i}^{\Omega} \epsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - \frac{1}{4} \sum_{ij}^{\Omega} v_{ij} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j \tag{1}$$

The ground state of the Hamiltonian (1) is approximated by a PBCS condensate defined by

$$|PBCS\rangle = (\Gamma^\dagger)^N |-\rangle \tag{2}$$

where $\Gamma^\dagger = \sum_i x_i (a_i^\dagger a_{\bar{i}}^\dagger)^{J=0}$ is the collective pair operator. The mixing amplitudes x_i are obtained variationally by minimizing the average of the Hamiltonian on the PBCS state and imposing the normalisation condition for the latter.

In this study we analyse the excited states of zero seniority built on time-reversed states. In the PBCS formalism the simplest way to construct such excitations is to break a pair from the PBCS condensate and to replace it with an excited pair. These states are defined by

$$|0_k\rangle = \tilde{\Gamma}_k^\dagger (\overline{\Gamma}^\dagger)^{N-1} |-\rangle \tag{3}$$

where $\overline{\Gamma}^\dagger = \sum_i y_i (a_i^\dagger a_{\bar{i}}^\dagger)^{J=0}$ and $\tilde{\Gamma_k}^\dagger = \sum_i z_i^{(k)} (a_i^\dagger a_{\bar{i}}^\dagger)^{J=0}$. The y_i and $w_i^{(k)}$ are determined variationally such that $\langle PBCS|0_k\rangle = 0$ and $\langle 0_k|0_{k'}\rangle = \delta_{k,k'}$.

In a similar manner, one can construct excited states with more broken pairs. Of special interest here are the excited states obtained by breaking all the pairs from the pair condensate and replacing them with identical excited pairs. Such excited pair condensate (EPC) states are defined by

$$|EPC(k)\rangle = (\hat{\Gamma}_k^\dagger)^N |-\rangle \tag{4}$$

where

$$\hat{\Gamma_k}^\dagger = \sum_i w_i^{(k)} (a_i^\dagger a_{\bar{i}}^\dagger)^{J=0} \tag{5}$$

The amplitudes $w_i^{(k)}$ are determined variationally such that the average of the Hamiltonian is minimized with the additional constraints $\langle PBCS|EPC(k)\rangle = 0$.

In all the states introduced so far the collective pairs are built on time-reversed orbits. Therefore, if the orbits have spherical symmetry, the states defined in Eqs. 2-4 have angular momentum $J = 0$.

3. – Applications

As an example, we discuss here the properties of the excited states (3) and (4) in the case of ^{108}Sn. These states are obtained with a state-dependent pairing interaction derived from G-matrix [8]. The matrix elements and single-particle energies are provided in [9].

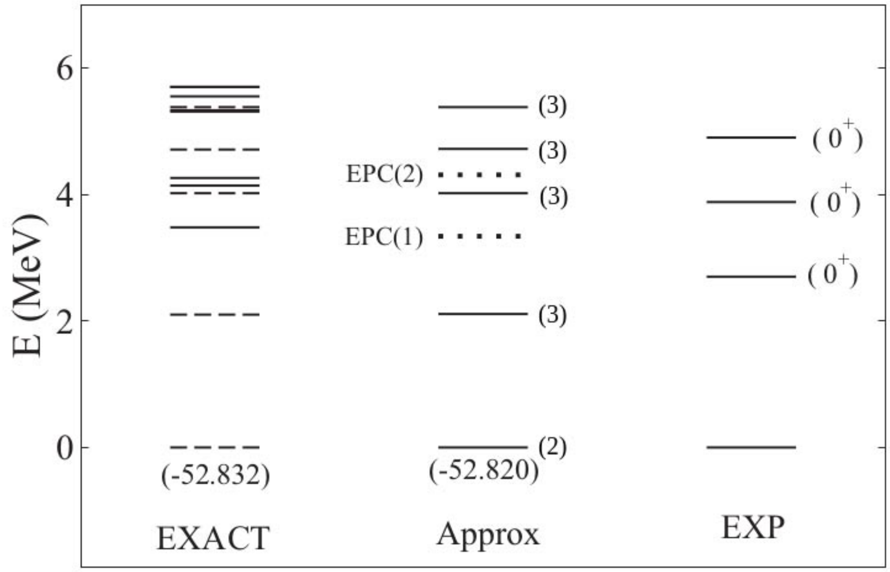

Fig. 1. – Energies corresponding to the states (2,3) compared to the exact spectrum and experimental energies [10]. The exact energies which correspond to the states (2,3) are indicated by dashed lines. The numbers within round brackets are the ground state energies, in MeV. The energies corresponding to the states (4) are indicated by the labels EPC(1) and EPC(2).

