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# Probing the tau lepton magnetic moment at future lepton colliders(*)

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**Summary.** —
The anomalous magnetic moments of leptons are excellent candidates not only to test the Standard Model predictions, but also to investigate possible new physics effects. The long-standing discrepancy between the Standard Model prediction and the experimental measure of the muon $g-2$ motivates the study of non-standard effects also in the electron and tau $g-2$. In our work, we show the potential sensitivity of future lepton colliders (FCC-ee or a high-energy Muon Collider) to probe the tau $g-2$. We point out that these facilities can generate processes like the radiative Higgs decay $h \to \tau^+\tau^-\gamma$ or the Drell-Yan processes $\ell^+\ell^- \to \tau^+\tau^-(h)$ enabling to test the tau $g-2$ with a resolution of $\mathcal{O}(10^{-5} - 10^{-4})$ that is orders of magnitude better than the current LEP sensitivity.

## 1. – Introduction

The anomalous magnetic moment of the muon has provided, over the last ten years, an enduring hint for new physics (NP) therefore motivating a large number of theoretical speculations. Indeed, when comparing the Standard Model (SM) prediction for $a_\mu \equiv (g-2)_\mu/2$ [1] with the current experimental world average, based on the measurements by the E821 experiment at BNL [2] and the Muon $g-2$ experiment at Fermilab [3, 4], one observes a significant $\sim 5\,\sigma$ discrepancy

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 244(45) \times 10^{-11} \tag{1}$$

If NP is really emerging in the leptonic sector, a precise measurement of $a_\tau$ would represent an outstanding opportunity to probe it. In fact, in the conservative naive scaling scenario [5], where $\Delta a_\tau/\Delta a_\mu = m_\tau^2/m_\mu^2$, the discrepancy (1) would imply $\Delta a_\tau \approx 10^{-6}$.

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However, large enhancements in the tau sector are predicted by a wide variety of beyond the SM (BSM) scenarios through larger couplings to the third lepton generation [5].

Unluckily, the short lifetime of $\tau$ lepton prevents the $(g-2)_\tau$ extraction directly from the measurement of the $\tau$ spin precession in a magnetic field, as it was the case for electrons and muons. Indirect determinations of $a_\tau$ must be then devised, exploiting for instance high-energy processes that display tau leptons in the final state. This is the case for the present PDG limit, which reports the bound extracted from the total cross-section measurements of $e^+e^- \to \tau^+\tau^-(e^+e^-)$ by the DELPHI collaboration. The corresponding limit at 95% CL is [6]

$$-0.052 < a_\tau^{\rm EXP} < 0.013\,, \tag{2}$$

which is roughly one order of magnitude below the one-loop QED effect $\alpha/2\pi \simeq 0.001$.

Several experimental proposals have been suggested to improve the sensitivity on $a_\tau^{\rm EXP}$ such as radiative tau decays [7], measurements at the LHC using bent crystals [8] or the reaction $\gamma\gamma \to \tau^+\tau^-$ in ultraperipheral heavy ion collisions [9]. Nonetheless, in either cases, the experimental sensitivity is expected at the per-cent level.([1])

If present-day colliders are unlikely to significantly improve the LEP bound on $a_\tau$, we will show that the situation could drastically change at future lepton colliders such as FCC-ee [13] or a high-energy Muon Collider (MC) [14]. Assuming that NP is heavy, we will show that these facilities can probe $a_\tau$ with a resolution of $\mathcal{O}(10^{-5} - 10^{-4})$ through the processes $h \to \tau^+\tau^-\gamma$ and $\ell^+\ell^- \to \tau^+\tau^-(h)$.

