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Colloquia: COMEX7

Nuclear equation of state from nuclear collective excited state properties

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Summary. — An overview on some selected investigations of the excitation energy and sum rules in Giant Resonances and its impact on the nuclear equation of state around saturation density is presented.

1. – Introduction

Giant Resonances are collective excitations in nuclei that have been known for several decades [1]. The excitation energy and strength distribution of such resonances depends on the underlying nuclear interaction and have been very useful in characterizing the nuclear Equation of State (EoS, sect. **2**) [2]. Sum rules (sect. **3**) are of special relevance in this context since, in some special cases, allow for a direct access to basic nuclear properties. In this respect, one of the most paradigmatic examples is the incompressibility of the finite nucleus [3].

In the present contribution, a selection of recent analysis of the non-charge exchange Giant Monopole (GMR) and Dipole (GDR) resonances (sects. **4** and **5** respectively), as well as of the charge exchange Isobaric Analog Resonance (IAR) and the Spin-Dipole Resonance (SDR) (sects. **6** and **7**, respectively) are reviewed.

2. – The nuclear equation of state

The nuclear Equation of State (EoS) is defined as the energy per particle ($e \equiv E/A$) of an unpolarized infinite system of neutrons and protons at zero temperature and where the Coulomb interaction is neglected. It is customarily written in terms of the neutron and proton densities $e(\rho_n, \rho_p)$ or, equivalently, in terms of the total density $\rho \equiv \rho_n + \rho_p$ and relative difference $\delta \equiv (\rho_n - \rho_p)/\rho$ as $e(\rho, \delta)$.

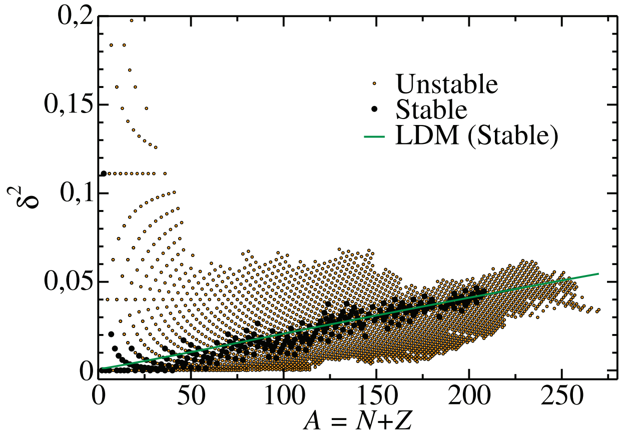

Fig. 1. – Square of the relative difference between the neutron and proton densities (δ) of experimentally known nuclei. For guidance the Liquid Drop Model (LDM) estimate of the stability line is also shown.

Stable nuclei typically show small values of δ (cf. fig. 1). Due to this and assuming isospin symmetry, it is useful to expand $e(\rho, \delta)$ for small δ as,

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2 + O[\delta^4] \ , \tag{1}$$

where it has been shown that around saturation, the parabolic expansion is already a very good approximation even for $\delta = 1$ (cf. fig. 1 of ref. [4]). The first term in the right hand side of the equation is the so called symmetric matter EoS while the second term is the so called symmetry energy, a penalty energy for departing from the most stable configuration $e(\rho, 0)$.

The interior of heavy nuclei is sensitive to densities around the nuclear saturation density ($\rho_0 = 0.16$ fm^{-3}). Expanding the symmetric matter EoS and the symmetry energy around ρ_0 allows to define different coefficients that would characterize the EoS and that can be calculated with most of the nuclear models available in the literature. That is,

$$e(\rho, 0) = e(\rho_0, 0) + \frac{1}{2}K\epsilon^2 + O[\epsilon^3] \tag{2}$$

$$S(\rho) = J - L\epsilon + \frac{1}{2}K_{\rm sym}\epsilon^2 + O[\epsilon^3] \ , \tag{3}$$

where $\epsilon \equiv (\rho_0 - \rho)/3\rho_0$.

In the present contribution, a discussion on how the values of K, J and L have been —or could be— estimated from the theoretical analysis of the experimental data on the isoscalar (IS) GMR, isovector (IV) GDR as well as on two charge exchange resonances: the IAR and the SDR; is presented.

3. – Strength function and sum rules

The strength function is defined as

$$S(E) \equiv \sum_\nu |\langle\nu|\mathcal{O}|0\rangle|^2 \delta(E - E_\nu - E_0) \tag{4}$$

where $\mathcal{O}$ is the transition operator that models the specific excitation proved in experiment, $|0\rangle$ is the ground state and $|\nu\rangle$ an excited state.

