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# Spectroscopy of hadrons with heavy quarks from lattice QCD

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**Summary.** — Lattice QCD results on hadrons with heavy quarks are briefly reviewed. The focus is on the spectrum of conventional and exotic hadrons. Structure of certain conventional hadrons is addressed as well.

## 1. – Introduction

Experiments revealed that hadrons with the following minimal contents exist: mesons $\bar{q}q$, baryons $qqq$, tetraquarks $\bar{q}q\bar{q}q$, pentquarks $\bar{q}qqqq$ and hybrid mesons $\bar{q}Gq$. The first two sectors correspond to conventional hadrons, while the last three are referred to as exotic hadrons. We will briefly review lattice results on some of these states(1).

## 2. – Hadron spectroscopy with lattice QCD

The *spectrum of hadrons* (below, near, or above threshold) is extracted from the energies $E_n$ of QCD eigenstates $|n\rangle$ on a finite and discretized lattice in Eucledian space-time. The eigen-energies $E_n$ are determined from the time dependence of two-point correlation functions $\langle O_i(t_E)O_j^\dagger(0)\rangle = \sum_n \langle O_i|n\rangle\, e^{-E_n t_E}\langle n|O_j^\dagger\rangle$, where operators $O$ create/annihilate the hadron system with a given quantum number of interest.

The masses of strongly stable hadrons well below threshold are obtained as $m = E_n|_{\vec{p}=0}$. These have already been determined and agree well with the experiment, *e.g.*, [1,2].

The masses of hadrons near threshold and hadronic resonances have to be inferred from the scattering of two hadrons $H_1H_2$, which is encoded in the scattering amplitude $T(E)$. The simplest example is a one-channel scattering in partial wave $l$ sketched in

(1) Only few references are cited due to the page limit. Other references are listed in the slides available at `https://agenda.infn.it/event/33110/contributions/196709/attachments/106240/149858/prelovsek_spectroscopy_lattice_hadron23.pdf`.

 

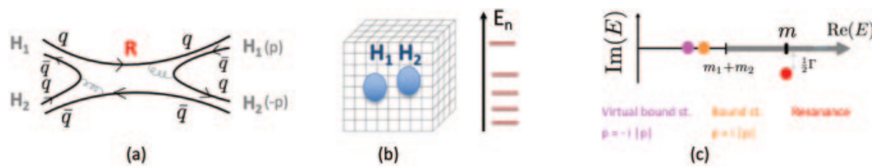


Fig. 1. – Extracting resonances and near-threshold bound states from one-channel scattering.

fig. 1. Lüscher has shown that the energy $E$ of a two-hadron eigenstate in finite volume renders $T(E)$ at that energy in infinite volume [3]. This relation leads to $T(E)$ for real $E$, which is then analytically continued to complex energies. A pole in $T(E)$ indicates the presence of a state, while its position renders its mass $m = \text{Re}(E)$ and the width $\Gamma = -2\ \text{Im}(E)$. A resonance corresponds to a pole away from the real axes. A bound state corresponds to a pole below the threshold: the state is referred to the bound state if the pole occurs for positive imaginary momenta $p = i|p|$ and a virtual bound state if it occurs for $p = -i|p|$, where $p$ denotes the magnitude of the 3-momentum in the center-of-mass frame. The majority of the scattering studies are still performed at $m_\pi > m_\pi^{phy}$ and at a single lattice spacing.

The resonances that decay via several strong decay channels $R \to H_1H_2,\ H_1'H_2', \ldots$ have to be extracted from the coupled-channel scattering (fig. 2). The scattering matrix for two coupled channels contains three unknown functions of energy. It is customary to parametrize their energy dependence in order to extract them from the eigen-energies using the Lüscher's formalism.

The formalism and progress in addressing decays $R \to H_1H_2H_2$ were reviewed at the last edition of this conference [4].

## 3. – Spectroscopy of various hadron sectors

The majority of the discovered exotic hadrons contain heavy quarks as those are more likely to form quasi-bound states due to small kinetic energies. Most of them decay strongly and the theoretical challenge to study them increases with the number of decay channels.

