IL NUOVO CIMENTO **47 C** (2024) 170
DOI 10.1393/ncc/i2024-24170-4

Colloquia: HADRON2023

Molecular states of $D^*D^*\bar{K}^*$ and $B^*B^*K^*$ nature

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received 21 December 2023

Summary. — We report the theoretical study of the three-body system composed of $D^*D^*\bar{K}^*$ and $B^*B^*K^*$. We study the interaction of two D^* (or two $\bar{B}^*$) and one $\bar{K}^*$ by using the fixed center approximation to the Faddeev equations to search for bound states of the three-body system. Since the D^*D^* interaction is attractive and the $D^*\bar{K}^*$ interaction is also attractive, we can expect to obtain the bound state of the three-body system $D^*D^*\bar{K}^*$, which is manifestly exotic state with ccs open quarks. Using the same analogy of the $D^*D^*\bar{K}^*$ system, we also study the $\bar{B}^*\bar{B}^*\bar{K}^*$ system containing the bbs open quarks since both interactions of $\bar{B}^*\bar{B}^*$ and $\bar{B}^*\bar{K}^*$ are attractive. We obtain the bound states of isospin $I = 1/2$, negative parity, and total spin $J = 0$, 1 and 2.

1. – Introduction

Many exotic mesons, which cannot be explained as the ordinary mesons of $q\bar{q}$, have been observed in the experiments. The recent experimental findings of the $X_0(2900)$ in the $D\bar{K}$ invariant mass and the $T_{cc}(3875)$ in the $DD\pi$ spectrum revealed clear exotic mesonic structures, since one has cs quarks in the first case and cc quarks in the second one. Based on the theoretical interpretations of the molecular picture, the $X_0(2900)$ and the $T_{cc}(3875)$ are identified as the $D^*\bar{K}^*$ and D^*D bound states, respectively.

In this article, we report the theoretical studies [1, 2] of the three-body systems $D^*D^*\bar{K}^*$ and $\bar{B}^*\bar{B}^*\bar{K}^*$, which are manifestly exotic bound states with ccs and bbs open quarks. The reason to choose the systems is that the D^*D^* and $\bar{B}^*\bar{B}^*$ interactions with $I(J^P) = 0(1^+)$ were found to bind in refs. [3, 4], and the $D^*\bar{K}^*$ and $\bar{B}^*\bar{K}^*$ interactions were also found to be attractive in refs. [5, 6]. Especially, the $D^*\bar{K}^*$ bound state with

$J^P = 0^+$ in ref. [5] is identified as the $X_0(2900)$. Therefore, these exotic three-body systems are expected to exist and we calculate the binding energy and width of the possible bound states.

The three body-systems of molecular nature have also been studied recently. One of the methods to solve the three-body system is the fixed center approximation (FCA) to the Faddeev equations. In the study of the $D\bar{D}K$ system, the FCA has been compared to the variational method and similar results have been found in refs. [7, 8]. Thus, we use the FCA to study the $D^*D^*\bar{K}^*$ and $\bar{B}^*\bar{B}^*\bar{K}^*$ systems.

2. – Formalism

First, we briefly explain the FCA formalism of the $D^*D^*\bar{K}^*$ system. In this picture, we assumed that there is a cluster of two bound particles D^*D^*, and the third one ($\bar{K}^*$) collides with the components of this cluster without modifying its wave function. The D^*D^* system was found to be bound with about 4–6 MeV in $I(J^P) = 0(1^+)$ in ref. [3]. In fig. 1, we show the corresponding diagrams. The total three-body scattering amplitude T is written by the sum of the partition functions T_1 and T_2. T_1 is the sum of all diagrams in the upper part of fig. 1, where the $\bar{K}^*$ collides first with the particle 1 of the cluster, while T_2 is the sum of all diagrams in the lower part of fig. 1, where the $\bar{K}^*$ collides first with the particle 2 of the cluster. We can write this as

$$\begin{aligned} T &= T_1 + T_2, \\ T_1 &= t_1 + t_1 G_0 T_2, \\ T_2 &= t_2 + t_2 G_0 T_1, \end{aligned} \tag{1}$$

where G_0 is the $\bar{K}^*$ propagator folded with the cluster wave function and t_i is the amplitude for two-body scattering $D^*(i)\bar{K}^*$ $(i = 1, 2)$. For the evaluation of the two-body t_i amplitudes, we consider the combination of the isospin and the spin decomposition of the $D^*\bar{K}^*$. We use the $D^*\bar{K}^*$ amplitude in ref. [5] for the different isospin $I = 0, 1$ and

