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Flavour anomalies, correlations, hadronic uncertainties, and all that

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Summary. — We present a short overview of the so-called flavour anomalies, discussing their significance and the connections with QCD issues discussed at the HADRON 2023 conference.

1. – Introduction

The Standard Model (SM) of fundamental interactions has achieved an astonishing success, both in precision and in extension, in describing the dynamics of the constituents of nature. Nevertheless, there are arguments suggesting that it is not the ultimate theory. Difficulties concern the neutrino masses and mixings, the baryon-antibaryon asymmetry in the universe and the dark matter. There are also conceptual issues to address, as the large number of unrelated parameters of the theory, the dynamics of the scalar sector and its impact on the fermion and gauge sectors, the arbitrariness of the quantum numbers assignment, not to say the connection with gravity. Many difficulties are rooted in the flavor sector of the model, where a few anomalies have emerged recently. The implications and perspectives of such anomalies deserve a discussion. They are connected with issues discussed at this conference, since the control of the hadronic uncertainties is of prime importance to assess the significance of the observed deviations.

2. – A list of tensions between SM expectations and measurements

Lepton Flavour Universality in exclusive semileptonic $b \to c$ transitions. – In SM the coupling of the gauge bosons to leptons is the same for all families. The *Lepton Flavour Universality (LFU)* property has, among others, the consequence that the ratios of semileptonic decay rates of B to charmed mesons can be precisely predicted if the hadronic form factors of the $b \to c$ current are efficiently constrained. Measurements of the ratios $R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)}\tau\bar{\nu}_\tau)}{\Gamma(B \to D^{(*)}\ell\bar{\nu}_\ell)}$, with $\ell = \mu, e$, produce a combined result which is 3.4σ away from SM [1-3], a result of debated interpretation. If the effect is attributed

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to a BSM phenomenon, the interpretation is easier in an effective field theory approach, allowing for $D = 6$ operators not present in the low-energy SM Hamiltonian

$$
\begin{aligned}
H_{\rm eff}^{b\to c\ell\nu} =& \frac{G_F}{\sqrt{2}} V_{cb} \Big[(1+\epsilon_V^\ell)\,(\bar c\gamma_\mu(1-\gamma_5)b)\,\big(\bar\ell\gamma^\mu(1-\gamma_5)\nu_\ell\big) \\
&+ \epsilon_R^\ell\,(\bar c\gamma_\mu(1+\gamma_5)b)\,\big(\bar\ell\gamma^\mu(1-\gamma_5)\nu_\ell\big) + \epsilon_S^\ell\,(\bar c b)\,\big(\bar\ell(1-\gamma_5)\nu_\ell\big) \\
&+ \epsilon_P^\ell\,(\bar c\gamma_5 b)\,\big(\bar\ell(1-\gamma_5)\nu_\ell\big) + \epsilon_T^\ell\,(\bar c\sigma_{\mu\nu}(1-\gamma_5)b)\,\big(\bar\ell\sigma^{\mu\nu}(1-\gamma_5)\nu_\ell\big) \Big] + \text{H.c.},
\end{aligned} \tag{1}
$$

and experimentally constraining the Wilson coefficients. In (1) G_F is the Fermi constant and V_{cb} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. $H_{\rm eff}$ involves four-fermion operators with left-handed neutrinos and complex lepton-flavour–dependent coefficients ϵ_i^ℓ. The SM is recovered for $\epsilon_{V,R,S,P,T}^\ell = 0$. After constraining the coefficients using the B modes, related effects are foreseen in different channels for mesons (B_s, B_c) and baryons (Λ_b), exclusive and inclusive. Observing correlated deviations from SM would be a smoking gun for the existence of the effect: we discuss below what is expected for Λ_b decays. The correlations could overcome the uncertainties in the hadronic matrix elements [4-6]. On the other hand, the treatment of the hadronic quantities is advocated to challenge the new physics (NP) interpretation of the tension [7].

Determination of $|V_{ub}|$ and $|V_{cb}|$ from exclusive and inclusive B decay modes. – In SM the elements of the CKM matrix are parameters to be determined experimentally. A long-standing issue concerning the measurement of $|V_{ub}|$ and $|V_{cb}|$ is that the most precise determinations, done using B decays, are in tension if inclusive or exclusive modes are exploited [3]. As we shall see below, the inclusive measurements rely on a systematic QCD expansion in the heavy-quark mass and in the strong coupling, also using moments of the lepton energy spectrum [8]. Third-order corrections to the moments in $B \to X_c\ell\nu$ have been considered [9], as well as the electromagnetic corrections [10]. The exclusive determinations rely on processes such as $B \to D^{(*)}\ell\nu$, for which the uncertainty related to the hadron form factors can be treated invoking arguments based on QCD symmetries, such as the heavy-quark symmetry holding in the large m_b limit [11], producing high-precision results [12-16]. The treatment of the hadronic form factors, including dispersive bounds, can remove the tension [7]; differently, a possible connection with the $b \to c$ semileptonic anomaly has been investigated [17].