*Charmonium-like states $\bar{c}c$, $\bar{c}c\bar{q}q'$, $\bar{c}cuud$.* – The spectrum of charmonium-like states with $I = 0$ was extracted from the coupled channels $\bar{D}D - \bar{D}_sD_s$ at $m_\pi \simeq 280\,\text{MeV}$ [5] (fig. 3). All the states except for two (indicated by magenta arrows) appear to be conventional charmonia $\bar{c}c$. In addition, two exotic scalar states are predicted near both thresholds. The heavier one has a large coupling to $\bar{D}_sD_s$ and a small coupling to

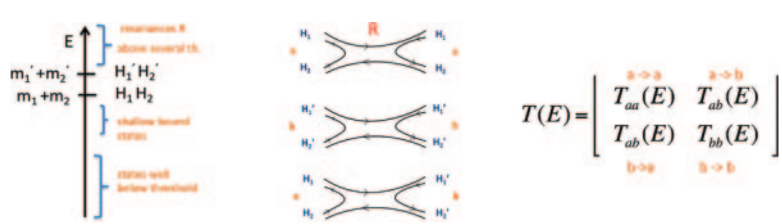


Fig. 2. – A resonance that decays via two channels $R \to H_1H_2,\ H_1'H_2'$ has to be inferred from the scattering of two coupled channels.

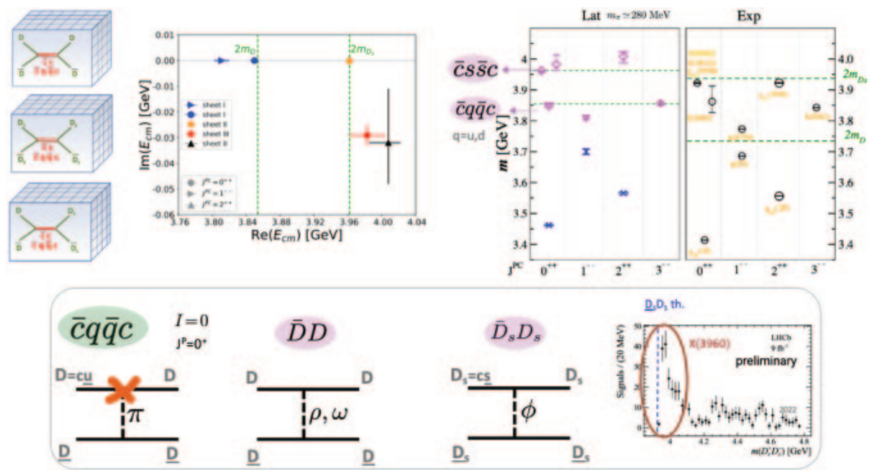


Fig. 3. – Poles and masses for a charmonium-like system with $I = 0$ from lattice [5]. LHCb discovery of a $X(3960)$ [6] composed of $\bar{c}s\bar{s}c$. Possible binding mechanism.

$\bar{D}D$ —it likely corresponds to a state $X(3960)$ composed of $\bar{c}s\bar{s}c$ recently discovered by LHCb [6]. The two additional scalars were not found at $m_\pi \simeq 390$ MeV in [7].

A candidate for pentaquark $P_c = \bar{c}cud$ with $J^P = 1/2^-$ was found $6 \pm 3$ MeV below $\bar{D}\Sigma_c$ threshold [8]. It appears as a bound state in one-channel scattering amplitude $\bar{D}\Sigma_c$ (fig. 4), where the lower-lying channel $J/\psi p$ was omitted.

