

The D_s^+ decay into $\pi^+ K_S^0 K_S^0$ reaction and the $I = 1$ partner of the $f_0(1710)$ state

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Summary. — Two identified decay modes of the $D_s^+ \rightarrow \pi^+ K^{*+} K^{*-}, \pi^+ K^{*0} \bar{K}^{*0}$ reactions producing a pion and two vector mesons are discussed in this talk. The posterior vector-vector interaction generates two resonances that we associate to the $f_0(1710)$ and the $a_0(1710)$ recently claimed, and they decay to the observed $K^+ K^-$ or $K_S^0 K_S^0$ pair, leading to the reactions $D_s^+ \rightarrow \pi^+ K^+ K^-, \pi^+ K_S^0 K_S^0$. The results depend on two parameters related to external and internal emission. We determine a narrow region of the parameters consistent with the large N_c limit within uncertainties which gives rise to decay widths in agreement with experiment. With this scenario we make predictions for the branching ratio of the $a_0(1710)$ contribution to the $D_s^+ \rightarrow \pi^0 K^+ K_S^0$ reaction, finding values within the range of $(1.3 \pm 0.4) \times 10^{-3}$. We obtain predictions in good agreement with the BESIII measurements, confirming the new $a_0(1710)$ ($a_0(1817)$) resonance. This is an important state and will shed light into the structure of scalar mesons in light quark sector and other relevant issues currently under debate in hadron physics.

1. – Motivation

An isospin $I = 0$, $f_0(1710)$ resonance has been known for quite some time [1]. From the recent BESIII experiments, the following branching fraction was found [2]:

$$\text{Br}[D_s^+ \rightarrow \pi^+ "f_0(1710)"; "f_0(1710)" \rightarrow K^+ K^-] = (1.0 \pm 0.2 \pm 0.3) \times 10^{-3},$$

and in another work it was found that [3]

$$\text{Br}[D_s^+ \rightarrow \pi^+ "f_0(1710)"; "f_0(1710)" \rightarrow K_S^0 K_S^0] = (3.1 \pm 0.3 \pm 0.1) \times 10^{-3},$$

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where “ $f_0(1710)$ ” was supposed to be the $f_0(1710)$ resonance. Thus one finds

$$(1) \quad R_1 = \frac{\Gamma(D_s^+ \rightarrow \pi^+ “f_0(1710)” \rightarrow \pi^+ K^0 \bar{K}^0)}{\Gamma(D_s^+ \rightarrow \pi^+ “f_0(1710)” \rightarrow \pi^+ K^+ K^-)} = 6.20 \pm 0.67.$$

But, it is easy to prove that if “ $f_0(1710)$ ” was the $f_0(1710)$ resonance, this latter ratio should be 1. Therefore, hidden below or around the $f_0(1710)$, there should be an $I = 1$ resonance responsible for this surprising large ratio. We think a mixture of the two resonances and their interference would be responsible for a different $K^+ K^-$ or $K^0 \bar{K}^0$ production, due to

$$(2) \quad \begin{aligned} |K\bar{K}, I = 0, I_3 = 0\rangle &= -\frac{1}{\sqrt{2}}(K^0 \bar{K}^0 + K^+ K^-), \\ |K\bar{K}, I = 1, I_3 = 0\rangle &= \frac{1}{\sqrt{2}}(K^0 \bar{K}^0 - K^+ K^-). \end{aligned}$$

As we know in the chiral unitary approach, $a_0(980)$ is dynamically generated as the interaction of the coupled channels $\pi\eta$ and $K\bar{K}$. Then an extension of these ideas to the interaction of vector mesons was done, and interestingly two resonances of $f_0(1710)$ and $a_0(1710)$ were predicted qualifying roughly as $K^* \bar{K}^*$ molecules [4, 5].

2. – Formalism

Figure 1 shows the Cabibbo-favored decay mode of D_s^+ at the quark level and the hadronization with the vacuum quantum numbers ($\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$). Figure 2 shows the internal emission and hadronization, which is suppressed by a color factor $1/N_c$. The external and internal emissions will produce the $f_0(1710)$ and $a_0(1710)$ resonances [6].

From figs. 1 and 2 and because the $G_{\omega\phi}$ and $G_{\rho\phi}$ loop functions are remarkably similar to $G_{K^* \bar{K}^*}$, finally the amplitudes \tilde{t}_{f_0} and \tilde{t}_{a_0} can be written

$$(3) \quad \begin{aligned} \tilde{t}_{f_0} &= A\{-\sqrt{2} G_{K^* \bar{K}^*}(M_{\text{inv}}) g_{f_0, K^* \bar{K}^*} + G_{\phi\phi}(M_{\text{inv}}) \sqrt{2} g_{f_0, \phi\phi} \\ &\quad - \sqrt{2} \gamma' G_{K^* \bar{K}^*}(M_{\text{inv}}) g_{f_0, K^* \bar{K}^*}\}, \\ \tilde{t}_{a_0} &= -A\sqrt{2} \delta' G_{K^* \bar{K}^*}(M_{\text{inv}}) g_{a_0, K^* \bar{K}^*}, \end{aligned}$$

