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Quark mass dependence of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ states

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Summary. — We study the light and heavy quark mass dependence of the low-lying charmed mesons in the framework of one-loop HHχ PT. The low-energy constants are determined by analyzing the available lattice data from different LQCD simulations. Model selection tools are implemented to determine the relevant parameters as required by data with a higher precision. We analyze the HSC energy levels for DK scattering in $I = 0$ for different boosts and two pion masses and perform a global fit with these levels in addition to DK and D^*K scattering energy levels from RQCD and Prelovsek *et al.* Finally we extract the pion mass dependence of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ resonances.

1. – Introduction

The discovery of exotic hadrons in the heavy quark sector, which cannot be accomodated in terms of $q\bar{q}$ mesons or qqq baryons, like those with a tetraquark or pentaquark structure, has manifested the relevance of hadronic loops in order to explain the masses and other properties of many states in the hadron spectrum [1].

This results in a couple of bound states with binding energy of ~40 MeV with respect to the DK and D^*K thresholds, with a probability of 70% for the DK and D^*K components in the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ states, respectively. Given the large amount of parameters in one-loop HMχPT it has not been possible yet to pin down precisely the parameters of the theory. Because of the recent progress in lattice QCD it is worth coming back to this issue. In the first section we study the quark mass dependence of the ground state charmed mesons, D, D^*, D_s and D^*_s within one-loop HMχPT [2] by analyzing the available lattice data.

In the second section, the data of [3] are analyzed considering explicitly the coupling of DK components to $c\bar{s}$ states, including the mass of the latter as fitting parameter.

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The meson-meson interaction is given by the hidden gauge formalism. At leading order, this interaction is of the same type as the one of HMχPT. The quark mass dependence of the $D^*_{s0}(2317)$ properties in both, the light and charm quarks is extracted [4]. In order to perform the extrapolation, we use the analysis of charmed meson masses [5] from the previous section. Moreover, we study the sensitivity in the compositeness of Weinberg to the quark masses within the energy range considered. We compare with previous lattice QCD analyses, and, finally, we perform a global LQCD data fit of [3, 6-9], extracting accurately the quark mass dependence of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$.

2. – Quark mass dependence of the low-lying charmed mesons at one loop in HHχPT

The formulas for the low-lying charmed meson masses (D, D^*, D_s and D^*_s) are calculated at one loop in HHχPT in the appendices (A.1) and (A.10) of [2]. These are

$$\begin{aligned}(1)\quad &\frac{1}{4}(M_{D_a}+3M_{D^*_a}) = \\ &m_H+\alpha_a-\sum_{X=\pi,K,\eta}\beta_a^{(X)}\frac{M_X^3}{16\pi f^2}+\sum_{X=\pi,K,\eta}\left(\gamma_a^{(X)}-\lambda_a^{(X)}\alpha_a\right)\frac{M_X^2}{16\pi^2 f^2}\log\left(M_X^2/\mu^2\right)+c_a\end{aligned}$$

and

$$(M_{D^*_a}-M_{D_a})=\Delta+\sum_{X=\pi,K,\eta}\left(\gamma_a^{(X)}-\lambda_a^{(X)}\Delta\right)\frac{M_X^2}{16\pi^2 f^2}\log\left(M_X^2/\mu^2\right)+\delta c_a. \tag{2}$$

The meson masses are fitted to the lattice data collected. Given the large number of parameters, we apply the Least Absolute Shrinkage and Selection Operator (LASSO) [10] method to eliminate the less relevant parameters of the model. In order to do that we will add a penalty term to the χ^2 of the form $P=\frac{\lambda}{10}\sum_i^n |p_i|$, where p_i are the parameters of the model. As the penalty coefficient λ increases, the LASSO method will set the less important parameters to zero in order to minimize the full penalized $\chi^2_F=\chi^2+P$.

We use automatic differentiation [11] to evaluate the errors propagation with the Julia ADerrors package. We can see in fig. 1 the final result of D and D^* meson masses fit to the lattice data.

