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Testing effective dark energy models in gravitationally bound systems: The case of Non-local Gravity(*)

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Summary. — The forthcoming generation of astrophysical and cosmological surveys are expected to generate an extensive and groundbreaking collection of data. The unmatched quantity and quality of these observations should boost our knowledge of the Universe, especially in regards to the dark sector. The true nature of the dark energy, either gravitational or related to fundamental fields, may therefore be unveiled in the next decades. Within this framework, the following contribution investigates the viability of a dark energy model based upon Non-local Gravity. The main cosmological features of the model are presented, and the impact of non-locality within the non-linear regime is analysed through astrophysical tests.

1. – Introduction

The origin of modern cosmology is tightly interwoven with the Einstein's breakthrough in gravitational physics. General Relativity (GR) enables the description of an expanding, homogeneous and isotropic Universe, in agreement with the primary observations. Therefore, GR is usually picked as the main pillar of cosmological theories, leading to the establishment of the so-called Λ Cold Dark Matter (ΛCDM) paradigm as the standard model of cosmology. Building on the predictions of GR, this model explains the puzzling astrophysical and cosmological observations through the introduction of two dark fluids, which should account for the $\sim 95\%$ of the matter-energy content of the Universe [1]. A dark matter component, constituting the $\sim 27\%$ of the total energy budget, is responsible for the structure formation; a dominant dark energy component ($\sim 68\%$) is introduced to account for the late time cosmic acceleration. Since the nature of the dark fluids is still unknown, the physics of the associated dark sector represents the most intriguing research area for both cosmology and particle physics.

Nevertheless, several shortcomings and recently risen tensions make room for alternative cosmological models as well as extended theories of gravity. The presence of singularities, along with the inconsistency at quantum level, plagues GR on ultraviolet (UV) scales. In addition, the ineffective effort to find out the true nature of the dark fluids as well as the so-called cosmological tensions pose severe challenges to the reliability of

(*) IFAE 2023 - "Cosmology and Astroparticles" session the ΛCDM paradigm in the infrared (IR) regime [2]. To tackle these issues, one potential approach is to reinterpret the discrepancy between observations and GR predictions as a missing geometry rather than a missing matter/energy. Such an opportunity is ensured by the Bianchi identities, that provide the direct correspondence between the geometric Degrees of Freedom (DoF) and the DoF of the stress-energy tensor.

According to the Lovelock's theorem, there exist only five options to extend the geometrical content of our theory of gravity [3]: (i) consider other fundamental fields, along with or instead of the metric tensor; (ii) move to higher-order field equations; (iii) consider a n-dimensional spacetime, with $n \neq 4$; (iv) accept field equations that contain either different than rank-(2, 0) tensors, non symmetric objects, or non-divergenceless terms; (v) abandon the locality principle. In this contribution, I investigate the physical consequences of the latter option, *i.e.*, of the non-locally extended theories of gravity.

In sect. **2**, the main aspects of Non-local Gravity theories are presented. Then, I focus on the theoretical features of a specific non-local model, originally proposed in [4]. In sect. **3**, I overview the pivotal results of this model within the cosmological framework. Finally, the viability tests of the non-local model at different astrophysical scales are discussed in sect. **4**. The conclusions are drawn in sect. **5**.

2. – Non-local Gravity

Two approaches exist for inducing non-locality into the gravitational interaction. On the one hand, the theory can be non-local at the kinematic level, specifically in the definition itself of spacetime. The introduction of a minimal length scale and, therefore, the discretization of the spacetime is the most natural way to account for the non-local kinematics. On the other hand, it is possible to give up locality at the dynamical level through the addition of non-local operators into the gravitational lagrangian.