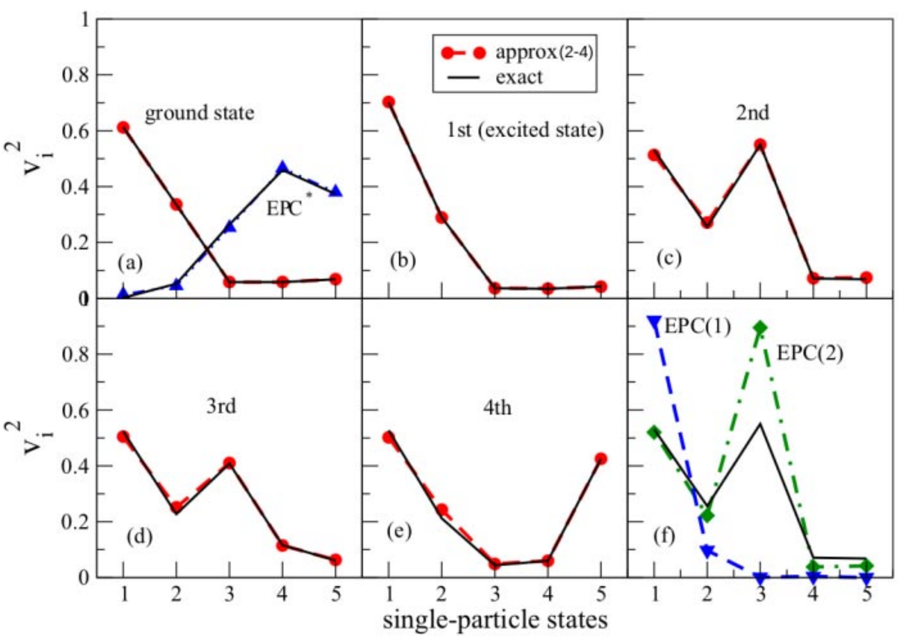

Fig. 2. – Occupation probabilities of single-particle orbits corresponding to the states (2,3,4) and to the exact states which have the closest similarity with the former. In fig. 2(a) are also shown the occupation probabilities corresponding to the EPC* state (see the text).

The energies for the states (3) and (4) are shown in fig. 1, in the middle column. They are compared to the exact J=0 states obtained by diagonalisation and the experimentally known excited J=0 states in ^{108}Sn. First of all we can observe that the energies of the states (3) agree rather well with the experimental levels. On the other hand, the states (3) are very close in energy and structure with the exact J=0 states shown by dashed lines. The fact that these states have a similar structure is supported by the occupation probabilities of the single-particle orbits shown in fig. 2.

From this comparison we can also notice that only a few exact states correspond to seniority zero states. The other ones, shown by full lines, have a more complicated structure, in which the particles are not distributed in time-reversed states.

In fig. 1 we also show the energies of the first EPC states (4) obtained from variational calculations. The energies of these states appear at surprisingly low energies. In the energy region of these states we could not find exact states with a similar structure.

One of the most interesting results of this study is the fact that at an excitation energy of around 21 MeV there is an exact state which has the structure of a pair condensate. The comparison between the occupation probablities of this exact state and the excited pair condensate, denoted by EPC*, is shown in fig. 2(a). From this figure one can notice that the EPC* state is mainly built on the high energy orbits $1h_{11/2}$ and $2d_{3/2}$. This is the reversed situation compared to the ground state condensate, in which the low energy orbits $1g_{7/2}$ and $2d_{5/2}$ have the highest occupation probabilities. Since between the lowest and the highest orbits there is a large energy gap, the EPC* state has properties in common with a giant pairing vibration.

4. – Conclusions

We discussed two types of excited states of zero seniority based on a PBCS pair condensate. Namely, one-broken-pair excited states and excited pair condensate (EPC) states. As an example we considered the J=0 excited states of ^{108}Sn described by a state-dependent pairing interaction. We have shown that the lowest one-broken-pair states agree well with the experimental J=0 states and also with the exact J=0 states obtained by digonalisation. Concerning the EPC states determined variationally, we have found that they appear at relatively low energies and that they cannot be associated to the exact states of the pairing Hamiltonian. However, at a much higher energy, of about 21 MeV, we have found an exact state which has the structure of a pair condensate and which has similar properties to the giant pairing vibration.

REFERENCES

[1] Ring P. and Schuck P., *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin) 1980.
[2] Bayman B. F., *Nucl. Phys.*, **15** (1960) 33.
[3] Blatt M., *Progr. Theor. Phys.*, **24** (1960) 851.
[4] Talmi I., *Nucl. Phys. A*, **172** (1971) 1.
[5] Bonsignori G., Savoia M., Allaart K., van Egmond A. and Te Velde G., *Nucl. Phys. A*, **432** (1985) 389.
[6] Allaart K., Boeker E., Bonsignori G., Savoia M. and Gambhir Y. K., *Phys. Rep.*, **169** (1988) 209.
[7] Popa Th., Sandulescu N. and Sambataro M., *Phys. Rev. C*, **107** (2023) 034318.
[8] Holt A., Engeland T., Hjorth-Jensen M. and Osnes E., *Nucl. Phys. A*, **634** (1998) 41.
[9] Zelevinsky V. and Volya A., *Phys. At. Nucl.*, **66** (2003) 1781.
[10] `https://www.nndc.bnl.gov/`.