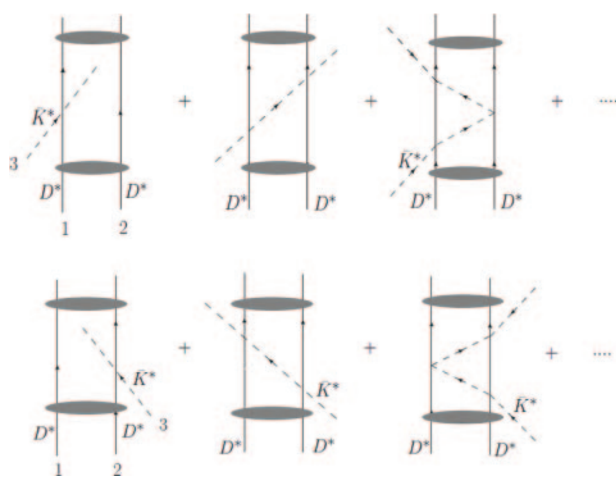

Fig. 1. – Diagrams involved in the fixed center approximation (FCA) for the collision of the $\bar{K}^*$ with the cluster of D^*D^*.

spin $J = 0, 1, 2$. In the $D^*D^*\bar{K}^*$ system, we can have three total spins $J = 0, 1, 2$, and we obtain the final contribution of t_i for different total spin J in ref. [1]. In addition, we consider the normalization of the amplitudes when mixing two-body amplitudes with three-body amplitudes in the same expression. We replace t_i into $\tilde{t}_i = \frac{m_C}{m_{D^*}} t_i$ with the cluster mass m_C and the D^* mass m_{D^*}, thus eq. (1) leads to

$$\tilde{T}_1 = \tilde{t}_1 + \tilde{t}_1 \tilde{G}_0 \tilde{T}_1; \quad \tilde{T}_1 = \frac{1}{\tilde{t_1}^{-1} - \tilde{G}_0}; \qquad \tilde{T} = \tilde{T}_1 + \tilde{T}_2 = 2\tilde{T}_1, \tag{2}$$

where we used $t_1 = t_2$, then $T_1 = T_2$. We plot $|\tilde{T}|^2$ for the three-body invariant mass energy $\sqrt{s}$ and we look for the peaks to deduce the mass and width of the bound states.

In the $\bar{B}^*\bar{B}^*\bar{K}^*$ system, it was found that the $\bar{B}^*\bar{B}^*$ in $I(J^P) = 0(1^+)$ was bound with a binding energy of about 40 MeV in ref. [4]. In addition, the $\bar{B}^*\bar{K}^*$ was also found to be strongly attractive in ref. [6]. Thus, we perform a similar calculation to the $D^*D^*\bar{K}^*$.

3. – Numerical results and discussions

In fig. 2, we show the calculated three-body amplitude $|\tilde{T}|^2$ for the $D^*D^*\bar{K}^*$ system as a function of the three-body invariant mass energy $\sqrt{s}$. For the total spin $J = 0$, we find a clear peak around 4845 MeV, about 61 MeV below the $D^*D^*\bar{K}^*$ threshold. The width is about 80 MeV. The D^*D^* state is bound by about 4–6 MeV, while the $D^*\bar{K}^*$ state, corresponding to the $X_0(2900)$, is bound by about 30 MeV. This means that the interaction of $\bar{K}^*$ with two D^* would lead to a binding about twice as big as that of $D^*\bar{K}^*$. We also calculated the wave function for the $\bar{K}^*$ in the $D^*D^*\bar{K}^*$ system at rest in ref. [1]. We found that the mean square radius is about 1 fm, which is larger than the mean square radius of the proton, 0.84 fm, and smaller than that of the deuteron, 2.1 fm. For the total spin $J = 1, 2$, we can see two peaks indicating two states. We can easily trace the origin of the peaks from the $D^*\bar{K}^*$ amplitude t_i as discussed in ref. [1]. This is because the calculation of the three-body total spin $J = 1$ is tied to the $J = 0, 1, 2$ of $D^*\bar{K}^*$. On the other hand, the calculation of the total spin $J = 0$ appears as one peak because it has only $J = 1$ of $D^*\bar{K}^*$. Thus, in total, we find five states for the total spin $J = 0, 1, 2$ and summarize their binding energy and width in table I (top).