In this contribution, I focus on the latter class of gravitational theories. Dynamical non-locality can be seen as either a fundamental feature of gravity or an effective approach. In the standard framework of Quantum Field Theory (QFT) is not possible to obtain a theory of gravity that is ghost-free, renormalizable and local at once [5]. As a consequence, one of these principles has to be abandoned, provided that no new ingredients are added to QFT. Since it is almost unavoidable to ask for unitarity and renormalizability, locality may be given up and non-locality could therefore represent an intrinsic feature of gravity. Nevertheless, non-locality emerges in QFT when we move to one-loop effective actions, as the signal that we are reaching the energy scale beyond which the effective theory is no longer viable and additional fields have to be introduced. Accordingly, the non-local corrections may be included in the action as an effective approach to ameliorate the behaviour of the gravitational interaction both in the UV and IR regime, where the General Relativity might be breaking down.

Regardless of the interpretation of the dynamical non-locality, the non-locally extended metric theories of gravity can be grouped according to the form of the additional operators. On the one hand, the Infinite Derivatives theories of Gravity (IDGs) are characterized by entire analytic functions of a differential operator. Due to the derivative nature of the non-local operators, the IDGs introduce non-locality on small scales. They are therefore used to fix the UV shortcomings of GR, such as the lack of unitarity caused by bad ghosts and the presence of singularities. On the other hand, the Integral Kernel theories of Gravity (IKGs) show non-local operators made of transcendental functions of the geometric fields. These operators can always be rewritten as the integral kernels of differential operators, such as

$$\Box^{-1}R(x) \equiv \int d^4x' G(x,x')R(x') \,. \tag{1}$$

This kind of non-local operators are inspired by quantum corrections, resulting from either the application of non-perturbative methods to the dimensional regularization of QFTs on curved spacetime, or the spontaneous breaking of the conformal symmetry in the early Universe. Due to the integral nature of IKG operators, the Integral Kernel theories give rise to long-range non-localities that are usually applied for addressing the IR issues of GR, *e.g.*, the late time cosmic acceleration.

In this contribution, I overview the cosmological and astrophysical features of a curvature-based IKG, originally proposed in [4] to explain the late time cosmic acceleration

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R\big[1+f(\Box^{-1}R)\big]\right\}. \tag{2}$$

The non-local correction is here inserted in the Hilbert-Einstein Lagrangian by adding a distortion function, namely a general function of the inverse d'Alembert operator. Moreover, the non-locality can be encoded in two auxiliary scalar fields [6]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R\big(1+f(\eta)-\xi\big) - \nabla^\mu\xi\,\nabla_\mu\eta\right], \tag{3}$$

where

$$\Box\eta = R \qquad\qquad \Box\xi = -R\,\frac{df(\eta)}{d\eta}\,, \tag{4}$$

hence the non-local model in eq. (2) can be rewritten as a biscalar-tensor theory. The associated gravitational field equations therefore are

$$\begin{aligned} G_{\mu\nu} \;=\; & \frac{1}{1+f(\eta)-\xi}\Bigg\{\kappa\,T^{(m)}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,\partial^\alpha\xi\,\partial_\alpha\eta \\ & + \frac{1}{2}\Big(\partial_\mu\xi\,\partial_\nu\eta + \partial_\mu\eta\,\partial_\nu\xi\Big) - \Big(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\Big)\Big[f(\eta)-\xi\Big]\Bigg\}. \end{aligned} \tag{5}$$

3. – Non-local Cosmology

The non-local model from eq. (2) and eq. (3) shows an extremely appealing cosmological phenomenology. It indeed provides a viable mechanism to account for the late time cosmic acceleration through a delayed response to the radiation-to-matter dominance transition [4]. The non-local contribution, when evaluated in the Friedmann-Lemaître-Robertson-Walker metric, vanishes during the radiation epoch, and then starts to grow after t_{eq} (*i.e.*, after the transition to the matter epoch)