Unitarity relations in the Cabibbo-Kobayashi-Maskawa matrix. – A deficit has been observed in the unitarity relations involving the elements of the first row and the first column of the CKM matrix [1]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005, \quad |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970 \pm 0.0018.$$

Due to the small values of V_{ub} and V_{td}, the deficit concerns the first two terms of the relations (*Cabibbo angle anomaly*). The attention is focused on the determinations of V_{ud} from nuclear β decays with the role of the radiative corrections [18-21], and on the extraction of V_{us} from leptonic and semileptonic K^+ decays [22-25]. Determinations in τ decays are also scrutinized. The possible origin from BSM phenomena has been considered [26], namely investigating the effects of modified neutrino couplings [27].

Observables in $b \to s\ell^+\ell^-$ *processes.* – Processes as those induced by $b \to s\ell^+\ell^-$ and the other Flavour Changing Neutral Current (FCNC) transitions, which in SM occur at loop level, are sensitive to heavy-quanta contributions. Anomalies have been detected in decay rates, such as $\Gamma(B \to K\mu^+\mu^-)$ and $\Gamma(B_s \to \phi\mu^+\mu^-)$ [28-31] and in observables constructed from the $B \to K^*\mu^+\mu^-$ angular distributions [32,33]. Also semi-inclusive transitions show difficulties [34]. On the other hand, the $B_s \to \mu^+\mu^-$ decay rate is compatible with SM [35]. An enhancement of the rate of another FCNC process, $B \to K\nu\bar{\nu}$ with respect to the SM expectation has been preliminarily observed by the Belle-II Collaboration [36], a result requiring a confirmation.

Anomalous magnetic moment of the muon. – The measurement in [37] combined with the results in [38, 39] has provided a determination of the muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ with the precision of 0.20 ppm. It deviates from the SM result quoted in the White Paper [40],

$$a_\mu^{\rm exp} = 116\,592\,059\,(22) \times 10^{-11}, \quad a_\mu^{WP} = 116\,591\,810\,(43) \times 10^{-11},$$

a tension the origin of which is under scrutiny. Improvement in the measurement is foreseen at Fermilab. The main uncertainty in the SM determination is in the hadronic contributions to a_μ. New evaluations of the hadronic light-by-light contributions, *e.g.*, in [41], confirm the previous results obtained by dispersive analyses (see [42] and references therein). The hadronic vacuum polarization (HVP) contribution is determined by dispersive analyses with the measured $e^+e^- \to$ hadron cross-section as an input [40]. A value of a_μ more consistent with experiment is obtained in a lattice QCD computation of the HVP contribution [43]. A tension is found in the comparison of the R ratio measurements in the low-s range with lattice QCD determinations [44]. Moreover, a measurement of the $e^+e^- \to \pi^+\pi^-$ cross-section from threshold to 1.2 GeV disagrees with previous results [45]. The situation is intriguing, improved analyses are ongoing, but the issue stresses the role of controlling the hadronic effects in precision observables.

3. – Interplay between flavour sector and hadron physics

As we have seen, there is an interplay between the tensions in SM observables and the hadron physics discussed at this conference. A few new examples are given below.

Inclusive semileptonic and radiative decays of b baryons. – The inclusive $b \to c$ semileptonic decay of a baryon H_b comprising a single b quark,

$$H_b(p,s) \to X_c(p_X)\ell^-(p_\ell)\bar{\nu}_\ell(p_\nu), \tag{2}$$

with s the spin of the decaying baryon, is induced by the generalized low-energy Hamiltonian (1) written as

$$H_{\rm eff}^{b\to c\ell\nu} = \frac{G_F}{\sqrt{2}} V_{cb} \sum_{i=1}^{5} C_i^\ell \, J_M^{(i)} \, L^{(i)M} + {\rm H.c.} \tag{3}$$

