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Connecting low-lying dipole modes to nuclear structure and equation of state

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Summary. — We review some recent studies devoted to shed light on the properties of low-lying dipole modes in nuclei, particularly exploring how their features are connected to nuclear structure properties, also providing valuable information on the nuclear matter equation of state (EOS). We perform time-dependent mean-field calculations, in the small-amplitude limit, to investigate dipole excitations in several nuclei. Employing standard Skyrme-like energy density functionals (EDFs), we show how the relative weight of the excitations emerging in the low-energy region evolves with nuclear global features, such as density profile and nuclear surface, reflecting in turn important properties of the effective interaction and EOS. Moreover, we suggest a novel interpretation of the nature of the Pygmy Dipole Resonance and of its correlation with the neutron skin thickness in neutron-rich nuclei. Within the framework of the modern density functional theory, we also discuss recent developments introduced to refine the EDF approaches in the low-density regime. On the one hand, by considering new class of EDFs, inspired by effective field theory and benchmarked on ab-initio calculations, which were recently devised to improve the description of neutron matter and to provide a more refined description of the surface in finite systems. On the other hand, by possibly extending the EDFs by accounting for the existence of many-body correlations and clustering phenomena.

1. – Introduction

The development of collective motion, as a coherent pattern of particles in phase-space, constitutes a fundamental feature of many-body systems and its study reveals helpful to shed light on the properties of the underlying interaction among the constituents. However, the understanding of the microscopic emergence of a collective motion from the single-particle one, being favoured by kinematic conditions, represents a true challenge. Among the isovector (IV) dipole excitations in nuclei, the most prominent example of collective excitation is the Giant Dipole Resonance (GDR), which is a well established collective state, described in a classical picture in terms of neutrons and protons moving against each other. It has a large response function and provides valuable information on the IV channel of the nuclear matter equation of state (EOS), of crucial importance also in the modelization of compact stellar objects.

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In the last years, a large amount of works concentrate on the properties of low-lying dipole modes in nuclei, particularly exploring how their features evolve with the isospin-asymmetry of the systems. In neutron-rich systems, indeed, one generally observes a strong fragmentation of the IV response, in an energy domain below the GDR. The nature of these excitations, which are associated to the so-called Pygmy Dipole Resonance (PDR), and their interpretation in terms of collective motion of the neutron excess at the surface, namely the neutron skin, is still under debate. Moreover, pygmy mode and neutron skin are related to the IV terms of nuclear effective interactions, and so to the symmetry energy contribution in the nuclear EOS. In particular, a strong correlation between the neutron skin thickness and the density slope parameter of the symmetry energy at saturation L was since long known [1].

The aim of the present work is then to get a deeper insight on these features, to better assess the nature of the low-lying dipole modes and their connection with the nuclear structure properties and the EoS. We rely to the framework of phenomenological models, based on energy density functionals (EDFs), which constitutes nowadays the reference choice to undertake the study of the collective excitations. In their standard form, EDFs are usually derived within a self-consistent mean-field approximation, which maps the many-body to a one-body problem, replacing the bare interaction with an effective one, averagely reproducing the mutual interaction among the constituents. The parameters of the effective interaction are usually fitted in order to reproduce several properties of both nuclear matter and finite nuclei. In such a way, these functionals on one side provide the EOS in the equilibrium limit, on the other allow one to describe the ground and collective states along the whole nuclear chart, in a complementary manner with ab-initio approaches, which make use of realistic interactions or derived from effective field theory (EFT), based on diagrammatic expansions. Accidentally, we note that several attempts are currently ongoing to bridge the two classes, ab-initio and EDF, of approaches [2, 3].

In sect. **2**, we revise some mean-field dynamical models, employed to investigate the small amplitude dipole response, which make use of standard EDFs based on non-relativistic Skyrme-like effective interactions. The corresponding results are discussed in sect. **3**. In sect. **4**, we finally discuss recent developments introduced to refine the EDFs in the low-density regime, to improve the description of the surface in finite systems and account for the existence of many-body correlations and clustering phenomena.