$$
\begin{aligned}
\left[\Box^{-1}R\right](t) &= \int_0^t dt' \frac{1}{a^3(t')} \int_0^{t'} dt'' a^3(t'') R(t'') \\
&= -\frac{6s(2s-1)}{3s-1}\left[\ln\left(\frac{t}{t_{eq}}\right) - \frac{1}{3s-1} + \frac{1}{3s-1}\left(\frac{t_{eq}}{t}\right)^{3s-1}\right],
\end{aligned}
\tag{6}
$$

where $a(t) \sim t^{\,s}$. However, the evolution of the non-local cosmological term is significantly slow, and it therefore remains subdominant until very low redshift. Around $z = 1$, the gravitational correction finally becomes non-negligible and drives the onset of the cosmic acceleration, thus preventing the introduction of any fine-tuned cosmological constant.

The non-local model is therefore able to reproduce the ΛCDM expansion history through an effective dark energy contribution from the gravitational correction. However, despite the equivalent background evolution, the non-locality affects the growth of the cosmic structures [7]. The growth tension, associated to the early- and late-time estimates of the S_8 parameter, is alleviated accordingly [8].

4. – Astrophysical tests of Non-local Gravity

As highlighted in sect. **3**, the additional DoF of the non-local model affect the non-linear regime, hence the cosmic structures. The astrophysical systems therefore emerge as the optimal framework to benchmark non-local gravity against GR and the associated ΛCDM model.

In order to yield the non-local theoretical predictions to be compared with the astrophysical data, it is necessary to specify the form of the distortion function. Two approaches exist: $f(\eta)$ can be derived either phenomenologically by matching the cosmic expansion history, or theoretically by building upon first principles. Here I focus on the latter approach, thus selecting the distortion function through the existence of Noether symmetries in a spherically symmetric spacetime. Note that the spherical symmetry holds for each of the astrophysical systems that will be presented in the following. The distortion function therefore reads [9]

$$
f(\eta) = 1 + e^{\eta}\,. \tag{7}
$$

Once provided an explicit form for the non-local action in eq. (3), the weak field limit can be performed to derive the Newtonian potentials

$$
\Phi(r) = -\frac{GM}{r} + \frac{G^2M^2}{2c^2r^2}\left[\frac{14}{9} + \frac{18r_\xi - 11r_\eta}{6r_\eta r_\xi}\,r\right], \tag{8}
$$

$$
\Psi(r) = -\frac{GM}{3r} - \frac{G^2M^2}{2c^2r^2}\left[\frac{2}{9} + \frac{3r_\eta - 2r_\xi}{2r_\eta r_\xi}\,r\right]. \tag{9}
$$

The magnitude of the higher order non-local corrections depend on the non-local length scales, r_η and r_ξ, associated to the auxiliary scalar fields. These additional gravitational radii represent the main astrophysical signature of Non-local Gravity.

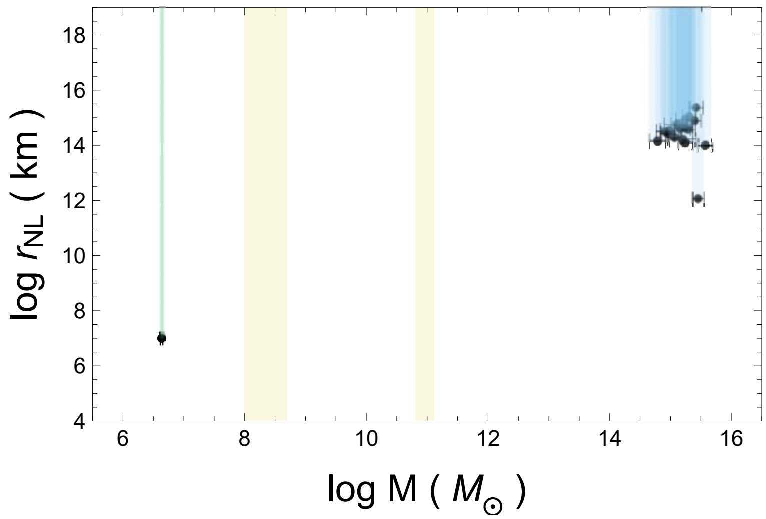

Fig. 1. – Lower bounds on the non-local gravitational radius from the analysis of the orbits of the star S2 around SgrA* [9] (in green), and the gravitational lensing by the CLASH clusters [10] (in blu). The kinematic of UDGs in the framework of non-local gravity is currently under investigation, and should provide new constraints at intermediate mass scales (in yellow).