$J_M^{(i)}$ is the hadronic and $L^{(i)M}$ the leptonic current in each operator, with M the set of Lorentz indices contracted between J and L. The decay width of (2) is obtained from

$$\mathrm{d}\Gamma = \mathrm{d}\Sigma\,\frac{G_F^2|V_{cb}|^2}{4m_H}\sum_{i,j} C_i^* C_j (W^{ij})_{MN}(L^{ij})^{MN}, \tag{4}$$

with phase-space $\mathrm{d}\Sigma$. By the optical theorem, the hadronic tensor $(W^{ij})_{MN}$ is related to the discontinuity of the forward amplitude,

$$(T^{ij})_{MN} = i\int \mathrm{d}^4x\, e^{-i\,q\cdot x}\langle H_b(p,s)|T[J_M^{(i)\dagger}(x)\,J_N^{(j)}(0)]|H_b(p,s)\rangle, \tag{5}$$

across the cut corresponding to the process (2). This can be computed by an operator product expansion (OPE) in the inverse b quark mass [46, 47]. The resulting expression involves H_b matrix elements of QCD operators of increasing dimension,

$$\mathcal{M}_{\mu_1\dots\mu_n} = \langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle, \tag{6}$$

(a, b Dirac indices), given in terms of nonperturbative parameters, the number of which increases with the dimension of the operators. The matrix elements needed for the expansion at $\mathcal{O}(1/m_b^3)$ keeping the s_μ dependence are given in [5]. With such expressions one can compute, *e.g.*, the distributions in the charged lepton energy and in the angle θ_P between $\vec{p}_\ell$ and $\vec{s}$ in the H_b rest frame. In fig. 1 the deviation correlated to the

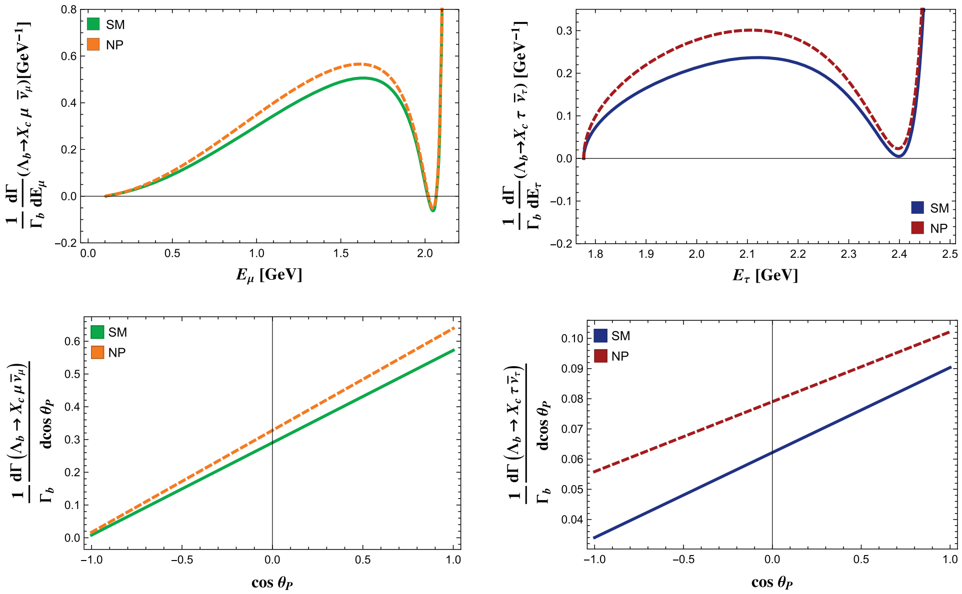

Fig. 1. – Charged lepton energy spectrum (top panels) and $\frac{1}{\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta_P}$ distribution (bottom panels) for $\Lambda_b \to X_c\ell\bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line is the SM result, the dashed line the result for NP at a benchmark point for the Wilson coefficients compatible with $B \to D^*\ell\nu_\ell$ data [4, 5].

anomaly in semileptonic $B \to D^{(*)}\tau\nu_\tau$ decays can be appreciated [4, 5]. Λ_b with sizable polarization are expected to be produced at the new lepton colliders, with the b quarks coming from Z^0 and top quark decays. Inclusive $b \to u$ semileptonic modes and rare radiative modes can be described by the same approach. The treatment of the singular terms in the distributions involves a nonperturbative shape function, a new expression of which has been derived in [6].