## 2. – Leptonic $g-2$ in the Standard Model Effective Field Theory

Under the assumption that the new degrees of freedom beyond the SM ones are heavy, emerging at a typical NP scale $\Lambda \gg v = 246$ GeV, their interactions at energies $E \ll \Lambda$ can be parametrized in a model-independent fashion by the Standard Model effective field theory (SMEFT). The SMEFT is built out of a tower of non-renormalizable operators that are invariant under the SM gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ [15,16]. Among such operators, the ones that have an impact on the leptonic anomalous magnetic moment are encoded in the following Lagrangian term:

$$\begin{aligned}\mathcal{L} = \frac{C^\ell_{eB}}{\Lambda^2}\,(\bar\ell_L\sigma^{\mu\nu}e_R)HB_{\mu\nu} + \frac{C^\ell_{eW}}{\Lambda^2}\,(\bar\ell_L\sigma^{\mu\nu}e_R)\tau^I HW^I_{\mu\nu}\\ + \frac{C^\ell_T}{\Lambda^2}(\bar\ell^a_L\sigma_{\mu\nu}e_R)\varepsilon_{ab}(\overline{Q}^b_L\sigma^{\mu\nu}u_R) + \text{h.c.}\end{aligned} \tag{3}$$

Starting from eq. (3), the resulting expression for $\Delta a_\tau$ is given by [17]

$$\Delta a_\tau \simeq \frac{4m_\tau v}{e\sqrt{2}\Lambda^2}\left(C^\tau_{e\gamma} - \frac{3\alpha}{2\pi}\frac{c_W^2 - s_W^2}{s_W c_W}\,C^\ell_{eZ}\log\frac{\Lambda}{m_Z}\right) - \frac{4m_\tau m_t}{\pi^2}\frac{C^{\tau t}_T}{\Lambda^2}\,\log\frac{\Lambda}{m_t}, \tag{4}$$

([1]) So far, the most ambitious proposal to extract $a_\tau$ is at Belle II, through transverse and longitudinal asymmetries in tau pair events [10-12], which requires a longitudinally polarized electron beam. It has to be seen if the claimed outstanding resolution of $\sim 10^{-6}$ is indeed feasible.

where $s_W(c_W)$ is the sine (cosine) of the weak mixing angle, $C_{e\gamma} = c_W C_{eB} - s_W C_{eW}$ and $C_{eZ} = -s_W C_{eB} - c_W C_{eW}$. From a direct inspection of eq. (3) and eq. (4), it is apparent that the very same operators that can induce an effect on the $g-2$ of the tau will on general grounds also generate decay and scattering processes involving a physical Higgs field which can be used in order to indirectly probe $\Delta a_\tau$. Indeed, while the effects on $(g-2)_\tau$ in eq. (4) stem from the selection of the electroweak vacuum expectation value $v$ from $H \propto (h+v)$, rare decays such as $h \to \tau^+\tau^-\gamma$ and the scattering processes $\mu^+\mu^- \to \tau^+\tau^-(h)$ are obtained when a Higgs field is considered.

All of these processes can be conveniently studied at future lepton colliders such as FCC-ee or MC. Indeed, both the high luminosity and the large center-of-mass energy attainable at such facilities would positively impact on the relative significance of the NP signal over the SM background. In particular, besides the larger statistics due to a higher luminosity, larger NP effects can be fueled by the dependence of the contact dipole operators on the energy injected in it. Larger energies are then expected to suprress the SM contributions ($\propto 1/s$) while enhancing some NP ones ($\propto s/\Lambda^4$).

Before moving to the discussion of the specific processes and the relative bounds on $\Delta a_\tau$, a comment on the validity of our EFT has to be made. The fundamental assumption of our approach is indeed that the energy $E$ entering each one of the effective vertices is significantly smaller than the cutoff scale $\Lambda \gtrsim 1$ TeV. In the case of higgs decays, our discussion will be senseful as long as $m_h \ll \Lambda$, which is always satisfied in the SMEFT. Instead, for the process $\mu^+\mu^- \to \tau^+\tau^-(h)$ our EFT approach is valid as long as $\sqrt{s} \ll \Lambda$.