The gravitational potential from eq. (8) has been used in [9] to test the non-local model through the orbits of the star S2 around the galactic centre. Comparing the two-body simulations with the astronomic observations by the New Technology Telescope and the Very Large Telescope, the authors have been able to set constraints on the non-local length scales (see fig. 1). Moreover, they have shown that the Non-local Gravity model is able to reproduce the orbits of S2 around SgrA* at the same level of statistical significance as GR.

Furthermore, the gravitational and the metric potential, from eq. (8) and eq. (9) respectively, have been used to test the non-local scenario at galaxy cluster scales [10]. These systems, being the most prominent density peaks of the large-scale structure in the Universe, act as powerful cosmic lenses, hence privileged systems to test both the standard cosmological model and the underlying theory of gravity. Gravitational lensing is a very well modelled phenomenon indeed, which can be regarded as a primary probe since it is mainly sensitive to gravity, whereas the complex baryonic effects slightly affect it. The modelling of the lensing phenomenology within the Non-local Gravity framework yields [10]

$$\kappa(R) \equiv \frac{1}{c^2}\frac{D_{LS}D_L}{D_S}\int_{-\infty}^{+\infty}\nabla_r^2\left(\frac{\Phi(R,z)+\Psi(R,z)}{2}\right)\mathrm{d}z\,, \tag{10}$$

where the Newtonian potentials have been generalized to extended mass distributions, accounting for the fact that the Gauss theorem does not hold in Non-local Gravity. The theoretical predictions from eq. (10) have been then compared with the lensing data from the CLASH survey [11]. Our sample is composed of nineteen massive galaxy clusters, that span a redshift range of $0.187 \leq z \leq 0.686$, with a median redshift of $z_{med} = 0.352$. The resolution limit of the mass reconstruction, set by the *Hubble Space Telecsope* and the Subaru Telescope lensing data, is 10 arcsec, hence $\sim 35\,h^{-1}$ kpc at z_{med}. The Markov Chain Monte Carlo analysis of the non-local model in light of the CLASH data provides lower bounds on the non-local radii (see fig. 1). Moreover, our analysis clearly shows that there is no evidence in favour of General Relativity with respect to Non-local Gravity in the context of galaxy cluster lensing.

We are currently carrying out additional test of the non-local model at galactic scales as to test its viability in intermediate mass systems. The analysis of the kinematics of Ultra-Diffuse Galaxies (UDGs), both dark matter- and baryon-dominated, should also yield new constraints on the non-local gravitational radii at intermediate mass scales. An empirical relation $r_{NL}(M)$ may be derived accordingly.

5. – Conclusions

Non-locality emerges as an interesting feature in the gravitational framework to address both the UV and the IR issues of the ΛCDM model. Throughout this contribution, I have presented a non-local metric model of gravity, which is able to perfectly reproduce the cosmic expansion history without any additional dark energy component. While waiting for the data of the IV generation cosmological surveys, that may break the degeneracy among dark energy models, the gravitationally bound systems represent a rewarding test bench for those models based upon extensions of GR. The Non-local Gravity model under consideration has been tested on a wide range of astrophysical scales, ranging from the galactic centre up to massive galaxy clusters, showing no evidence for spoiling effects. Moreover, the additional gravitational radii provided by the theory may be constrained through future analysis at intermediate mass scales, thus yielding a fundamental tool to investigate the structure formation within the Non-local Gravity framework.

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