2. – Theoretical framework

In the present work, we investigate the small-amplitude nuclear dipole response, by employing mean-field models based on the Time-Dependent-Hartree-Fock (TDHF) equation. In the TDHF theory, the evolution of the one-body density matrix ρ is determined by

$$i\hbar\partial_t\rho(t) = \left[h\left[\rho\right], \rho(t)\right], \tag{1}$$

where $h\left[\rho\right] = \mathbf{p}^2/2m + U\left[\rho\right]$ is the non-relativistic single-particle Hamiltonian with $U\left[\rho\right]$ being the self-consistent mean-field potential [4]. Although the TDHF approach is applied in nuclear physics to describe various aspects of nuclear dynamics [5], we restrict here ourselves to the study of small amplitude fluctuations from the equilibrium, which can be determined either by solving explicitly the time-dependent evolution given by eq. (1) or by linearizing the TDHF equation, leading to the Random Phase Approximation (RPA) approach. Within these approaches, we study the E1 (isoscalar (IS) and IV) response of

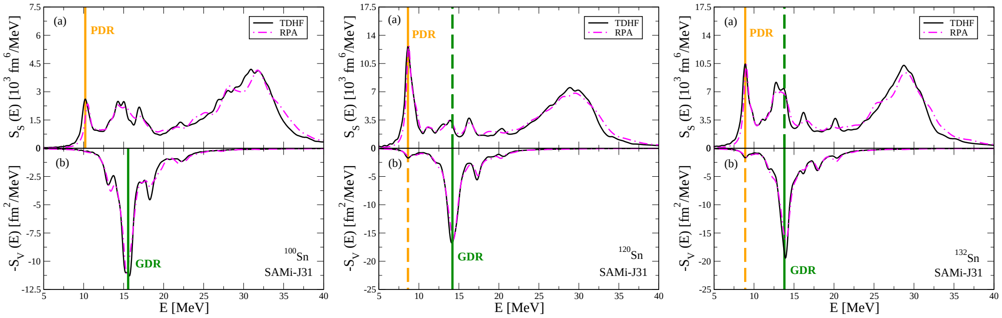

Fig. 1. – The strength functions $S_k(E)$ of the IS ($k = S$) (top panels) and IV ($k = V$) (bottom panels) responses for three different nuclei which belong to the tin isotopic chain: ^{100}Sn (left panels), ^{120}Sn (centered panels), and ^{132}Sn (right panels), as obtained in TDHF or in RPA calculations with SAMi-J31 effective interaction. The vertical lines indicate the energy of the IV GDR (green full lines) and IS PDR (orange full lines) modes and their projections onto the IS and IV spectrum, respectively.

nuclear systems, considering initial conditions determined by IS (k = S) or IV (k = V) dipole excitations $\hat{D}_k$ along the z direction:

$$\hat{D}_S = \sum_{i=1}^{A} \left(r_i^2 - \frac{5}{3}\langle r^2 \rangle \right) z_i, \qquad \hat{D}_V = \sum_{i=1}^{A} \left[\tau_i \frac{N}{A} - (1-\tau_i)\frac{Z}{A} \right] z_i \tag{2}$$

where N and Z indicate neutron and proton number, $A = N + Z$, $\tau_i = 1(0)$ for protons (neutrons) and $\langle r^2 \rangle$ denotes the mean square radius of the nucleus considered. We then extract the corresponding strength functions at the excitation energy E, $S_k(E) = \sum_{n>0} |\langle n|\hat{D}_k|0\rangle|^2 \delta(E - (E_n - E_0))$, where E_n is the excitation energy of the state $|n\rangle$ and E_0 is the energy of the ground state $|0\rangle$. Moverover, for a given mode of energy E, we also correspondingly extract the transition densities, to get information also on the spatial structure of the modes.

3. – Results

In this section, we show the main results concerning the study of the dipole response in some nuclei along the tin isotopic chain. In such a way, one aims at clarify how both IS and IV dipole energy spectrum evolves with the neutron/proton content.

Figure 1 displays in particular the energy spectrum of the strength functions $S_k(E)$ of both IS and IV responses for three different isotopes: ^{100}Sn, ^{120}Sn and ^{132}Sn, as obtained in TDHF or in RPA calculations with SAMi-J31 effective interaction. First of all, in fig. 1, a nice comparison between the approaches here considered is recovered, by convoluting the RPA transition probability with a Lorentzian function of width equal to 0.5 MeV to reproduce a similar spread as the one obtained in the TDHF results, in light of the finite time interval of the dynamics and the adopted smoothing procedure. It is worthwhile to notice that a satisfactory comparison with semi-classical calculations of ref. [6] was also achieved (see refs. [4, 7]). Moreover, one observes that the pygmy is mostly an IS-like mode since, for any system, its strength is mostly pronounced in the IS response. However, for asymmetric systems, some strength emerges at the same excitation energy