Exclusive B_c decays to charmonium. – The semileptonic $b \to c$ exclusive decays of B_c to negative- and positive-parity charmonia are of particular interest, since the matrix elements of the hadron currents $\bar{c}\Gamma b$ can be expressed near the zero-recoil point invoking the heavy quark spin symmetry [48, 49]. The consequence is that different processes can be related. The symmetry can be used to reconstruct the form factors of new physics operators starting from those computed, *e.g.*, by lattice QCD [50]. Moreover, B_c decay processes provide us with methods to investigate the nature of debated charmonia such as $X(3872)$ [51], using observables in semileptonic [52] and nonleptonic channels [53]. Due to the heavy quark spin symmetry, the four P-wave charmonium states χ_{ci} $(i = 1, 2, 3)$ and h_c belong to a spin 4-plet, and this holds also for the first radial excitations. If $X(3872)$ can be identified with $\chi_{c1}(2P)$, its production in semileptonic and nonleptonic B_c modes would be precisely correlated to the production of the other members of the charmonium multiplet in the same process; such correlations can be experimentally tested. An example is in fig. 2, in which ratios of semileptonic decay distributions are plotted *versus* the variable $w = v \cdot v'$, with v and v' the four-velocities of B_c and of the produced charmonium state.

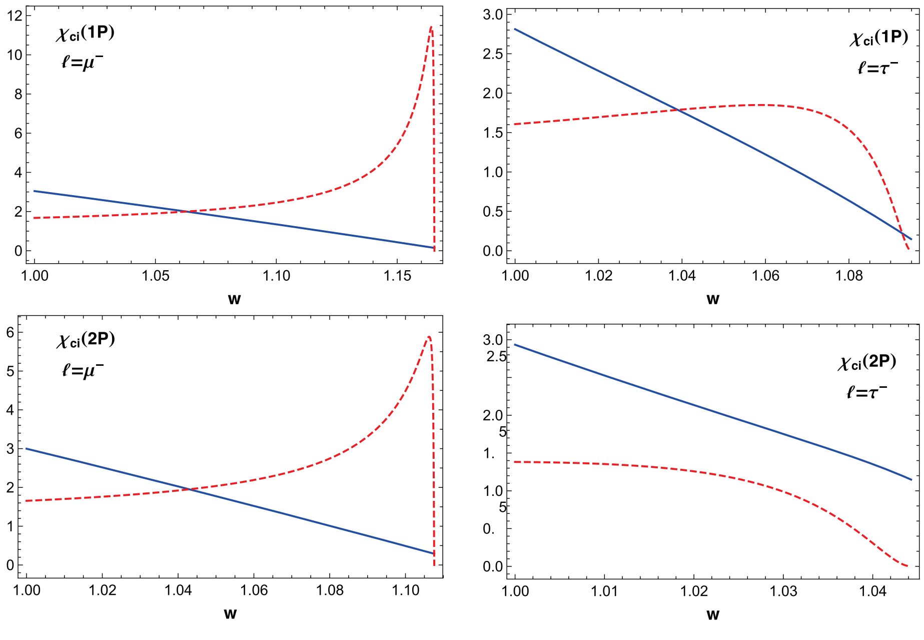

Fig. 2. – Ratios of distributions $\frac{\mathrm{d}\Gamma(B_c \to \chi_{c1}\ell\bar{\nu})/\mathrm{d}w}{\mathrm{d}\Gamma(B_c \to \chi_{c0}\ell\bar{\nu})/\mathrm{d}w}$ (solid lines) and $\frac{\mathrm{d}\Gamma(B_c \to \chi_{c2}\ell\bar{\nu})/\mathrm{d}w}{\mathrm{d}\Gamma(B_c \to \chi_{c1}\ell\bar{\nu})/\mathrm{d}w}$ (dashed lines) in SM, for $\ell = \mu$ (left) and $\ell = \tau$ (right), and for the $1P$ (top panels) and $2P$ final charmonia (bottom panels). The meson masses are quoted in [1, 52]. The LO relations among form factors obtained in [52] are extrapolated to the full kinematic range.

