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New theory highlights on B_c decays(*)

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Summary. — We present recent results in semileptonic and non-leptonic exclusive B_c decays to charmonium states both in S-wave, J/ψ and η_c, and in P-wave, χ_{cJ} and h_c. The analysis, based on the heavy quark spin symmetry (HQSS), produces relations among form factors that parametrize the hadronic matrix elements in the amplitudes of the decays. These relations are helpful to control the hadronic uncertainty affecting these processes. Furthermore, B_c decays allow us to get hint on the structure of states like $\chi_{c1}(3872)$, whose exotic or ordinary charmonium nature is debated.

1. – Introduction

We focus on the heavy hadron decays induced by the $b \to c$ transition at the quark level. These decays allow us to measure a fundamental parameter of the Standard Model (SM), the element $|V_{cb}|$ of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The analyses of several processes induced by the same quark transition but involving different hadrons in the initial or final states (exclusive/inclusive) must provide compatible results. This has not been achieved, yet, considering the $|V_{cb}|$ determinations from inclusive and exclusive B decays which fuel discussions on possible effects within the SM or beyond [1, 2]. These decays provide also information on fundamental properties of the Standard Model, namely Lepton Flavour Universality (LFU). Signals of LFU violation have been detected in B decays induced by this transition [3]. Such an observation would hint to physics beyond SM, the structure of which can be constrained by the analysis of various decay observables [4-6].

Furthermore, these decays allow us to obtain information about the strong interaction between the quarks composing the mesons. For decays involving hadrons comprising a single heavy quark Q, a double expansion in powers of $1/m_Q$ and of α_s can be derived in QCD, providing a powerful method to classify the hadronic matrix elements, both for

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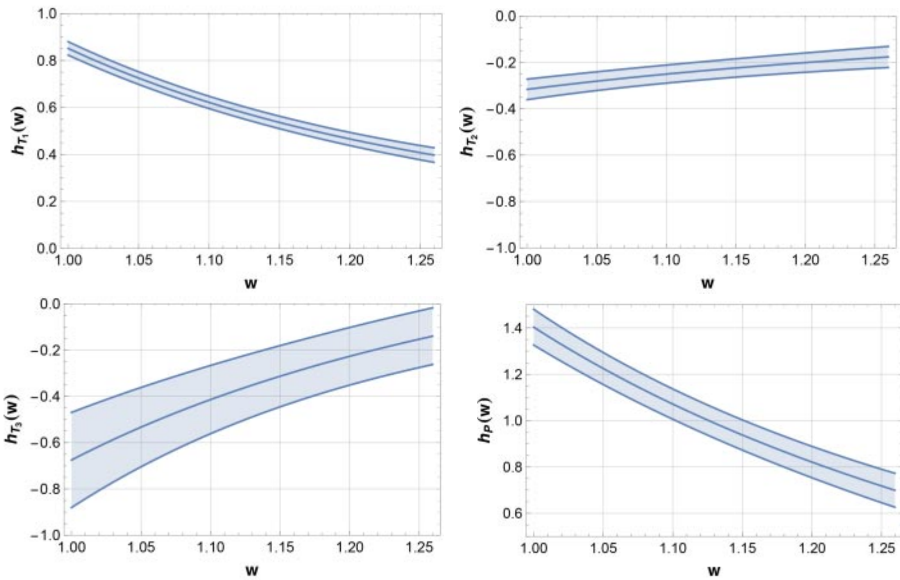

Fig. 1. – Tensor and pseudoscalar $B_c \to J/\psi$ form factors in the full kinematical range and using lattice QCD results as input.

exclusive and inclusive transitions [7, 8]. This is the basis for a control of the theoretical uncertainty in the measurements. For mesons comprising two heavy quarks such as B_c, the expansion parameter is the relative three-velocity of the heavy quarks, with counting rules given by Non-relativistic QCD (NRQCD) [9].

2. – Semileptonic B_c decays

In [10] relations have been obtained among the form factors governing $B_c \to J/\psi(\eta_c)\ell\bar{\nu}_\ell$ near the zero-recoil point. Using the form factors h_{A_1},h_{A_2}, h_V determined by lattice QCD [11] and exploiting such relations, other form factors have been computed. As an example, fig. 1 shows the tensor and the pseudoscalar form factors for the transition to J/ψ obtained from the relations in [10] and the lattice QCD results [11]. The analysis has been also extended to the second order in $1/m_Q$ [10].