*Hadrons with two heavy quarks: $QQ\bar{q}\bar{q}'$.* – The $bb\bar{u}\bar{d}$ and $bb\bar{s}\bar{d}$ tetraquarks with $J^P = 1^+$ are expected to reside significantly below strong decay thresholds (fig. 5). This is a reliable conclusion based on a number of lattice simulations and model-based calculations (listed in the slides). The hadron $bb\bar{u}\bar{d}$ with such a deep binding is expected to have a small size, which indicates the dominance of $[bb]^{S=1}_{\bar{3}_c}[\bar{u}\bar{d}]^{S=0}_{3_c}$. So far, this the only tetraquark where lattice finds a strong support for the dominance of the diquark antidiquark Fock component.

Tetraquarks $QQ\bar{q}\bar{q}'(Q = c, b, q = u, d, s)$ with other flavors and spin-parities are expected near or above strong decay thresholds $H_1H_2$, as suggested also by fig. 5. In order to prove the existence of such a state, one needs to extract the scattering matrix and establish a pole in it as shown in fig. 1.

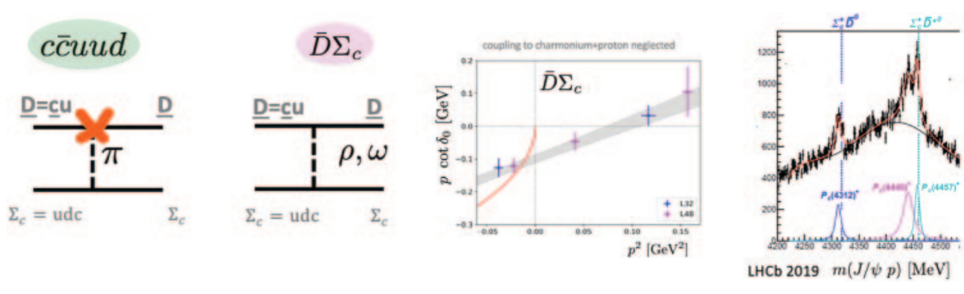


Fig. 4. – $\bar{D}\Sigma_c$ scattering in $P_c$ channel [8], LHCb discovery and possible binding mechanism.

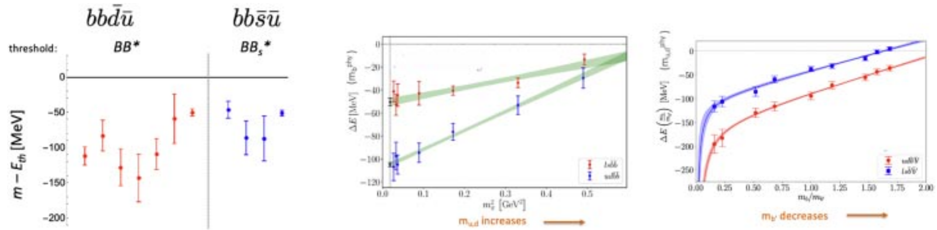


Fig. 5. – Left: the binding energies of $bb\bar{u}\bar{d}$ and $bb\bar{u}\bar{s}$ with $J^P = 1^+$ from lattice (see references in the slides). Right: the dependence of the binding energy on $m_b$ and $m_{u,d}$ [9, 10].

The doubly charm tetraquark $T_{cc} = cc\bar{u}\bar{d}$ was discovered by LHCb just 0.4 MeV below $DD^*$ threshold [14], it has $I = 0$ and most likely $J^P = 1^+$. The lattice results on $DD^*$ scattering from three simulations at different $m_\pi$ are shown in fig. 6. All simulations find significant attraction; the attraction decreases with increasing $m_\pi$. This implies that $T_{cc}$ would-be bound state from experiment converts to virtual bound state or a resonance at heavier $m_\pi$. Virtual bound state poles are indeed found in simulations [11-13] when assuming effective range approximation. Relaxing this approximation and taking into account the effect of left-hand cut from one-pion exchange leads to a pair of virtual bound states or a resonance [15]. The long-range potential is dominated by $\pi\pi$ exchange in [11]. The dominant attraction in this channel is attributed to $\rho$ exchange in [13].

