IL NUOVO CIMENTO **47 C** (2024) 53
DOI 10.1393/ncc/i2024-24053-8

Colloquia: COMEX7

Temperature evolution of the nucleon effective mass and symmetry energy coefficient in the $^{68-78}$Ni isotopic chain

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received 31 October 2023

Summary. — The effective mass is an essential characteristic of nuclear matter and finite nuclei. The temperature evolution of the effective mass plays a significant role in understanding the temperature evolution of the symmetry coefficient of the nuclear equation of state. In the present contribution, the single-(quasi)particle spectra for $^{68-78}$Ni isotopes at zero and finite temperature are obtained by solving the Dyson equation in the basis of Dirac spinors. While the static part of the self-energy of the Dyson equation has its origin from a self-consistent mean field generated by the effective mesons, the dynamical part takes into account the coupling between (quasi)particles and phonons. In the leading approximation beyond the mean field, the (quasi)particle-vibration coupling (qPVC) mechanism is responsible for the fragmentation of single-(quasi)particle spectra. The calculated spectra of nickel isotopes yield the temperature-dependent effective mass for the 0 to 2 MeV temperature interval, which is relevant for astrophysical modeling, such as core-collapse supernova simulations. The impact of the temperature dependence of the effective mass on the symmetry coefficient in the nickel isotopic chain is discussed.

1. – Introduction

Understanding the behavior of atomic nuclei and nuclear matter at finite temperature plays a key role in the astrophysical modeling, in particular, the r-process nucleosynthesis and core-collapse supernovae (CCSN) simulations. The key nuclear physics input for the r-process modeling are masses, beta-decay half-lives, and neutron capture rates [1]. For the CCSN, the electron capture rates and the equation of state parameters are mostly needed. The evolution of the nuclear shell structure with temperature underlies all the astrophysically relevant nuclear structure properties, in particular, nuclear level density,

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nucleon effective mass, and the symmetry energy [2], which are crucial for both the r-process and CCSN.

2. – Solution of Dyson equation at finite temperature

In the nuclear systems at finite temperature T, the motion of a nucleon through the heated correlated medium is governed by the Matsubara propagator defined as a thermal average [3]

$$\mathcal{G}_{k_1k_{1'}}(\tau_1-\tau_{1'}) = -\langle T_\tau \hat{\psi}_{k_1}(\tau_1)\hat{\psi}^\dagger_{k_{1'}}(\tau_{1'})\rangle, \tag{1}$$

where T_τ is the imaginary time, τ, ordering operator and $\hat{\psi}_{k_1}(\tau_1) \equiv e^{\hat{\mathcal{H}}\tau_1}\hat{\psi}_{k_1}e^{-\hat{\mathcal{H}}\tau_1}$, $\hat{\psi}^\dagger_{k_1}(\tau_1) \equiv e^{\hat{\mathcal{H}}\tau_1}\hat{\psi}^\dagger_{k_1}e^{-\hat{\mathcal{H}}\tau_1}$ represent the nucleonic field operators in the Wick-rotated picture. Here, $\hat{\mathcal{H}} = \hat{H} - \mu\hat{N}$, where $\hat{H}$ is the many-body Hamiltonian, μ is the chemical potential, and $\hat{N}$ is the particle number operator. The spectral representation of the correlated propagator (1), defined by

$$\mathcal{G}_{k_1k_2}(\varepsilon_\ell) = \int_0^{1/T} d\tau e^{i\varepsilon_\ell\tau}\mathcal{G}_{k_1k_2}(\tau), \quad \tau = \tau_1 - \tau_2, \quad \varepsilon_\ell = (2\ell+1)T, \tag{2}$$

satisfies the finite-temperature Dyson equation

$$\mathcal{G}_{k_1k_2}(\varepsilon_\ell) = \widetilde{\mathcal{G}}_{k_1k_2}(\varepsilon_\ell) + \sum_{k_3k_4}\widetilde{\mathcal{G}}_{k_1k_3}(\varepsilon_\ell)\Sigma^e_{k_3k_4}(\varepsilon_\ell)\mathcal{G}_{k_4k_2}(\varepsilon_\ell). \tag{3}$$