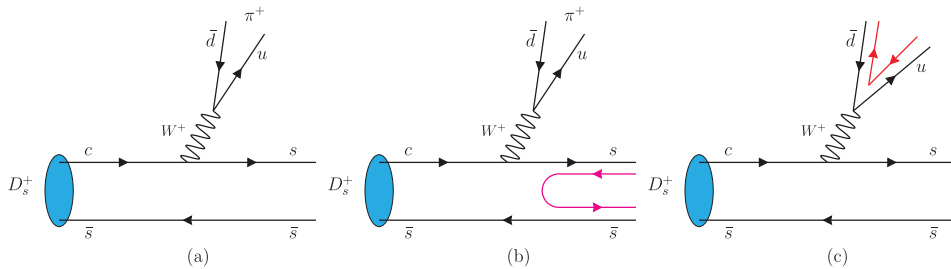
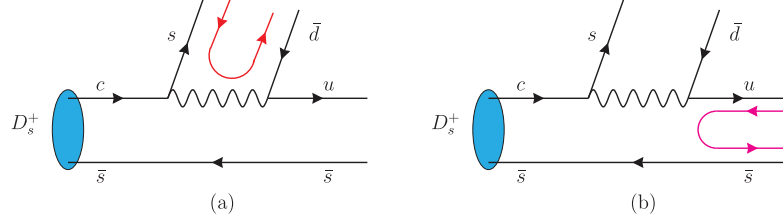


Fig. 1. – External emission of D_s^+ decay with π^+ production at the quark level (a) and hadronization of the $s\bar{s}$ component (b) and the $u\bar{d}$ component (c) with the vacuum quantum numbers.

Fig. 2. – Internal emission of D_s^+ decay and hadronization of the $s\bar{d}$ pair (a) and the $u\bar{s}$ pair (b).

with the two effective parameters

$$\gamma' = \gamma - \alpha \frac{g_{f_0, \omega \phi}}{g_{f_0, K^* \bar{K}^*}}, \quad \delta' = \delta - \beta \frac{g_{f_0, \rho \phi}}{g_{a_0, K^* \bar{K}^*}}.$$

The global factor A will disappear when we evaluate the ratios of production. We also need amplitudes of the two resonances decay into $K\bar{K}$, shown in fig. 3, with $K^* \bar{K}^* \rightarrow K\bar{K}$ transitions driven by π exchange, and $\phi(\rho, \omega, \phi) \rightarrow K\bar{K}$ transitions driven by K exchange, the weights are given by

$$(4) \quad W_{f_0} = \sum_i g_{f_0, i} \widetilde{W}_i G_i(M_{\text{inv}}), \quad W_{a_0} = \sum_i g_{a_0, i} \widetilde{W}_i G_i(M_{\text{inv}}),$$

where $g_{f_0, i}$ and $g_{a_0, i}$ are the couplings of the $f_0(1710)$ and $a_0(1710)$ resonances to the different coupled channels that build up the resonance, the \widetilde{W}_i coefficients can be evaluated from Lagrangian for $V \rightarrow PP$, and the sum over i goes over the channels of $I = 0$ and $I = 1$ respectively. Finally we obtain the t_i amplitudes as follows:

$$\begin{aligned} t_{K^+ K^-} &= -\tilde{t}_{f_0} \frac{1}{M_{\text{inv}}^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}} W_{f_0} \frac{1}{\sqrt{2}} g_{K\bar{K}} - \tilde{t}_{a_0} \frac{1}{M_{\text{inv}}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} \\ &\quad \times W_{a_0} \frac{1}{\sqrt{2}} g_{K\bar{K}}, \\ t_{K^0 \bar{K}^0} &= -\tilde{t}_{f_0} \frac{1}{M_{\text{inv}}^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}} W_{f_0} \frac{1}{\sqrt{2}} g_{K\bar{K}} + \tilde{t}_{a_0} \frac{1}{M_{\text{inv}}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} \\ &\quad \times W_{a_0} \frac{1}{\sqrt{2}} g_{K\bar{K}}, \\ t_{K^+ \bar{K}^0} &= \tilde{t}_{a_0} \frac{1}{M_{\text{inv}}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0} g_{K\bar{K}}, \\ t_{K^+ K_S^0} &= -\frac{1}{\sqrt{2}} t_{K^+ \bar{K}^0}. \end{aligned}$$

The differential decay width

$$(5) \quad \frac{d\Gamma_i}{dM_{\text{inv}}(K\bar{K})} = \frac{1}{(2\pi)^3} \frac{1}{4M_{D_s}^2} p_\pi \tilde{p}_k |t_i|^2.$$

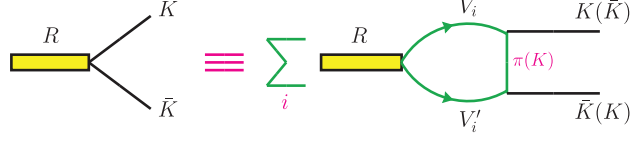


Fig. 3. – Amplitude for $R \rightarrow K\bar{K}$ for a resonance build up from the V_i, V_i' channels. Diagrams with $\bar{K}K$ instead of $K\bar{K}$ in the final state appear with ρ, ω, ϕ vector mesons but not for the $V_i, V_i' \equiv K^* \bar{K}^*$.