## 3. – Leptonic $g-2$ from rare Higgs decays

In this section, we discuss the relation existing between the lepton $g-2$ and the radiative Higgs decays $h \to \ell^+\ell^-\gamma$. As a consequence of the large luminosity, and the growth with energy of the vector-boson-fusion cross-section, a very large number number of Higgs bosons is expected to be produced at a high-energy lepton collider [18]. In particular, a MC running at $\sqrt{s} = 30$ TeV with an integrated luminosity of $90\,\text{ab}^{-1}$ will produce $\mathcal{O}(10^8)$ Higgs bosons. With the precision of Higgs couplings measurements most likely limited by systematic errors, the main advantage of having such a large number of events is the possibility to look for very rare decays of the Higgs.

The dipole operator $O_{e\gamma} = (v+h)/\sqrt{2}\ \text{Re}C^\ell_{e\gamma}/\Lambda^2\ \bar{\ell}\sigma^{\mu\nu}\ell$ contributes to the rare decay $h \to \ell^+\ell^-\gamma$ as encapsulated by the following expression:

$$\Gamma_{h\ell\ell\gamma} = \Gamma_{\text{SM}} + \frac{e y_\ell m_h^3}{64\pi^3}\frac{\text{Re}\,C^\ell_{e\gamma}}{\Lambda^2} + \frac{m_h^5}{768\pi^3}\frac{|C^\ell_{e\gamma}|^2}{\Lambda^4}. \tag{5}$$

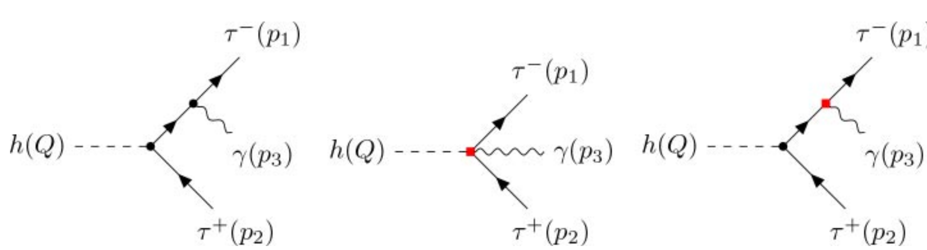


Fig. 1. – Examples of Feynman diagrams contributing to the decay $h \to \tau^+\tau^-\gamma$ at leading order. Red dots represent NP insertions, while black dots denote the insertion of a SM vertex.

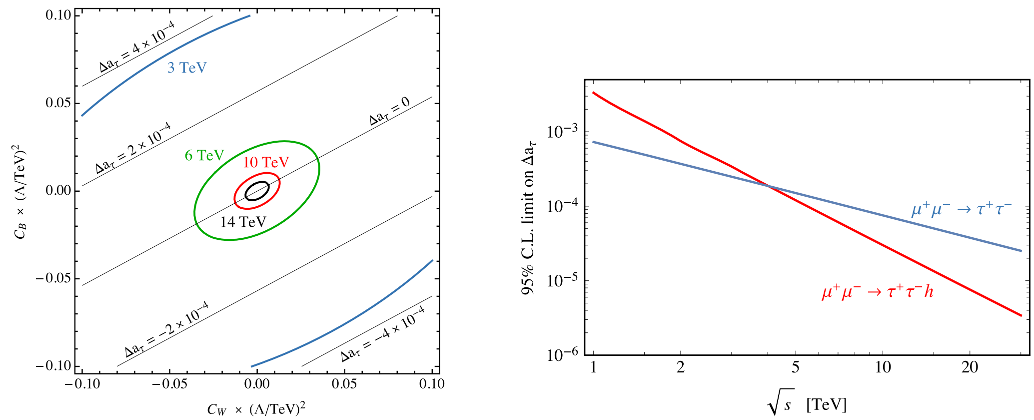


Fig. 2. – Left: 95% CL limits on the Wilson coefficients $C_{eW}$ and $C_{eB}$ from $\mu^+\mu^- \to \tau^+\tau^- h$ at muon colliders of different energies; the black isolines indicate the corresponding value of $\Delta a_\tau$. Right: limit on $\Delta a_\tau$ from $\mu^+\mu^- \to \tau^+\tau^-, \tau^+\tau^- h$ as a function of center-of-mass energy $\sqrt{s}$.