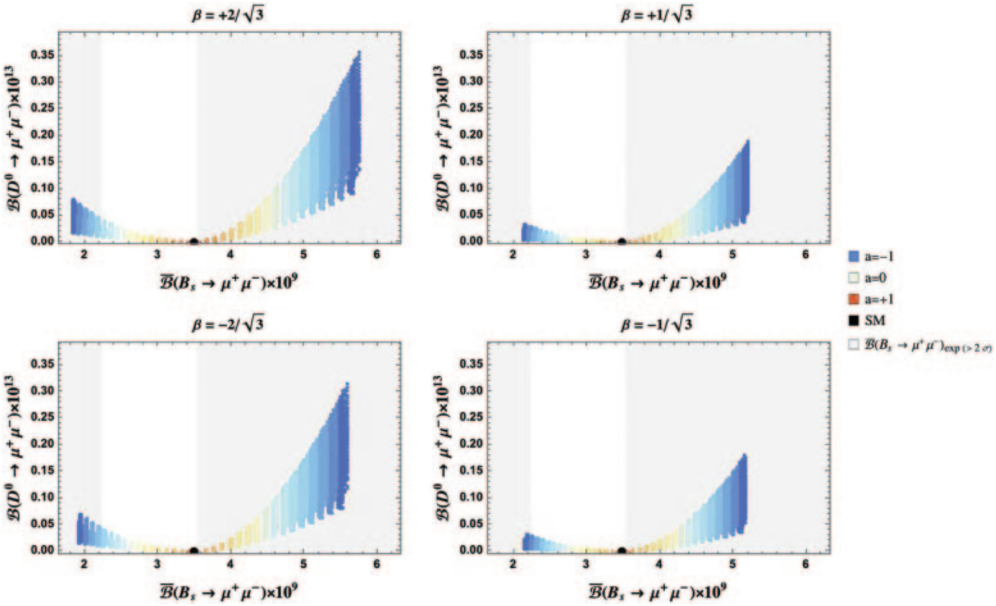

Fig. 3. – Correlation between $\mathcal{B}(D^0 \to \mu^+\mu^-)$ and $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ in 331 models for a set of model parameters, namely the parameter a entering in the $Z - Z'$ mixing [59]. The black dots are the SM result. The gray areas are regions excluded by the $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ measurements within 2σ [64].

4. – Example of a BSM construction: The 331 model. Correlations among FCNC processes in down- and up-quark sectors

Among the models constructed enlarging the SM gauge group, the *331 models* are based on $SU(3)_c \times SU(3)_L \times U(1)_X$, spontaneous broken first to $SU(3)_c \times SU(2)_L \times U(1)_Y$, then to $SU(3)_c \times U(1)_Q$ [54-56]. Five additional gauge bosons and new fermions are in the spectrum: the left-handed SM fermions belong to triplets or antitriplets, with the third component, usually a new heavy fermion. In 331 models the anomaly cancelation and the asymptotic freedom of QCD constrain the number of generations to be equal to the number of colours, a rationale for such theories. Moreover, the quark generations transform differently under $SU(3)_L$, two generations as triplets, one (usually the third generation) as an antitriplet, a difference that can be invoked as the origin of the large top quark mass.

The relation between the electric charge Q, the $SU(3)$ generators T_3 and T_8, and the generator X of $U(1)_X$: $Q = T_3 + \beta T_8 + X$ introduces a parameter β defining the variant of the model. For β multiple of $\frac{1}{\sqrt{3}}$ and of $\sqrt{3}$ the new gauge bosons $Y^{Q_Y^{\pm}}$ and $V^{Q_V^{\pm}}$ have integer charges. A neutral gauge boson Z' mediates tree-level FCNC in the quark sector, while the couplings to leptons are diagonal and universal. The extended Higgs sector comprises three $SU(3)_L$ triplets and one sextet.

In analogy with SM, the quark mass eigenstates are obtained rotating the flavour eigenstates by two unitary matrices, U_L for up-type and V_L for down-type quarks, with $V_{CKM} = U_L^\dagger V_L$. However, while in SM V_{CKM} only appears in charged current interactions and U_L and V_L never appear individually, in 331 models one can get rid of only one matrix, either U_L or V_L, expressed in terms of V_{CKM} and of the other matrix. The remaining rotation matrix enters in Z' couplings to quarks [57]. Considering the Z' interaction with ordinary fermions, correlations between observables in $B_{d,s}$ sectors and in the kaon sector can be established [57-62]. For $\beta = \pm\frac{2}{\sqrt{3}}$ and $\beta = \pm\frac{1}{\sqrt{3}}$ and the third generation quarks in an antitriplet, phenomenological constraints are satisfied, namely from $\Delta F = 2$ observables in the B_d, B_s, K systems and the electroweak precision observables, for Z' in the TeV range [58]. For $\beta = \frac{2}{\sqrt{3}}$ relevant contributions to ε'/ε are predicted [60]. The relation $U_L = V_L \cdot V_{CKM}^\dagger$ induces correlations between FCNC transitions in the up- and down-type quarks, a striking feature of the models. For example, $c \to u\nu\bar{\nu}$ induced processes, *e.g.*, $B_c \to B_u^{(*)}\nu\bar{\nu}$, can be related to $b \to s\nu\bar{\nu}$ and $s \to d\nu\bar{\nu}$ modes [63]. Figure 3 shows the correlations between $D^0 \to \mu^+\mu^-$ and $B_s \to \mu^+\mu^-$ [64].

5. – Conclusions

At present no undoubtable evidence is found of experimental deviations from SM: the above listed anomalies need to be confirmed with higher precision, with reduced hadronic uncertainty. The results of the new measurements and the arguments about the limitations of the theory will drive the analysis of the structure of fundamental interactions.

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