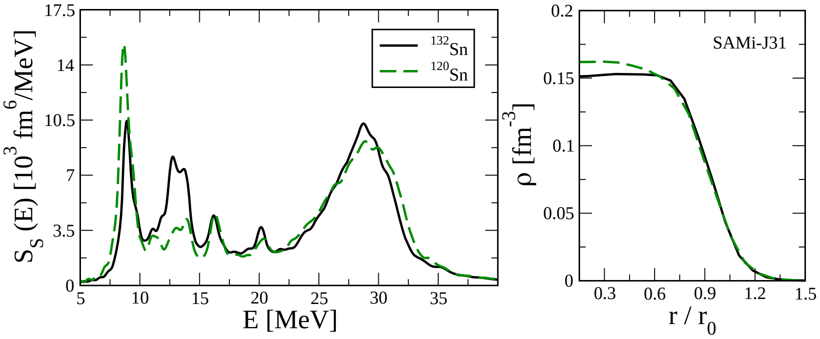

Fig. 2. – Left panel: The same IS strength functions as shown in the top (centered and right) panels of fig. 1, as obtained in TDHF calculations. Right panel: Total density profile $\rho(r)$, as obtained in the stationary HF limit of eq. (1) as a function of the reduced radius r/r_0 for two tin isotopes: ^{120}Sn and ^{132}Sn.

also in the IV response, by virtue of the coupling existing between IS and IV modes in asymmetric nuclear systems and also in asymmetric nuclear matter [8, 9]. A similar coupling also characterizes the GDR in fig. 1, which turns out to be an IV-like mode, also excited in the IS response of asymmetric systems. Nevertheless, this coupling does not characterize all modes, according to their specific nature and the structure component which they involve in the oscillation. The relative importance of the PDR in the IS response is enhanced for ^{120}Sn with respect to ^{132}Sn, at expenses of the other collective low-lying dipole modes. This is clearly evidenced by the left panel of fig. 2, where the IS strengths obtained within the TDHF approach for the two isotopes ^{120}Sn and ^{132}Sn, as given in the top (centered and right) panels of fig. 1, are plotted together to facilitate their comparison. The observed behavior in turn reflects the evolution of the density profile, when moving from ^{120}Sn to ^{132}Sn. As one sees from the right panel of fig. 2, where the total density profile $\rho(r)$, as obtained in the stationary Hartree-Fock (HF) limit of eq. (1), is plotted as a function of the reduced radius r/r_0 where $r_0 = 1.2A^{1/3}$ fm, the isotope ^{120}Sn has a more diffuse density profile with respect to the compact shape of closed-shell ^{132}Sn nucleus. Such a smoother density profile enhances then surface effects, which are connected to spatial gradient terms, leading to more robust oscillations in the lowest-energy region as compared to the ones obtained by the sharper radial evolution of the system ^{132}Sn. The analysis of the transition densities provides an alternative gateway to interpret the results. In fig. 3, we plot then the neutron and proton transition densities, as obtained within the TDHF approach for the three systems here considered, at the excitation energy specified by the orange full lines plotted in the top panels of fig. 1. First of all, this analysis confirms again the IS-like nature of the pygmy resonance. Indeed, in fig. 3, the two species move exactly in phase for the symmetric ^{100}Sn system, confirming the IS nature of the pygmy mode, whose IV character emerge only when moving towards asymmetric systems. Moreover, these low-lying modes deeply involve the outer part of the surface. As a result, a relevant role is played by the neutron excess which emerges in neutron-rich nuclei owing to the development of the neutron skin. This analysis also explains why, despite the increase of the neutron skin thickness, the percentage fraction of the Energy Weigthed Sum Rule (EWSR) exhausted in the PDR region of the IV response does not grow along the tin isotope chain, beyond N equal 70, as observed in

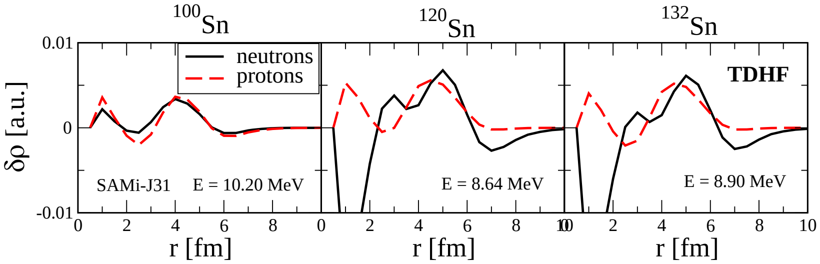

Fig. 3. – Radial behavior of the neutron and proton transition densities, as obtained at the excitation energies indicated by orange full lines of fig. 1, within the TDHF approach and employing the SAMI-J31 effective interaction, for three different tin isotopes.