The thermal mean-field propagator $\widetilde{\mathcal{G}}_{k_1k_2}(\varepsilon_\ell)$ is diagonal, *viz.*, $\widetilde{\mathcal{G}}_{k_1k_2}(\varepsilon_\ell) = \delta_{k_1k_2}\widetilde{\mathcal{G}}_{k_1}(\varepsilon_\ell)$, where $\widetilde{\mathcal{G}}_{k_1}(\varepsilon_\ell) = [i\varepsilon_\ell - \varepsilon_{k_1} + \mu]^{-1}$. The single-particle states k and energies ε_k are obtained by solving a set of thermal relativistic mean-field equation, which describes the independent motion of Dirac nucleons inside a self-consistent field generated by an effective meson-nucleon interaction at $T > 0$ [7, 8]. The dynamical kernel Σ^e is approximated by the (quasi)particle-vibration coupling ((q)PVC) model, which describes the coupling between (quasi)particles and phonons [8, 9]. The phonon energies and vertices are obtained by solving the finite-temperature relativistic (quasiparticle) random phase approximation equations. In the present work, we use NL3 forces [4] and take into account the pairing correlations at $T = 0$ in the framework of Bardeen-Cooper-Schrieffer (BCS) approximation [5].

In the diagonal approximation, $\Sigma^e_{k_1k_2}(\varepsilon) = \delta_{k_1k_2}\Sigma^e_{k_1}(\varepsilon)$ and eq. (3) can be recast into

$$[\varepsilon - \varepsilon_k + \mu - \Sigma^e_k(\varepsilon)]\mathcal{G}_k(\varepsilon) = 1 \tag{4}$$

for each state k. For each mean-field states k, the zeros of the function $f(\varepsilon) = \varepsilon - \varepsilon_k + \mu - \Sigma^e_k(\varepsilon)$ correspond to the energy fragments $\varepsilon^{(\lambda)}_k$ with $\lambda = 1,\ 2,\ ....$ The appearance of the energy fragments $\varepsilon^{(\lambda)}_k$ indicates that the qPVC mechanism induces the fragmentation of the single-particle mean-field states k. For each energy fragment $\varepsilon^{(\lambda)}_k$, the spectroscopic factor $S^{(\lambda)}_k$ is determined by

$$S^{(\lambda)}_k = \left[1 - \frac{d}{d\varepsilon}\Sigma^e_k(\varepsilon)\right]^{-1}_{\varepsilon=\varepsilon^{(\lambda)}_k}, \quad \text{where } \sum_k S^{(\lambda)}_k = 1 \text{ and } \sum_k \varepsilon^{(\lambda)}_k S^{(\lambda)}_k = \varepsilon_k. \tag{5}$$

3. – Temperature evolution of nucleon effective mass

The temperature-dependent nucleon effective mass $m^*(T)$ is defined by the relation $m^*(T)/M = (\widetilde{m}/M) \times (m_\omega(T)/M)$, where M, $\widetilde{m}$, and $m_\omega(T)$ stand for the bare nucleon mass, k mass, and ω mass, respectively [2, 6]. For the NL3 parametrization [4], the value of k mass is $0.6M$ with $M = 939$ MeV. While the k mass is nearly temperature-independent, the ω mass accounts for qPVC and finite temperature effects. To determine the value of $m_\omega(T)$ for each temperature T, we perform the following procedures [9]:

1) For each temperature T and single-particle state $k = \{(k), m_k\}$, we determine the quantity $\overline{m}_{(k)}(E,T)/M$ as a function of excitation energy E:

$$\frac{\overline{m}_{(k)}(E,T)}{M} = 1 - \frac{\partial}{\partial \varepsilon}\mathrm{Re}\Sigma^e_{(k)}(\varepsilon), \quad \varepsilon = E + i\Delta. \tag{6}$$

2) For each temperature T, the ω mass is calculated as the maximal value of $\overline{m}_{(k)}(E,T)/M$ averaged over the single-particle states k:

$$\frac{m_\omega(T)}{M} = \max_E \left[\frac{\sum_{(k)}(2j_{(k)}+1)\left(\overline{m}_{(k)}(E,T)/M\right)\left(1/v^2_{(k)}\right)}{\sum_{(k)}(2j_{(k)}+1)}\right], \tag{7}$$

where $v^2_{(k)}$ is the BCS occupation probability.

3) The temperature dependence of the ω mass is parametrized according to the relation $m_\omega(T)/M = 1 + \{(m_\omega(T=0)/M) - 1\}\exp\{-T/T_0\}$, where $m_\omega(T=0)/M$ and T_0 are the fitting parameters [2]. The best values of these parameters are summarized in table III of ref. [9].