3. – Quark mass dependence of $D^*_{s0}(2317)$ and $D_{s1}(2460)$

In order to extrapolate the $D_{s0}(2317)$ pole we need first to fit the indeterminate parameters of the $DK \to DK$ amplitude. The scattering amplitude in the infinite volume is given by the Bethe-Salpeter equation, that is,

$$T=\frac{V}{1-VG}, \tag{3}$$

where V is the scattering potential matrix and G the loop matrix. To compute the V potential for $DK \to DK$ we use the hidden gauge Lagrangian which corresponds to the hidden gauge formalism [12,13], and also the lowest order of HMχPT. For the $DK \to DK$

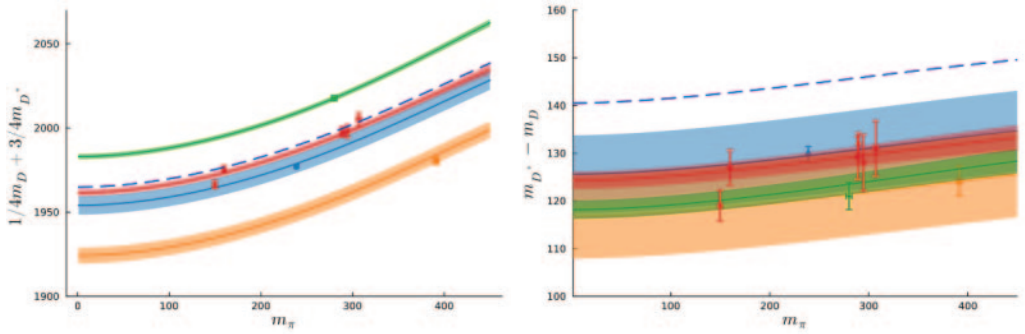

Fig. 1. – Results of the D and D^* meson masses for the global analysis.

scattering we only need the Lagrangian term of pseudoscalar-pseudoscalar-vector that is, $\mathcal{L}^{PPV} = ig'\text{Tr}[[\partial_i\Phi, \Phi]\mathcal{V}^i]$, where $g' = m_\rho/(2f_\pi)$, Φ is the $SU(4)$ pseudoscalar meson matrix given in [13] and $\mathcal{V}^i$ the vector meson matrix. With this, we have the following amplitude just before projecting to partial waves:

$$V_{DK} = -\frac{s-u}{2f^2}. \tag{4}$$

Note that one arrives also to this interaction through the LO HMχPT [14, 15]. A term associated with the interaction of DK with a bare $c\bar{s}$ states of the $J_l^P = \frac{1}{2}^+$ HQSS doublet can be added. At LO in the heavy quark expansion this gives [15]

$$V_{\text{ex}} = \frac{V_{c\bar{s}}^2}{s - m_{c\bar{s}}^2}\ , \qquad \text{with} \qquad V_{c\bar{s}}(s) = -\frac{c}{f}\sqrt{M_D m_{c\bar{s}}}\frac{s + m_K^2 - M_D^2}{\sqrt{s}}, \tag{5}$$

where $m_{c\bar{s}}$ is the mass of the bare $c\bar{s}$ component, and c is a dimensionless constant that provides the strength of the coupling of this component to the DK channel. In this work we will consider this coupling as a free parameter, together with $m_{c\bar{s}}$. Note that $m_{c\bar{s}}$ can vary for every LQCD simulation with a different setup. The potential $V(s)$ consistent with HQSS is then given by $V(s) = V_{DK}(s) + V_{\text{ex}}(s)$. To project to partial waves we use

$$V_l(s) = \frac{1}{2}\int_{-1}^{1} M(s,\theta)\mathcal{P}_l(\cos(\theta))\text{d}\cos(\theta), \tag{6}$$

where $\mathcal{P}_l(x)$ are the Legendre polynomials. However, as we are going to fit to lattice data we need to discretize the momenta $q = (2\pi/L)\vec{n}$ with $\vec{n} \in Z^3$. In the finite volume the scattering amplitude is given by $\widetilde{T} = V/(1 - V\widetilde{G})$. The final $\widetilde{G}$ that we insert to the Bethe-Salpeter equation is constructed with the elements

$$\widetilde{G}_{fin}^{(i)co} = G_{inf}^{(i)DR} + \lim_{q_{co}\to\infty}(G_{fin}^{(i)} - G_{inf}^{(i)co}), \tag{7}$$

where superscripts DR and co means that we have used the dimensional and the cut-off regularization method and the inf, fin subscripts means infinite or finite volume respectively. With this, the value of the cutoff is cancelled between the last two terms

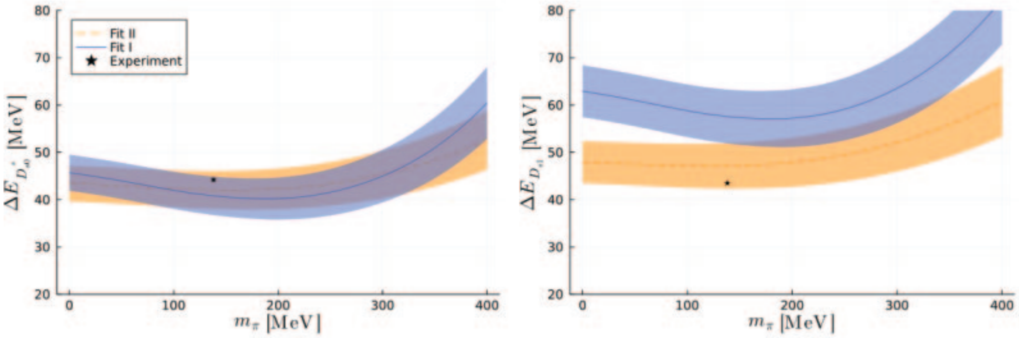

Fig. 2. – Binding energies of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ for physical charm and strange quark masses.