*Hadrons with a single heavy quark.* – The charmed scalar mesons would form a $SU(3)$ flavor triplet in fig. 7(a) according to the quark model. However, a new paradigm is supported by the effective field theories based on HQET and ChPT, combined with the lattice results as well as the experimental data (see slides for many references). According to this paradigm, the spectrum features $c\bar{q}$ as well as $c\bar{q}\ \bar{q}q$ Fock components ($q = u, d, s$). The latter decomposes to the multiplets $\bar{3}\oplus 6\oplus 5$ in the $SU(3)$ flavor limit. The attractive interactions within the anti-triplet and the sextet suggest the existence of hadrons with flavors indicated by cyan circles in fig. 7(b), with two pairs of poles for $I = 1/2$ charmed mesons. The lower one resides at 2.1–2.2 GeV in agreement with the lattice simulations and it is a natural partner of $D^*_{s0}(2317)$. The heavier pole at 2.4–2.5 GeV is suggested by the EFT re-analysis of the lattice and experimental data.

The simulation of $D^*\pi$ scattering finds one axial charmed meson dominated by s-wave and the other by d-wave coupling to $D^*\pi$ [16] (fig. 8). This is in line with HQET

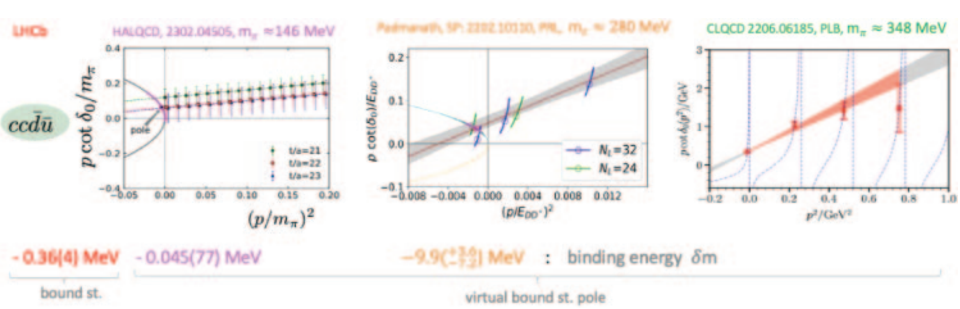


Fig. 6. – The $DD^*$ scattering in $T_{cc}$ channel and the pole locations from lattice [11-13].

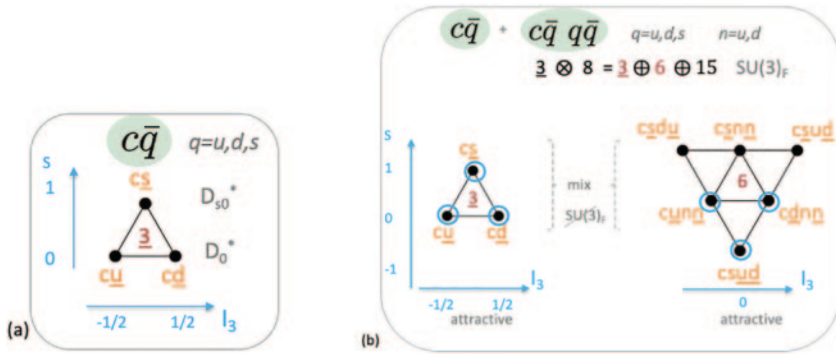


Fig. 7. – (Color online) Scalar charmed mesons according to the quark model (a) and the new paradigm (b).

prediction and significantly different experimental decay widths.

*Bottomonium-like states* $\bar{b}b$, $\bar{b}Gb$, $\bar{b}b\bar{q}q'$. – These systems can be studied using relativistic, non-relativistic or static $b$-quarks. Here I focus on the last option, where two heavy quarks and additional light degrees of freedom are investigated via the Born-Oppenheimer approximation. The eigen-energies at fixed distance between heavy quarks render the potential $V(r)$. Motion of heavy quarks within this potential is then studied with Schrödinger-type equation. The aim is to determine whether bound states or resonances exist.

