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The description of meson and glueball spectra within the graviton soft-wall model

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Summary. — In this paper the main predictions of the holographic graviton soft-wall model have been discussed. In particular, the glueball and meson spectra have been shown. Results are in very good agreement with lattice calculations and experimental data. Moreover, the model has been minimally modified to take into account the chiral symmetry-breaking mechanism to describe the pion.

1. – Introduction

In this contribution we summarize the results of the calculations of meson and glueball spectra within the graviton soft-wall (GSW) model [1-4]. Holographic approaches rely on a correspondence between a five-dimensional classical theory with an AdS metric and a supersymmetric conformal quantum field theory. Since the latter is not QCD, the five-dimensional classical theory is properly modified to try to reproduce non-perturbative properties of QCD [5-8]. In our model, the original soft-wall (SW) metric is properly modified to describe the glueball spectrum as a graviton propagating in this space. We also calculated the spectra of light and heavy scalar mesons. The results, obtained with only two fixed parameters are in good agreement with data and lattice analyses. The model has been also properly modified to describe the pion spectrum and its static properties [9]. Recently, the dependence of the glueball spectrum on the temperature has been also investigated [10, 11].

2. – The graviton soft-wall model

The most relevant difference between the GSW model and the well-known SW one is the background metric of the theory in the gravity sector. Similar approaches are those

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of refs. [12-19]. In the present case the metric reads

$$\mathrm{d}s^2 = e^{\alpha k^2 z^2} g_{MN} \mathrm{d}x^M \mathrm{d}x^N = e^{\alpha k^2 z^2} \frac{R^2}{z^2} (\eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2), \tag{1}$$

where g_{MN} is the usual AdS$_5$ metric used in the SW model [1, 8, 13, 20, 21]. For fields propagating in this new space, where a dilaton is included, the action is

$$\bar{S} = \int \mathrm{d}^5 x e^{k^2 z^2 \left(\frac{5}{2}\alpha - \beta + 1\right)} e^{-\phi_n(z)} \sqrt{-g} e^{-\phi_0(z)} \mathcal{L}(x_\mu, z). \tag{2}$$

The parameter α measures the modification of the metric and β, not a free parameter, is fixed to get the SW kinematic term in the action [1, 3, 4]. For scalar fields $\beta = \beta_s = 1 + \frac{3}{2}\alpha$ and for a vector $\beta = \beta_v = 1 + \frac{1}{2}\alpha$. An additional dilaton ϕ_n has been included [2] to guarantee that the corresponding equations of motions lead to bound states. This quantity does not contain any free parameter.

3. – The glueball spectra within the GSW model

In our model [1] we calculated the spectrum of the scalar glueball from that of a graviton propagating in the space eq. (1). The linearized Einstein's equation can be rearranged in a Schrödinger like equation,

$$-\frac{\mathrm{d}^2\phi(t)}{\mathrm{d}t^2} + \left(\frac{8}{t^2} e^{2t^2} - 15t^2 + 14 - \frac{17}{4t^2}\right)\phi(t) = \Lambda^2 \phi(t). \tag{3}$$

Here $t = \sqrt{\alpha k^2/2}\, z$ and $\Lambda^2 = (2/\alpha k^2)\, M^2$, being M the mode mass. The potential is uniquely determined by the modified metric. The only free parameter is the scale factor $\alpha k^2 \sim (0.37 \text{ GeV})^2$ fixed from the comparison with lattice QCD. As one can see in the left panel of fig. 1, the linear glueball spectrum is well reproduced. The predicted ground state is in agreement with the BESIII data of the J/Ψ decays [22, 23]. The model [4] also reproduces the lattice predictions of the Regge trajectories for even and odd glueball spin [24-26] if we consider the approach of refs. [13, 27, 28] to describe the spin-dependent spectrum of glueballs.