ref. [10]. The missing fraction could be attributed indeed to the decrease observed in the IS strength, when tin isotopes from N equal 70 to 82 are considered. Indeed, when considering the ratio between the EWSR fractions, to normalize the effect of the IS/IV mixing to the strength of the PDR that is mostly an IS-like mode, an almost linear increase *versus* neutron number N and neutron skin thickness is nicely recovered. It is wortwhile to notice that pairing correlations, which have been neglected in the present work, should be in principle taken into account, when looking at the open-shell spherical system ^{120}Sn. Nevertheless, in ref. [11] it has been shown that pairing effects have no influence on the dipole polarizability in the $^{116-132}$Sn isotopes, especially in the case of SAMi-J31, that is employed here. We then decided to neglect pairing correlations to facilitate a consistent comparison between the different adopted approaches.

4. – Further developments, outlooks and conclusions

The results of the previous section highlighted the crucial role played by the nuclear surface in shaping the details of the low-lying dipole modes, which are often employed to extract valuable information on the properties of the nuclear effective interaction and EOS. Then, it is reasonable to expect that these results might be strongly affected by a more refined treatment of the low-density regime, especially when the strongly isospin-asymmetric nuclear systems are concerned. In this last section, we focus thus on some possible perspectives which may open from two different lines of developments which are characterizing the EDF approach, as employed here in the study of the collective modes.

Benchmark ab-initio for low-density pure neutron matter (PNM). – An intense research activity was devoted in these last years to establish a deeper connection between phenomenological and ab-initio approaches, particularly improving the predicting power of the standard EDFs out of the domain on which the effective interaction was fitted, for instance in the dilute regime [2, 3, 12].

Indeed, as a common drawback, phenomenological EDFs are not able to reproduce the behavior in the very-low density regime, where the PNM lies close to the unitary limit of the interacting Fermi gas owing to the large value of the s-wave scattering length a_s. In this domain, an expansion in terms of $a_s k_F$, where k_F is the Fermi momentum, was derived within the chiral EFT, whose truncation to the second order is known as Lee-Yang expansion. Two different classes of EDFs were then recently developed, which reconcile by construction the regime around saturation (where the phenomenological interactions were usually fitted) with the Lee-Yang constraints in the very-low density limit [12]. The parameters of these EDFs, whose design is inspired by the systematic expansion of diagrammatic nature adopted in EFT, were benchmarked on ab-initio calculations, which are available for both PNM and neutron drops. The analysis performed in ref. [13] shows that these new EDFs provide a reasonable description of several ground state properties, such as binding energies and skin thickness, even though they are not adjusted directly to nuclei. Moreover, the analysis of ref. [13] highlights that, beside the well-known correlation of the neutron skin thickness with L, an interesting correlation exists between the tail of the density profile and the slope of the symmetry energy at densities below saturation, id est in the regime where the new class of EDFs was grounded on ab-initio calculations, differently than standard phenomenological EDF. The idea for the next future is then to implement these EDFs in dynamical mean-field models to study again the collective modes, taking advantage of this refined treatment of the nuclear surface.

Embedding many-body correlations and clustering phenomena. – Another line of development within the EDF framework aims to overcome the mean-field approximation and take explicitly into account some many-body correlations, which are responsible for the emergence of bound states of nucleons, namely nuclear clusters, in nuclear matter at low-density, of crucial importance also in the astrophysical context [14].

Recent works, performed within a generalized relativistic mean-field (RMF) model which introduces clusters as explicit degrees of freedom [15], show indeed that the formation of α clusters on the surface of heavy nuclei, as recently discussed in ref. [16], modifies the correlation of the skin thickness with L. Moreover, a recent work [17] has properly extended the RMF model to include, beside bound clusters also effective resonances, in particular quasi-deuterons, to account for the existence of neutron-proton short-range correlations in the neighbourhood of saturation density, as recently highlighted by nucleon knock-out experiment with inelastic electron scattering [18]. As a perspective, we aim then to embed all these features within a unified framework, to assess the role of light-clusters in dynamical processes, starting again from the study of collective excitation modes.

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