The method has been applied to the analysis of the B_c transitions to P-wave charmonia [12]. In a selected kinematical range the B_c and the P-wave charmonium matrix elements can be expressed as an expansion in the heavy quark relative three-velocity in the heavy hadrons, together with an expansion in the inverse heavy quark mass. In this range the form factors describing the decays of B_c to the four P-wave charmonia (the lowest lying or the radial excitations) are related. The relations among different modes can be experimentally verified. Their violation or confirmation can be used to obtain information on the nature of mesons belonging to the same spin multiplet. This is relevant for $\chi_{c1}(3872)$ (usually denoted as $X(3872)$). This meson shows features hinting to a non conventional charmonium structure. Due to the closeness of its mass to the $D^{*0}\bar{D}^0$ threshold and the large decay rate to $J/\psi\,\pi\pi$ $(J/\psi\,\rho)$ compared to $J/\psi\,\pi\pi\pi$ $(J/\psi\,\omega)$, proposals have been put forward that $X(3872)$ is an exotic state of multiquark structure [13]. On the

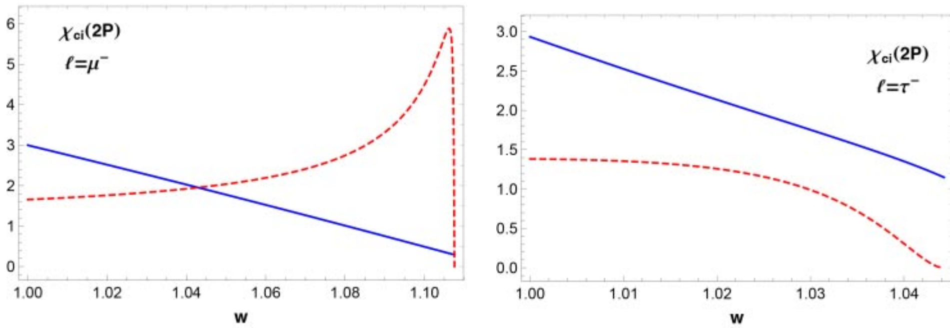

Fig. 2. – Ratios of the decay distributions $\frac{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c0}\ell\bar{\nu})/dw}$ (blue continuos line) and $\frac{d\Gamma(B_c\to\chi_{c2}\ell\bar{\nu})/dw}{d\Gamma(B_c\to\chi_{c1}\ell\bar{\nu})/dw}$ (red dashed line) in the case $\ell=\mu$ (left) and $\ell=\tau$ (right) for the $2P$ final charmonia, using the LO relations among form factors extrapolated to the full kinematical range.

other hand, the large ratio $\Gamma(X(3872)\to\gamma\,\psi(2S))/\Gamma(X(3872)\to\gamma\,J/\psi)$ and the production cross sections in e^+e^- and $\gamma\gamma^*$ can be better accommodated identifying $\chi_{c1}(3872)$ with $\chi_{c1}(2P)$ [14]. A confirmation of the relations obtained in [12] for $\chi_{c1}(3872)$ would point to the ordinary quark-antiquark structure. The behaviour expected for $\chi_{c1}(3872)$ considered as a radial excitation of χ_{c1} is shown in fig. 2.

3. – Non-leptonic B_c decays

The B_c non-leptonic transitions to P-wave charmonia complement the information on the structure of states as $\chi_{c1}(3872)$ from the semileptonic modes. Using the effective Hamiltonian describing the $b\to c\bar{q}_1q_2$ transition with $q_{1,2}$ light quarks, we have evaluated the decay widths of B_c to P-wave charmonium and a light pseudoscalar/vector meson. The LO relations for the form factors are used to predict several ratios of branching fractions. The effective Hamiltonian governing the non-leptonic B_c decays to charmonium and a light meson is given by

$$\mathcal{H}_{eff}=\frac{G_F}{\sqrt{2}}V^*_{cb}V_{uq}\left(C_1(\mu)Q_1(\mu)+C_2(\mu)Q_2(\mu)\right)+\text{h.c.},\tag{1}$$

where

$$\begin{aligned}Q_1&=\bar{u}_\alpha\gamma^\mu(1-\gamma_5)q_\alpha\bar{b}_\beta\gamma_\mu(1-\gamma_5)c_\beta,\\Q_2&=\bar{u}_\alpha\gamma^\mu(1-\gamma_5)q_\beta\bar{b}_\beta\gamma_\mu(1-\gamma_5)c_\alpha\,.\end{aligned}\tag{2}$$

α,β are colour indices, and q identifies the down-type quark of the final light meson. μ represents the scale dependence of the Wilson coefficients which encode the short-distance physics from energy greater than μ. It is cancelled in the amplitude by the μ dependence of the operators matrix elements. In the computation of B_c decays we set $\mu=m_b$. After a Fierz transformation and discharging the colour-octet operator, we obtain the Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} a_1(\mu) Q_1(\mu)\,, \tag{3}$$

where

$$a_1 = C_1 + \frac{1}{N_c} C_2. \tag{4}$$

For the processes $B_c^+ \to M_{c\bar{c}}(P)\, M_{P(V)}$, where $M_{c\bar{c}}(P)$ is one of the P-wave charmonium and P,V a light pseudoscalar or a vector meson, the decay width

$$\Gamma(B_c^+ \to M_{c\bar{c}}(P)\, M_{P(V)}) = \frac{|\mathbf{q}|}{8\pi m_{B_c}^2} |\mathcal{A}(B_c^+ \to M_{c\bar{c}}(P)\, M_{P(V)})|^2, \tag{5}$$

with $|\mathbf{q}| = \frac{1}{2m_{B_c}} \sqrt{\lambda(m_{B_c}^2, m_{M_{c\bar{c}}}^2, m_{P(V)}^2)}$ and $\lambda(x, y, z)$ the Källén function, involves the matrix element

$$\mathcal{A}(B_c^+ \to M_{c\bar{c}}(P)\, M_{P(V)}) = \langle M_{c\bar{c}}(P)\, M_{P(V)}|\, \mathcal{H}_{eff}\, |B_c^+\rangle\,. \tag{6}$$