The static potential for $\bar{b}b$ system with $I=0$ that accounts also for the coupling to a pair of heavy mesons [17] is shown in fig. 9(b). Coupled-channel Schrödinger equation with analogous potential (from an earlier calculation [19]) renders the poles related to bottomonium-like states and their composition in terms of $\bar{b}b$, $\bar{B}B$ and $\bar{B}_sB_s$ [18] (figs. 9(c), (d)). These are dominated by the conventional Fock component $\bar{b}b$, except for state $n=5$, which is likely related to unconventional $\Upsilon(10750)$ discovered by Belle.

The observed $Z_b$ resonances with flavor content $\bar{b}b\bar{d}u$ are challenging for rigorous treatment since the lowest decay channel is $\Upsilon_b\pi$, while they reside at the higher threshold $B\bar{B}^*$. This was taken into account in the extraction of the potential between $B$ and $\bar{B}^*$ in fig. 10, which is attractive at small distances [20, 27, 28]. This attraction is likely responsible for the existence of the exotic $Z_b$.

The excited potentials for $\bar{b}b$ with certain spin-parities in fig. 11 are relevant for hybrid mesons $\bar{b}Gb$. The masses from these potentials within the Born-Oppenheimer

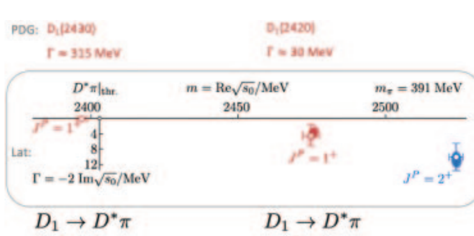


Fig. 8. – Poles related to charmed mesons with $J^P=1^+,2^+$ from [16].

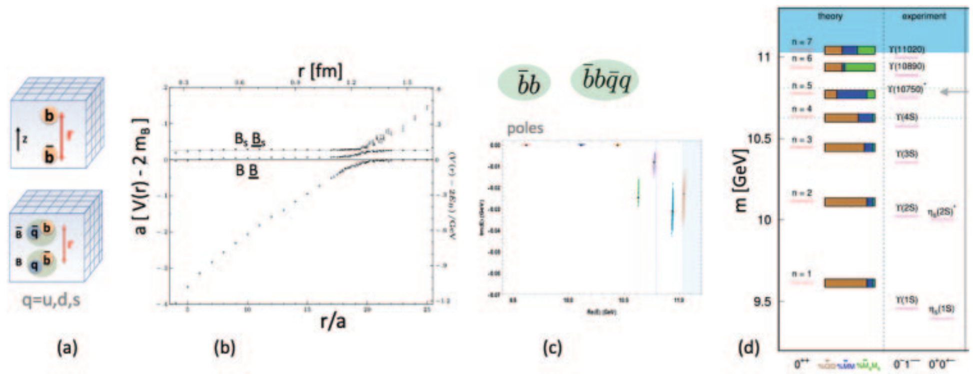


Fig. 9. – (a), (b): the static potential of the quarkonium system that accounts also for the coupling to a pair of heavy mesons [17]. (c), (d): poles and composition of the bottomonium-like states [18] from analogous static potential [19].

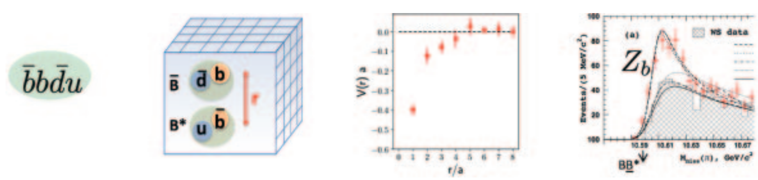


Fig. 10. – The attractive static potential between $B$ and $\bar{B}^*$ [20] is likely responsible for the existence of $Z_b \simeq \bar{b}b\bar{d}u$ [21].

approach [23, 24] agree with those obtained using relativistic $b$-quarks [26].