4. – The spectra of mesons

4·1. *Light and heavy scalar mesons.* – For scalar mesons the action is

$$\bar{S} = \int \mathrm{d}^5 x \, \sqrt{-g} e^{-k^2 z^2} \left[g^{MN} \partial_M S(x) \partial_N S(x) + e^{\alpha k^2 z^2} M_5^2 R^2 S(x) \right], \tag{4}$$

with $M_5^2 R^2 = -3$. Since the relative potential is not binding, an additional dilaton must be included $\exp[-k^2 z^2] \to \exp[-k^2 z^2 - \phi_n(z)]$, see details in ref. [2]. By keeping fixed $\alpha k^2 = 0.37 \text{ GeV}^2$, we found a reasonable good fit, see the left panel of fig. 1, for $0.51 \leq \alpha \leq 0.59$ (this represents the theoretical uncertainty in the calculations). For heavy mesons we added the quark mass contribution to the light scalar masses (M_l) [2, 4] in order to effectively include the heavy quark mass (M_h) dynamics [29-31]: $M_h = M_h + C$. We found that $C_c = 2400$ MeV, for the $c\bar{c}$ mesons, and for the $b\bar{b}$ mesons

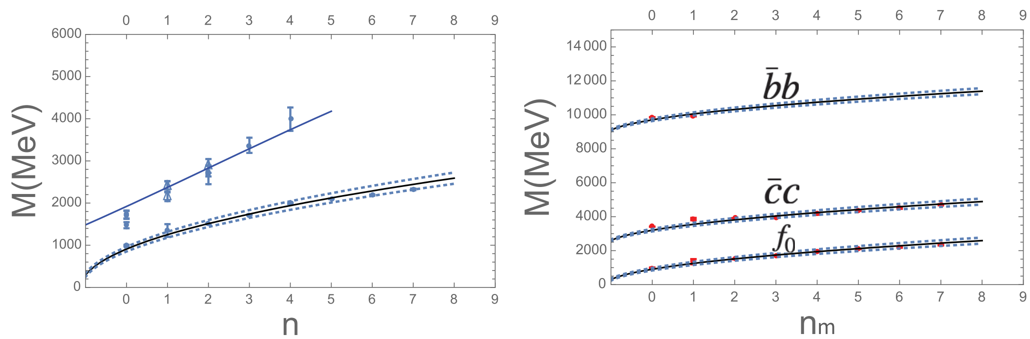

Fig. 1. – Left panel: GSW fit to the scalar lattice glueball spectrum and to the experimental scalar meson spectrum. Right panel: the scalar meson spectrum GSW fit to the data shown for all quark sectors. Data included in refs. [1,4].

$C_b = 8700\,\text{MeV}$. The successful comparison with data [4] is displayed in the right panels of fig. 1. One should notice that $C_c \sim 2m_c$ and $C_b \sim 2m_b$ as expected.

4·2. *The a_1 axial meson and ρ spectra.* – For a vector field one can assume the following action:

$$\bar{S} = -\frac{1}{2}\int \mathrm{d}^5x\sqrt{-g}e^{-k^2z^2-\phi_n}\left[\frac{1}{2}g^{MP}g^{QN}F_{MN}F^{PQ}M_5^2R^2g^{PM}A_PA_Me^{\alpha k^2z^2}\right]\,. \tag{5}$$

Here $M_5^2R^2 = -1$ [32,33]. Also in this case a modification of the dilaton is required [2]. With the above parameters we get the spectrum shown in the (a) panel of fig. 2. Our calculation favors that the $a_1(1930)$, $a_1(2095)$ and $a_1(2270)$ are axial resonances [34]. In the case of the ρ meson $M_5^2R^2 = 0$, as one can see in the (b) panel of fig. 2, the agreement is good, exception is $\rho(770)$.

4·3. *The η pseudo-scalar meson.* – For this system the EoM is formally equal to that of the scalar meson but with $M_5^2R^2 = -4$ [33]. As shown in the (c) panel of fig. 2, the comparison with the experimental data is very good. The GSW model predicts that i) $\eta(1405)$ and $\eta(1475)$ are degenerate, as discussed in PDG review; ii) two resonances between the $\eta(1760)$ and the $\eta(2225)$ could exist.

5. – The pion structure

In order to describe the pion structure, the GSW model must be properly modified to take into account the chiral symmetry-breaking mechanism. We propose the following strategy [9]: i) the additional dilaton ϕ_n leads to a massless pion by introducing a new parameter γ_π; ii) in order to break this symmetry the longitudinal dynamics has been introduced [9,35]. At the end two free parameters are requested, γ_π and the quark mass m_q. We proposed two ansatz: $\gamma_\pi = -0.6$ (-0.17) and $m_q = 45$ $(52)\,\text{MeV}$ called GSWL1 (GSWL2). Both parametrizations lead to very good description of, *e.g.*, the spectrum, the decay constant and the mean pion radius [9]. In fig. 2 we show the calculations of i) the form factor (FF), (d) panel; ii) the distribution amplitude (DA), (e) panel and the transition form factor (TFF), (f) panel. As one can see very good agreement was found.