The ratios are defined as

$$(6) \quad R_1 = \frac{\Gamma(D_s^+ \rightarrow \pi^+ K^0 \bar{K}^0)}{\Gamma(D_s^+ \rightarrow \pi^+ K^+ K^-)}, \quad R_2 = \frac{\Gamma(D_s^+ \rightarrow \pi^0 K^+ K_S^0)}{\Gamma(D_s^+ \rightarrow \pi^+ K^+ K^-)}.$$

3. – Results

For the two effective parameters, a narrow region of the parameters $\gamma' \in [-1, 0.1]$, $\delta' \in [-1.3, 1.3]$ is obtained and shown in fig. 4, which is consistent with the large N_c limit estimates within uncertainties.

Using the above parameters, we obtain the ratio of $R_1 = 6.20 \pm 0.67$, which is in good agreement with BESIII experimental data [2, 3].

Next, the big challenge of the approach is to make prediction of R_2 . In [6], $R_2^{\text{theo}} \simeq 1.31 \pm 0.12$ was obtained and from this ratio we have evaluated

$$(7) \quad \text{Br}[D_s^+ \rightarrow \pi^0 a_0(1710)^+; a_0(1710)^+ \rightarrow K^+ K_S^0] \simeq (1.3 \pm 0.4) \times 10^{-3},$$

which was a prediction before this ratio was measured.

We make a further analysis by taking $\gamma' = -0.5$, $\delta' = -0.75$ (middle of the allowed region) in fig. 5, finding that in the $K^0 \bar{K}^0$ mass distribution there has been a constructive interference from the two resonances of $I = 0$ and $I = 1$, while in the $K^+ K^-$ mass distribution the interference has been destructive. This is exactly the reason suggested in the experimental analysis to justify the existence of the $a_0(1710)$ resonance [2, 3],

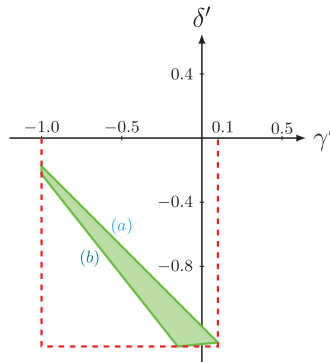


Fig. 4. – The range of two effective parameters.

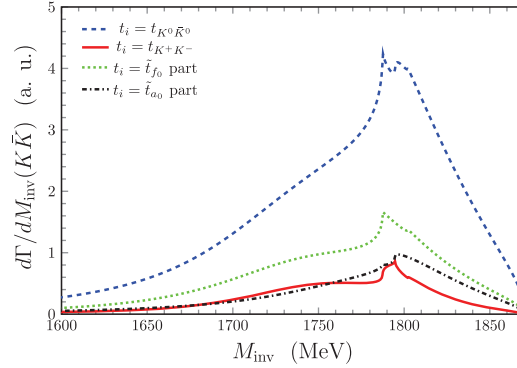


Fig. 5. – Mass distributions $d\Gamma/dM_{\text{inv}}$ for the cases of eq. (5).

because it should give the same K^+K^- or $K^0\bar{K}^0$ mass distributions should there be only the $f_0(1710)$ state. Hence, we give a boost to the molecular interpretation on the nature of these two $f_0(1710)$ and $a_0(1710)$ resonances.

4. – Summary

Based on the prediction of $f_0(1710)$ and $a_0(1710)$ as molecular states of $K^*\bar{K}^*$ and other vector-vector coupled channels, we investigate the two $D_s^+ \rightarrow \pi^+ K^+ K^-$ and $D_s^+ \rightarrow \pi^+ K_S^0 K_S^0$ reactions. Two effective parameters related to external and internal emission are obtained with a narrow region, which is consistent with the large N_c limit within uncertainties. Using the allowed parameters, we can reasonably explain the surprising large ratio of R_1 , in good agreement with recent BESIII experiments. We further made a prediction of $\text{Br}[D_s^+ \rightarrow \pi^0 a_0(1710)^+; a_0(1710)^+ \rightarrow K^+ K_S^0] \simeq (1.3 \pm 0.4) \times 10^{-3}$.

We obtain a fair prediction for the experimental branching fraction $\text{Br}[D_s^+ \rightarrow \pi^0 a_0(1710)^+; a_0(1710)^+ \rightarrow K^+ K_S^0] \simeq (3.44 \pm 0.52 \pm 0.32) \times 10^{-3}$ [7], confirming the existence of new $a_0(1817)$ resonance. Our predicted state of $a_0(1710)$ (new $a_0(1817)$) will shed light into the structure of scalar mesons in the light quark sector and other relevant issues currently under debate in hadron physics.

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