Of special interest are the moments of the strength function, also referred in the literature as sum rules,

$$m_k = \int dEE^k S(E) = \sum_\nu |\langle\nu|\mathcal{O}|0\rangle|^2 \delta(E_\nu - E_0)^k \ . \tag{5}$$

Assuming the completeness of the excitation spectra, one can rewrite the sum rules in a computational convenient way involving only an expectation value on the ground state. Relevant examples are: $m_0 = \langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle = \frac{1}{2}\langle 0|\{\mathcal{O}^\dagger, \mathcal{O}\}|0\rangle$ and $m_1 = \frac{1}{2}\langle 0|[\mathcal{O}^\dagger, [\mathcal{H}, \mathcal{O}]]|0\rangle$ The m_{-1}, directly related to the static response function, can be calculated by means of the dielectric theorem [5],

$$\frac{1}{m_{-1}} = -2\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\langle\mathcal{O}\rangle^2} \ . \tag{6}$$

where the original Hamiltonian $\mathcal{H}_0$ is pertubed using the operator $\mathcal{O}$: $\mathcal{H} = \mathcal{H}_0 + \lambda\mathcal{O}$; and the problem is solved by obtaining the ground state as a function of the values of λ —assumed to be small.

Based on these sum rules one can define two different excitation energies (E_x) of a Giant Resonance, the centroid and constrained E_x are defined as

$$E_x^{\mathrm{cent.}} \equiv \frac{m_1}{m_0} \quad \text{and} \quad E_x^{\mathrm{cons.}} \equiv \sqrt{\frac{m_1}{m_{-1}}} \ , \tag{7}$$

both coincide only in the case in which the sum rule is exhausted by a single peak and their difference can give a qualitative idea of the width of the resonance.

4. – Giant Monopole Resonance

The ISGMR ($\Delta L = 0$ and $\Delta S = 0$) can be modeled by the operator $\mathcal{O}_{GMR} = \sum_i^A r^2 Y_{00}(\hat{r})$. Assuming this operator, one can write

$$\left(E_x^{\mathrm{ISGMR}}\right)^2 = \frac{m_1}{m_{-1}} = 4\frac{\hbar^2}{m}\langle r^2\rangle\frac{\partial^2 E}{\partial\langle r^2\rangle^2} \equiv K_A\frac{\hbar^2}{m\langle r^2\rangle} \tag{8}$$

where the incompressibility of a finite nucleus K_A has been defined in an analogous way to the thermodynamic definition [2]. Hence, a theoretical proof of the the relation of E_x^{ISGMR} with the incompressibility of the infinite system $K = K_{A\to\infty}$. Based on this insight, many works have analyzed the experimental data on E_x^{ISGMR} in order to

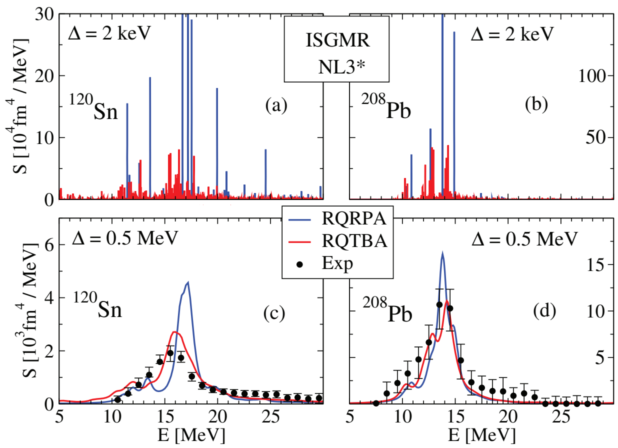

Fig. 2. – Strength function of the ISGMR in ^{120}Sn (left panels) and ^{208}Pb (right panels) as a function of the excitation energy. Predictions with NL3* functional are shown. Blue lines correspond to calculations based on the Relativistic Quasi-particle Random Phase Approximation RQRPA while redlines correspond to the Relativistic Quasi-particle Time Blocking Approximation (RQTBA). Top panels assume a smearing parameter of $\Delta = 2$ keV while in bottom panels assume a larger smearing parameter of $\Delta = 0.5$ MeV which is more commonly used in the literature. Figure reprinted from ref. [7].

characterize K. Note that the analysis is always done in finite-A systems and, thus, it will always require a modeling of the surface, isospin-asymmetry, etc.

One of the main problems that have been raised on the analysis of the ISGMR in connection with the K parameter of the EoS is that models that tend to describe E_x in closed shell nuclei such as ^{208}Pb overestimate the E_x in open shell nuclei such as Sn isotopes [3]. Different possibilities have been discussed in the literature (see refs. [2, 3] for details). Here, two of the main and new recent results [6, 7] are discussed. In these publications, the *softness* of the Sn isotopes is addressed by advocating for correlations beyond the mean-field.