4. – Temperature evolution of symmetry coefficient

The symmetry energy term in the nuclear equation of state (EOS) is defined as $E_S = S(T=0)\,[1-(2Z/A)]^2$, where $S(T=0)$ stands for the symmetry coefficient of the nuclear matter at $T=0$. The symmetry coefficient $S(T)$ at finite temperature takes the form:

$$S(T) = S(T=0) + \frac{\hbar^2 c^2 k_F^2}{6M}\left[\frac{M}{m^*(T)} - \frac{M}{m^*(T=0)}\right]. \tag{8}$$

For the NL3 parametrization [4], $S(T=0) = 37.4$ MeV and the nuclear matter density $\rho_0 = 0.148$ fm^{-3}. Since the value of $S(T=0)$ implicitly contains the contribution from the qPVC and pairing contributions at $T=0$, the subtraction term in the bracket aims to overcome the double counting. The corresponding Fermi momentum k_F is determined via $k_F = \left(\frac{3}{2}\pi^2\rho_0\right)^{1/3}$. The average values of $m_\omega(T=0)/M$ and T_0 over five Ni isotopes $(A = 68-76)$ are $m_\omega(T=0)/M = 1.39$ and $T_0 = 1.48$ MeV, respectively. The obtained values of the effective mass $m^*(T)$ and the symmetry coefficient $S(T)$ for $0 \le T \le 2$ MeV are plotted in fig. 1(a). It demonstrates a significant increase of the symmetry coefficient with temperature while the effective mass decreases considerably. Figure 1(b) shows the evolution of the symmetry coefficients across the Ni isotopic chain, which are associated

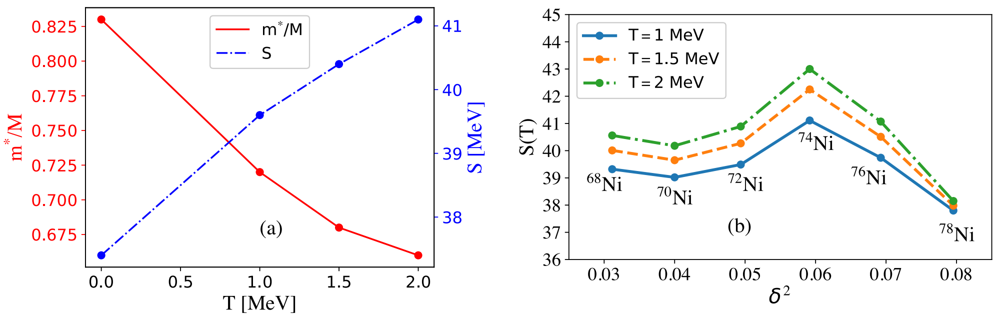

Fig. 1. – (a) The evolution of the total effective mass m^*/M and the symmetry coefficient S with temperature for $0 \leq T \leq 2$ MeV. (b) The dependence of the symmetry coefficient $S(T)$ on the asymmetry parameter δ^2 for $T = 1,\ 1.5,\ 2$ MeV [9].

with the asymmetry parameter δ^2, where $\delta = (N - Z)/A$, for $T = 1,\ 1.5,\ 2$ MeV. For all temperatures, the symmetry coefficient peaks for ^{74}Ni and has the lowest value for the doubly magic ^{78}Ni. This trend can be explained as follows. Equation (8) indicates that a higher value of total effective mass at $T = 0$, $m^*(T = 0)/M$, corresponds to a higher value of the symmetry coefficient $S(T)$. Since the k mass is temperature-independent, the trend of $m^*(T = 0)/M$ throughout the Ni isotopes is solely determined by the ω mass. According to eqs. (5)-(7), for each temperature T, the ω mass is inverse proportional to the dominant spectroscopic factors and the BCS occupation probabilities of the single-quasiparticle states around Fermi surface. As shown in ref. [9], the $^{72-76}$Ni isotopes exhibit the lower spectroscopic factors, which implies the stronger fragmentations, as compared to those of ^{68}Ni and ^{78}Ni isotopes. Furthermore, the ^{74}Ni isotope has the lowest BCS occupation probability, *i.e.*, $v^2_{(k)} = 0.53$ at the Fermi surface, which leads to the enhancement of the effective mass $m^*(T = 0)/M$, and, hence, the highest symmetry coefficient $S(T)$.

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The work of EL was supported by the US-NSF Grant PHY-2209376 and US-NSF CAREER Grant 1654379.

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