The amplitude is factorized as

$$\begin{aligned}\mathcal{A}(B_c^+ \to M_{c\bar{c}}(P)\, M_{P(V)}) =& \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} a_1(\mu)\, \langle M_{c\bar{c}}(P)|\, \bar{b}\gamma_\mu(1-\gamma_5)c\, |B_c^+\rangle \\ &\times \langle M_{P(V)}|\, \bar{u}\gamma^\mu(1-\gamma_5)q\, |0\rangle\,.\end{aligned}$$

This involves the current-particle matrix elements

$$\langle M_P(q)|\, \bar{u}\gamma^\mu(1-\gamma_5)q\, |0\rangle = -i f_P q^\mu \tag{7}$$

$$\langle M_V(q, \epsilon_V)|\, \bar{u}\gamma^\mu(1-\gamma_5)q\, |0\rangle = m_V f_V \epsilon_V^{*\mu}\,, \tag{8}$$

and $\langle M_{c\bar{c}}(P)|\, \bar{b}\gamma_\mu(1-\gamma_5)c\, |B_c^+\rangle$ for which we used the parametrization in [12]. The use of the naive factorization approach is based on the Bjorken's colour transparency argument [15]. It is possible to partially encode nonfactorizable effects replacing the coefficient $a_1(\mu)$ ($a_2(\mu)$ for colour suppressed processes), by the effective coefficient $a_1^{eff}(\mu)$ ($a_2^{eff}(\mu)$) treated as a phenomenological parameter.

For the hadronic form factors of B_c to χ_{cJ} and h_c, we use the results described in the previous section at LO in $1/m_Q$ [12]. Remarkably, at LO some form factors vanish: a consequence is that the B_c transition to χ_{c1} and a light pseudoscalar is suppressed.

The results for the branching fractions for π^+ in the final state are in table I, together with the results for the $2P$ excitations and for the case with K^+ in the final state.

The modes with $\chi_{c1}(1P)$ and $\chi_{c1}(2P)$ are suppressed: the observation of such a suppression for $\chi_{c1}(3872)$ would favour the identification of $X(3872)$ as an ordinary charmonium state. The same results are obtained for the ρ^+ and K^{*+} modes [16]. The conclusion is that the production of χ_{c1} is suppressed and of h_c is enhanced compared to the other charmonia.

TABLE I. – *Ratios of branching fractions of B_c decays to charmonium state and π^+ or K^+ meson.*

	$\frac{\mathcal{B}(B_c^+\to\chi_{c0}\,\pi^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,\pi^+)}$	$\frac{\mathcal{B}(B_c^+\to h_c\,\pi^+)}{\mathcal{B}(B_c^+\to\chi_{c0}\,\pi^+)}$	$\frac{\mathcal{B}(B_c^+\to h_c\,\pi^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,\pi^+)}$
$1P$	0.658	2.429	1.597
$2P$	0.583	2.746	1.601

	$\frac{\mathcal{B}(B_c^+\to\chi_{c0}\,K^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,K^+)}$	$\frac{\mathcal{B}(B_c^+\to h_c\,K^+)}{\mathcal{B}(B_c^+\to\chi_{c0}\,K^+)}$	$\frac{\mathcal{B}(B_c^+\to h_c\,K^+)}{\mathcal{B}(B_c^+\to\chi_{c2}\,K^+)}$
$1P$	0.663	2.482	1.645
$2P$	0.586	2.845	1.668

4. – Conclusions

We have investigated a method based on heavy quark spin symmetry and the use of NRQCD power counting to obtain a systematic treatment of the form factors for the B_c to charmonium decays. The formalism allows us to organize the states in doublets and multiplets, obtaining relations between form factors and consequently among observables in different decay modes. Using lattice QCD results [11] for a few form factors entering in the description of the decay we can obtain others, namely the form factors parametrizing the matrix element of the tensor and pseudoscalar operator for the transition to J/ψ, and combinations of them in the case of η_c. The relations for the P-wave states can be employed to get information on $\chi_{c1}(3872)$. If this state behaves according to our predictions, there is a hint to the ordinary charmonium structure.

The non-leptonic decays of B_c mesons have been computed using a naive factorization approach. We focused on the decays involving the lowest-lying and first radial excitations of the P-wave charmonium states together with π^+, K^+, ρ^+, and K^{*+}. By employing the leading-order (LO) relations among the form factors for $B_c \to \chi_{cJ}(h_c)$ transitions [12], we have predicted several branching ratios. We have found that the χ_{c1} channel, both in the $1P$ and $2P$ multiplets, is suppressed. If $\chi_{c1}(3872)$ is a conventional charmonium we should observe such a suppression.

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