In ref. [6], it is shown that pairing effects allow for a larger number of active configurations with respect to magic nuclei predicting a larger energy shift of the ISGMR when particle-vibrations effects are considered in open shell Sn isotopes (cf. fig. 2 in [6]). This feature paves the way to a unified description of the monopole resonance and to a coherent analysis that may shed some light on the value of K. Similar findings but within a relativistc approach are found in [7]. In the latter work only one parameterization NL3* is used while in [6] several Skyrme parametrizations are analyzed. In fig. 2, the strength function of the ISGMR in ^{120}Sn (left panels) and ^{208}Pb (right panels) as a function of the excitation energy are shown. Blue lines correspond to calculations based on the Relativistic Quasi-particle Random Phase Approximation QRPA while redlines correspond to the Relativistic Quasi-particle Time Blocking Approximation (RQTBA). In [6], the best description of the experimental data on the ISGMR is given by SV-K226 and KDE0 models, which are characterized by incompressibility values of 226 MeV and 229 MeV, respectively, at mean field level. In [7], the incompressibility predicted by NL3* is $K = 258$ MeV.

5. – Giant Dipole Resonance

The IVGDR ($\Delta L = 1$ and $\Delta S = 0$) is excited by the operator $\mathcal{O}_{GDR} = \frac{N}{A}\sum_i^Z rY_{1M}(\hat{r}_i) - \frac{Z}{A}\sum_i^N rY_{1M}(\hat{r}_i)$, where one must average over the magnetic quantum number M. Assuming this operator, one can apply the dielectric theorem to calculate m_{-1} and, thus, the electric dipole polarizability $\alpha_D = (8\pi e^2/9)m_{-1}$. Another option is to rely on the RPA or other more complex many-body approximations, such as the Particle Vibration Coupling (PVC). In order to gain a simple physical insight on this observable, in ref. [8] was proposed for guidance the Droplet Model (DM) expression (see [9] for more details),

$$\alpha_D^{\mathrm{DM}} \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left(1 + \frac{5}{3}\frac{L}{J}\epsilon_A\right) \ , \tag{9}$$

where $\epsilon_A \equiv (\rho_0 - \rho_A)/\rho_0$ and ρ_A is an average density probed in experiments measuring α_D provided this simple macroscopic approach captures the main features of the electric dipole polarizability (see a discussion in [2, 10] for more details). The latter equation points towards a correlation between $\alpha_D J$ and L that is fulfilled by nuclear Energy Density Functionals (EDFs) of the Skyrme and relativistic type (cf. fig. 2 of ref. [8] for the case of ^{208}Pb). This type of analysis based on EDFs has allowed to determine a linear relation between J and L on the basis of the experimental data on α_D in different even-even [10] and even-odd [11] nuclei.

6. – Isobaric Analog State

The Isobaric Analog State (IAS) is a collective mode associated to nuclear excitations with an isospin charge-exchange. The theoretical operator that models this transitions is $\mathcal{O}^{\pm}_{IAS} = \sum_i^A t_\pm(i) \equiv T_\pm$ where $t_\pm \equiv \tau_\pm/2$ and $\tau_\pm$ are the Pauli matrices in isospin space. The excitation energy of this resonance in the τ_- channel which is dominant in a neutron rich nucleus, can be calculated as

$$E_x^{\mathrm{IAS}} = \frac{m_1}{m_0} = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} \ . \tag{10}$$

That is, only terms that break isospin symmetry ($[\mathcal{H}, T_-] \neq 0$) contribute to E_x^{IAS}. The largest term is due to the Coulomb potential. However a small contribution from nuclear Isospin Symmetry Breaking (ISB) effects must be taken into account not only for a detailed study of the IAS and of the Nolen-Schiffer anomaly [12] but also for the study of the nuclear EoS [13].

The excitation energy of the IAS can be related to the neutron skin thickness ($\Delta r_{np} = \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$) and, thus, to the slope parameter L [14, 15] using a very simple model based on the fact that the Coulomb direct term will give the largest contribution to E_x^{IAS}. That is, assuming no isospin mixing ($\langle 0|T_+T_-|0\rangle = N - Z$) and a sharp sphere model for the neutron and proton densities one finds,

$$E_x^{\mathrm{IAS}} = \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \frac{1}{2}\frac{N}{N-Z}\frac{R_n - R_p}{R_p}\right) \ , \tag{11}$$

where $\Delta r_{np} = \sqrt{(3/5)}(R_n - R_p)$. This general trend of increasing E_x^{IAS} for decreasing neutron skin thickness is shown in fig. 3 by actual RPA calculations based on Skyrme