where $\Gamma_{h\ell\ell\gamma} \equiv \Gamma(h \to \ell\ell\gamma)$. This result can be combined with eq. (4) to give

$$\frac{\mathcal{B}_{h\tau\tau\gamma}}{\mathcal{B}^{\rm SM}_{h\tau\tau\gamma}} \approx 1 + 0.02\left(\frac{\Delta a_\tau}{10^{-4}}\right) + 2\times 10^{-4}\left(\frac{\Delta a_\tau}{10^{-4}}\right)^2 . \tag{6}$$

A sensitivity to $\Delta a_\tau$ of order $\Delta a_\tau \lesssim 10^{-4}$ could be attained through $h \to \tau^+\tau^-\gamma$. Interestingly enough, the leading NP effect in $\mathcal{B}_{h\tau\tau\gamma}$ is induced by the interference of the NP diagrams with the SM ones. The very same Wilson coefficients $C^\ell_{eB}$ and $C^\ell_{eW}$ contributing via their linear combination $C^\ell_{e\gamma}$ to $\Gamma_{h\ell\ell\gamma}$ will generate non-null contributions to the decays $h \to \ell^+\ell^- Z$ and $h \to W\ell\nu$. However, it turns out that the above processes could probe $\Delta a_\tau$ only with a resolution of $10^{-2}$ and $10^{-1}$, respectively, thus resulting in weaker bounds if compared to the photon channel [19].

## 4. – Leptonic $g-2$ from Drell-Yan processes

Another class of processes that can be studied in order probe NP effects on $\Delta a_\tau$ at a future high-energy lepton collider are the scattering processes $\mu^+\mu^- \to \tau^+\tau^-(h)$. To put constraints on $\Delta a_\tau$, we have computed both the differential and the total cross-section for the process $\mu^+\mu^- \to \tau^+\tau^-$ including both SM and NP contributions.

As far as the three-body final state scattering, we have both computed the analytical expressions for the differential cross sections and we have performed a MadGraph simulation of both the NP signal and the SM background at the parton level. In our simulation, we considered the Higgs boson decaying into a pair of $b$ quarks. At a lepton collider, the main sources of background are the SM irreducible contribution to $\mu^+\mu^- \to \tau^+\tau^- h$, and the reducible $\mu^+\mu^- \to \tau^+\tau^- Z$ process where the hadronically decaying $Z$ boson can be misidentified for a Higgs, for which we assume a mistag probability $\epsilon_{Z\to h} = 15\%$. In addition we included an 80% efficiency for tau identification, and a 50% efficiency for the reconstruction of a boosted Higgs decaying into $b\bar{b}$. We imposed basic acceptance cuts $\eta < 3$ for all final state particles (including the boosted Higgs boson), and further required $\Delta R_{\tau\tau} > 0.4$. In order to suppress the SM backgrounds we imposed the following analysis cuts:

$$p_{T,\tau} > E_{\rm cm}/10, \qquad p_{T,h} > E_{\rm cm}/10, \qquad M_{\tau\tau} > E_{\rm cm}/10, \tag{7}$$

where $p_{T,\tau}, p_{T,h}$ are the transverse momenta of the taus and of the Higgs boson, and $M_{\tau\tau}$ is the di-tau invariant mass. These cuts have an efficiency of roughly 70% on the signal at all energies, and reduce both the irreducible SM $\tau\tau h$ background and the reducible $Z\tau\tau$ by about one order of magnitude.