of eq. (7). Fixing the dimensional regularization scale $\mu = 1000\,\text{MeV}$ we only have the subtraction constant a as a free parameter. In order to extrapolate to the physical point we assume that the subtraction constant can be written as a first-order Taylor expansion of the squared pion mass as $a = a_1 + a_2 m_\pi^2$. We determine the two fitting parameters a_1, a_2. We perform a global fit analyzing the energy levels of [3] for DK scattering in $I = 0$ and the DK and D^*K scattering levels from [6, 7, 9]. As fitting parameters, we have c, $m_{c\bar{s}}$, a_1 and a_2 for the subtraction constants. We consider two situations, one where the bare $c\bar{s}$ component is not included, *i.e.*, $V_{\text{ex}} = 0$, and other with $V_{\text{ex}} \neq 0$.

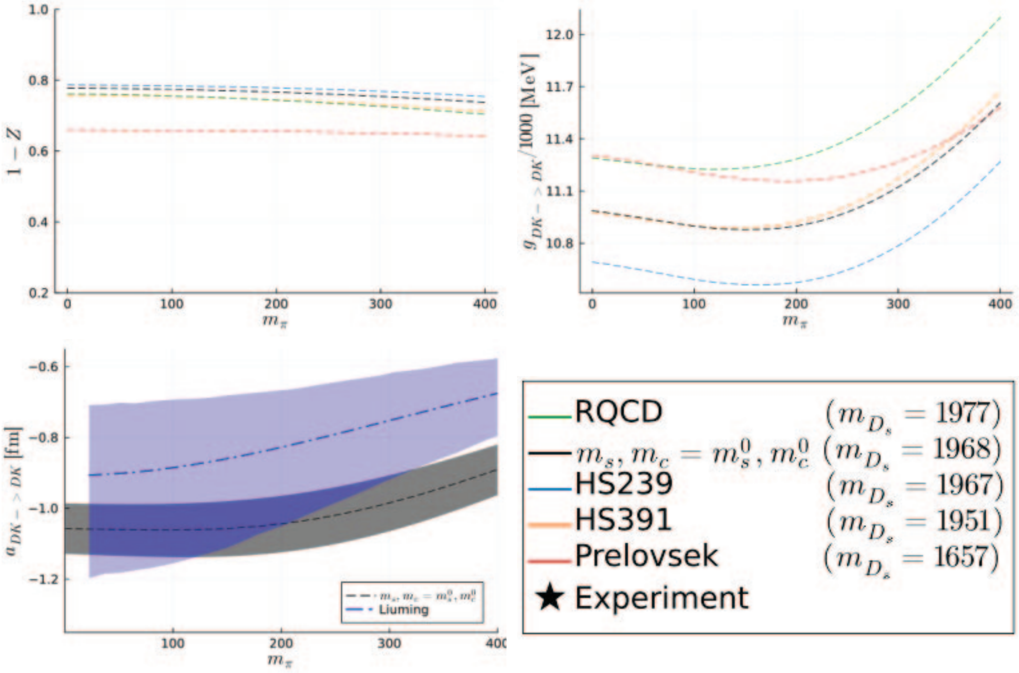

Fig. 3. – Fit II: result of the global fit for the scattering length as a function of the pion mass in comparison with the result obtained in [14] (dashed line with grey band).

In fig. 2 we plot the binding energies and mass splitting of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ states as a function of the pion mass in both fits. The binding energies of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ are compatible with the experimental data for Fit II, while for the Fit I we predict the $D_{s1}(2460)$ state with less than 20 MeV from the physical value. They also increase with the pion mass for pion masses above 200 MeV, showing a quadratic behaviour. To plot fig. 2 we have used an extrapolation of the bare mass to the physical point.

Finally, in fig. 3 we also show the pion mass dependence of the compositeness, the coupling constant and the scattering length. The compositeness is lower for the Prelovsek data set. The scattering length is in reasonably good agreement with previous works [8, 14]. However, the errors extracted here are smaller. The scattering length obtained here is lower than in [14].

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