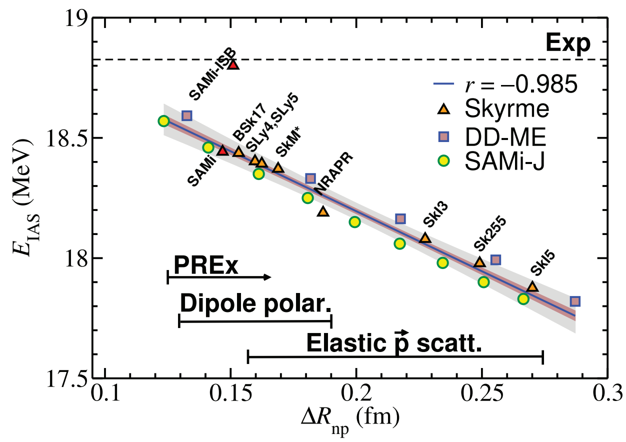

Fig. 3. – Energy of the IAS as a function of the neutron skin thickness in ^{208}Pb. The arrow and bars indicate the experimental results from polarized proton elastic scattering, parity violating elastic electron scattering, and from the electric dipole polarizability (see ref. [13] for details). Figure reprinted from ref. [13].

and covariant EDFs (see ref. [13] for details on the calculations and the experimental constraints shown as black arrows). The models used in this figure contain only Coulomb direct and Coulomb exchange —in Slater approximation— contributions to the IAS energy and none of them is able to describe the experimental value (shown as a black dashed line in fig. 3). What is missing are Coulomb corrections plus some contribution from nuclear ISB terms, the latter being model dependent and unknown in the nuclear medium. Hence, the determination of ISB in the medium can shed light into the EoS parameter L provided the fact that we accurately know the E_x^{IAS} in many nuclei.

7. – Spin-Dipole Resonance

The SDR is a collective charge exchange mode ($\Delta L = 1$ and $\Delta S = 1$). The theoretical operator that model spin-dipole transitions is $\mathcal{O}^{\pm}_{SDR} = \sum_i^A \tau_{\pm}(i) r_i^L [Y_{1M}(\hat{r}_i) \otimes \boldsymbol{\sigma}(i)]_{JM}$. Theoretically, the connection of this type of nuclear excitations and the EoS is particularly simple: the difference of the non-energy weighted sum rules in the two isospin-channels $m_0(\mathcal{O}^{-}_{SDR}) - m_0(\mathcal{O}^{+}_{SDR})$ is proportional to $N\langle r_n^2\rangle^{1/2} - Z\langle r_p^2\rangle^{1/2}$. This expression can be written in terms of the charge radius and the neutron skin thickness. Different measurements are available in the literature [16]. We show in fig. 4 the experimental results for the SDR in ^{90}Zr [17] (red symbols) that agree well with theoretical calculations — Skyrme EDFs with and without tensor terms— and would predict via the present sum rule approach a neutron skin thickness in ^{90}Zr of 0.07 ± 0.04 fm.

8. – Conclusions

Nuclear collective excitations have been used along the years to learn about the nuclear equation of state around saturation density. Specifically, sum rules of some selected modes have been instrumental for this aim [2]. In this contribution, some examples of particular interest for this topic and that, at the same time, have revitalized the field in the last years have been presented. With the advent of new experimental techniques such type of studies are now a reality also in exotic nuclei. On the other side, robust

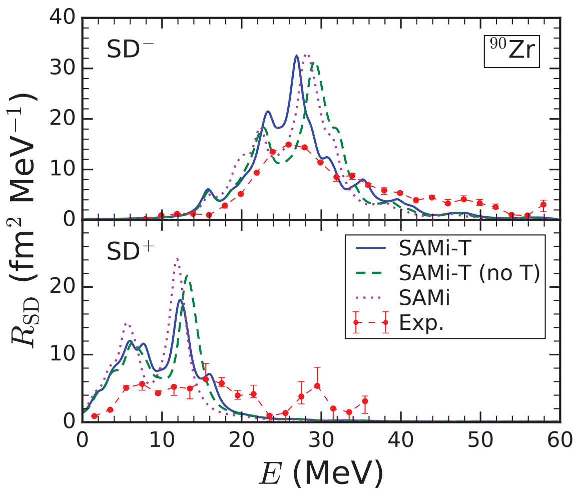

Fig. 4. – SDR strength function ($R_{\rm SD}$) of the τ_- (top) and τ_+ (bottom) channel in ^{90}Zr calculated by SAMi-T with and without tensor, in comparison with experimental and SAMi functional (see ref. [18] for details on the calculations) . Figure reprinted from ref. [18].

theoretical methods as well as clear interpretations support recent analysis that connects all modes presented here with some basic parameters of the nuclear equation of state.

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