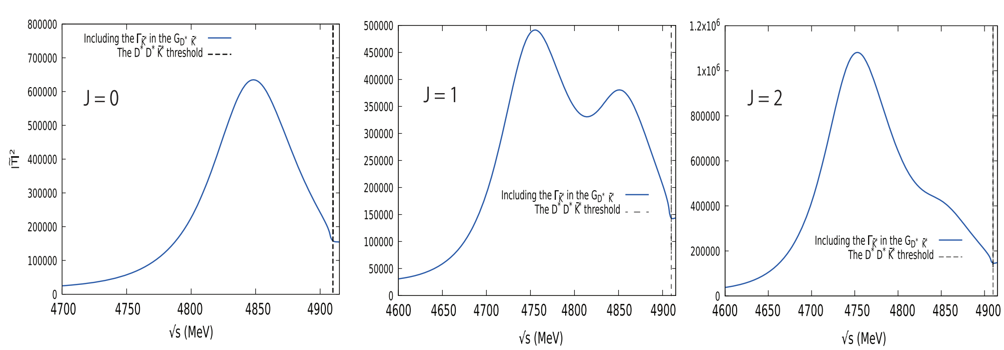

Fig. 2. – The three-body amplitude $|\tilde{T}|^2$ for the $D^*D^*\bar{K}^*$ system as a function of the three-body invariant mass energy $\sqrt{s}$ for the different total spin J. The dotted vertical line indicates the $D^*D^*\bar{K}^*$ threshold $(2m_{D^*} + m_{\bar{K}^*})$.

Table I. – *The calculated binding (B), width (Γ) of the three-body systems $D^*D^*\bar{K}^*$ and $\bar{B}^*\bar{B}^*\bar{K}^*$ states for the different possible total spins J. The binding energy B is obtained with respect to the threshold energy, $2m_{D^*} + m_{\bar{K}^*}$ and $2m_{\bar{B}^*} + m_{\bar{K}^*}$ for $D^*D^*\bar{K}^*$ and $\bar{B}^*\bar{B}^*\bar{K}^*$ respectively. Numbers are taken from refs.* [1,2].

	J	B [MeV]	Γ [MeV]
$D^*D^*\bar{K}^*$	0	61	80
	1 (State I)	56	94
	1 (State II)	152	100
	2 (State I)	66	85
	2 (State II)	151	100
$\bar{B}^*\bar{B}^*\bar{K}^*$	0	109–150	72–104
	1	118–158	106–153
	2	130–174	103–149

In table I (bottom), we summarize the binding energy and width for the three-body systems $\bar{B}^*\bar{B}^*\bar{K}^*$ obtained. In the $\bar{B}^*\bar{B}^*\bar{K}^*$ system, one bound state is obtained for each J, which is different from the case of the $D^*D^*\bar{K}^*$ system. This is the effect of an overlap of the different states due to the $\bar{B}^*\bar{K}^*$ large width. We also find that the binding energy and width for the $\bar{B}^*\bar{B}^*\bar{K}^*$ are relatively larger than those of the $D^*D^*\bar{K}^*$.

4. – Conclusion

We have reported the theoretical study of a search for possible bound states of the three-body systems $D^*D^*\bar{K}^*$ and $\bar{B}^*\bar{B}^*\bar{K}^*$ based on refs. [1,2]. The D^*D^* and $\bar{B}^*\bar{B}^*$ interactions with $I(J^P) = 0(1^+)$ were found to bind, and the $D^*\bar{K}^*$ and $\bar{B}^*\bar{K}^*$ interactions were also found to be attractive. For this, we applied the FCA to Faddeev equations where the $\bar{K}^*$ interact with each of the particles in the D^*D^* and $\bar{B}^*\bar{B}^*$ cluster. From the numerical results, we found that the bound states obtained have relatively large binding energy for different total spin $J = 0, 1, 2$. Thus, we hope that these exotic mesons, with open strange and double-charm(bottom) flavors, can be experimentally found in the near future.

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This work was partly supported by JSPS KAKENHI Grant No. JP P19K14709.

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