Such analyses on both the analytic formulas and on the simulated data allowed us to maximise the signal-to-noise ratio and perform a thorough significance study on the expected signal. Its result in turn were used to put our final bounds in fig. 2.

In the left panel of fig. 2 we show the 95% CL contours on the Wilson coefficients $C_{eW}$ and $C_{eB}$ at various collider center-of-mass energies. The black isolines show the corresponding value of $\Delta a_\tau$; a 3 TeV muon collider would be sensitive to $\Delta a_\tau \approx 3\times 10^{-4}$, while a 10 TeV collider would reach values $\Delta a_\tau \approx 3 \times 10^{-5}$. In the right panel of the same figure we show the reach on $\Delta a_\tau$ as a function of center-of-mass energy $E_{\rm cm}$, where the red line thows the constraint from $\mu^+\mu^- \to \tau^+\tau^- h$, while the blue line is the limit from $\mu^+\mu^- \to \tau^+\tau^-$ pair production.

The interesting feature of these results is that there exists an energy below which it is the $2 \to 2$ decay process that allows one to put the best bounds on $\Delta a_\tau$, but above which the most competitive process is rather the $2 \to 3$ one. This can be ascribed to the growth with the energy of the NP contribution which eventually overcomes the phase-space suppression experienced by the $2 \to 3$ scattering process around 4 TeV. Such a feature can be directly seen from the leading contributions to the differential scattering amplitudes for the two cases:

$$\frac{d\sigma_{\tau\tau}}{d\,c_\theta} = \frac{d\sigma^{\rm SM}_{\tau\tau}}{d\,c_\theta} + \frac{\alpha}{4}\frac{v^2}{\Lambda^4}\left(|C^\tau_{e\gamma}|^2 + \frac{|C^\tau_{eZ}|^2}{16c_W^2 s_W^2}\right)s_\theta^2 + \frac{v^2 y_\tau}{\Lambda^2 s}\alpha\, eC^\tau_{e\gamma}\left(1+\frac{c_\theta}{16c_W^2 s_W^2}\right) \tag{8}$$

where $\theta$ is the scattering angle between the two final-state $\tau$ and

$$\frac{d\sigma_{\tau\tau h}}{dx_1 dx_2} = \frac{d\sigma^{\rm SM}_{\tau\tau h}}{dx_1 dx_2} + \frac{y_\tau \alpha}{\Lambda^2}\frac{e{\rm Re}\, C^\tau_{e\gamma}}{24\pi^2} + \frac{\alpha}{48\pi^2}\frac{s}{\Lambda^4}\left(|C^\tau_{e\gamma}|^2 + \frac{|C^\tau_{eZ}|^2}{16c_W^2 s_W^2}\right)(1-x_1-x_2+2x_1x_2)\,. \tag{9}$$

Here $x_i \equiv Q\cdot k_i/Q^2$, where $Q$ is the momentum injected in the center of mass by the pair of colliding leptons, while $k_i$ is the momentum of one of the three final-state particles (here 1 and 2 label the two-final state tau leptons).

## 5. – Conclusions

In our work we investigated the possibility to probe heavy NP effects on the tau $g-2$ as induced by dipole operators in the SMEFT. Such effects can be studied indirectly by considering decay and scattering processes involving final-state tau leptons, like the Higgs radiative decays $h \to \tau^+\tau^-\gamma$, or the scattering processes $\ell^+\ell^- \to \tau^+\tau^-(h)$. The high luminosity and the large center of mass energy that will be available at future lepton colliders (FCC-ee or muon colliders) make these facilities the best possible ones where to generate a large number of such processes. In particular, we showed that studying the aforementioned processes at future lepton colliders would allow us to test NP effects to $(g-2)_\tau$ with a resolution of $\mathcal{O}(10^{-5}-10^{-4})$, thus improving the current bounds by two to three orders of magnitude.

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