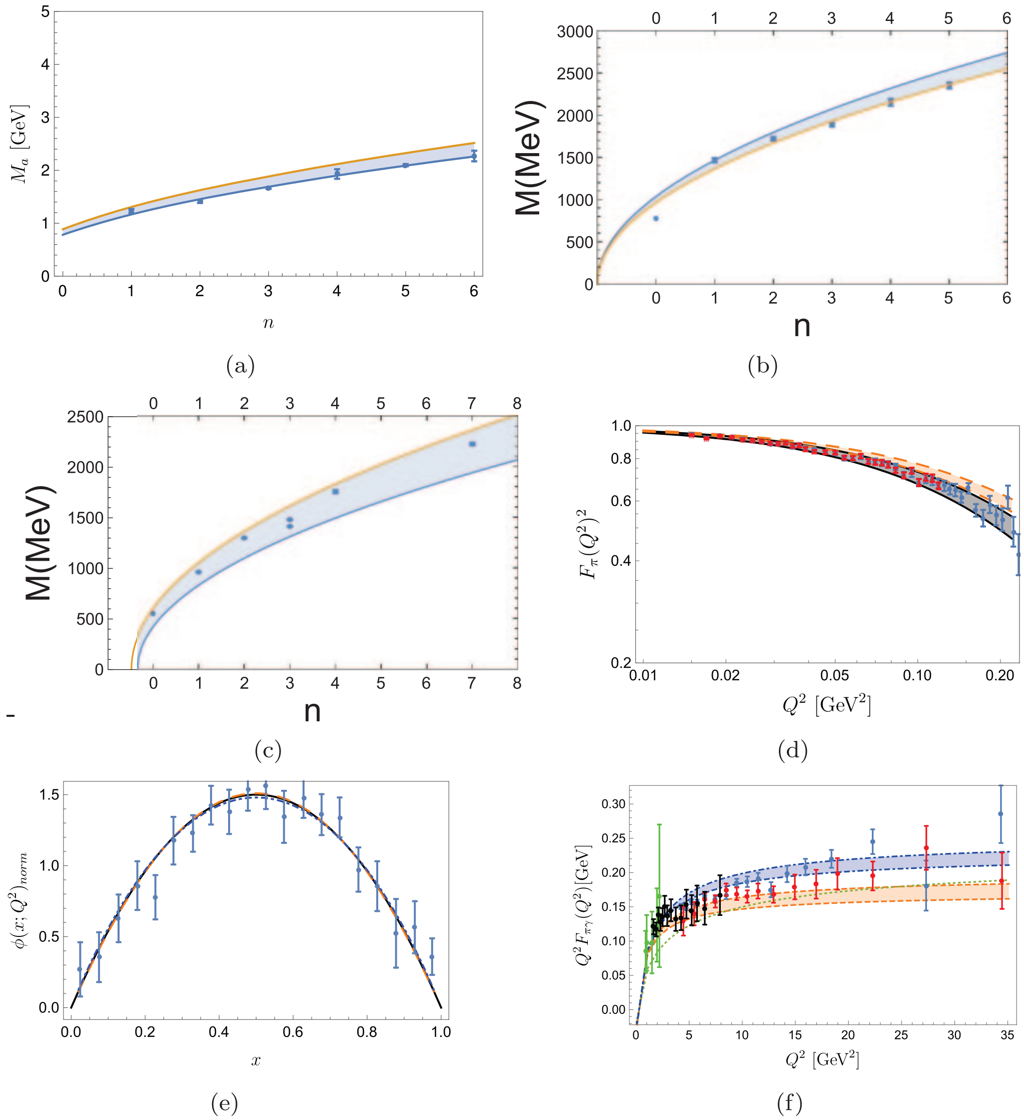

Fig. 2. – (a) The a_1 spectrum. (b) The ρ mass plot as a function of mode number. (c) The η spectrum. (d) The pion ff. Full line for GSWL2 and the dashed one for GSWL1. (e) The pion DA at $Q = 3.16$ GeV. (f) The pion TFF. Dashed line for GSWL2 and dot-dashed line for GSWL1. All data included in refs. [2, 9].

6. – Conclusions

In this contribution we presented the main predictions of the GSW model. A several amount of experimental data of different observables for different hadrons have been described with few fixed parameters. For the pion, the model has been modified and also in this case the comparison with data is quite good. We conclude by remarking the predicting power of the model.

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