## 4. – Structure of hadrons with heavy quarks

The charge distribution of charmonia was probed via EM and other currents in [29], while various decay constants that probe the wave function were extracted in [2]. The mass decomposition with respect to various terms of the QCD Hamiltonian was determined for charmed baryons in [30] (fig. 12).

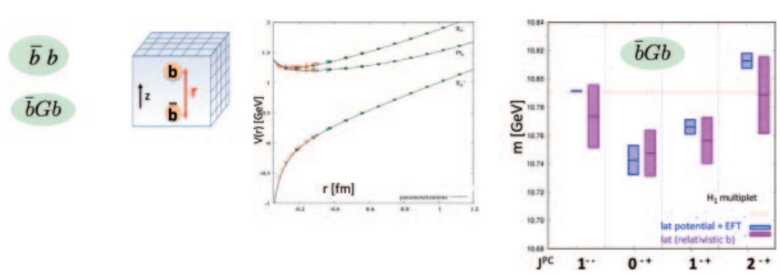


Fig. 11. – (Color online) The potentials related to hybrids from [22]. Masses of $\bar{b}Gb$ hybrids (update of [23, 24]) from older potentials [25] (blue) and from relativistic $b$ quarks [26] (violet).

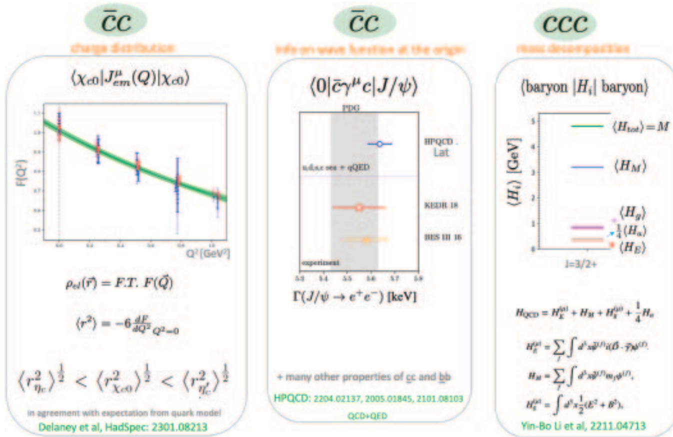

Fig. 12. – Probing structure of hadrons with charm quarks in simulations [2, 29, 30].

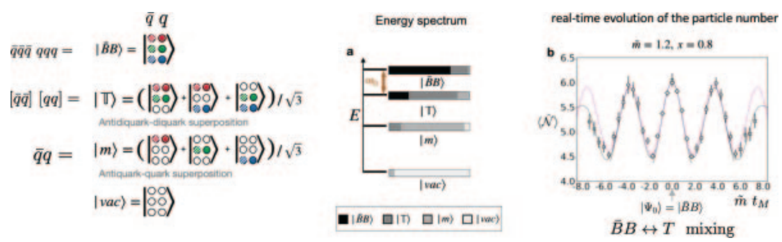


Fig. 13. – Tetra- and pentaquarks in one-dimensional QCD from quantum computer [31].

## 5. – Looking ahead

The drawback of spectroscopy on Eucledian lattice is suppressed contribution $e^{-E_n t_E}$ of the excited states. The evolution of the QCD systems in real Minkowsky time $e^{-iE_n t_M}$ is one of the long-term goals for quantum computers. Such time evolution for tetraquark and pentquark systems has already been studied in one-dimensional QCD with one quark flavor on a quantum computer [31] (fig. 13).

## 6. – Conclusions

Experiments have provided great discoveries of new conventional as well as around thirty exotic hadrons. I have reviewed the theoretical challenge to understand the spectroscopic properties of various hadron sectors from lattice QCD. This approach renders masses of hadrons that are strongly stable, as well as most of the hadrons that are slightly below the strong decay threshold or decay strongly via one decay channel. The theoretical challenge increases with the number of open decay channels. It seems impossible to address the high-lying states like $Z_c(4430)$ with current lattice methods, while many interesting physics conclusions are already available for certain lower-lying states.

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