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Mass spectrum of three-quark and five-quark singly heavy baryons from a chiral model

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Summary. — We construct a chiral effective model for three-quark and five-quark singly heavy baryons (SHBs) which is heavy-quark spin singlet, focusing on the $U(1)_A$ axial anomaly effects. Based on the model, we find that the anomaly effects induce the inverse mass hierarchy of negative-parity three-quark SHBs, where Λ_c becomes heavier than Ξ_c. On the contrary, the anomaly effect is found to provide no effects for the mass spectrum of five-quark SHBs. We also present a predicted mass spectrum of the SHBs in the presence of the mixing between three-quark and five-quark states. The predicted five-quark dominant $\Lambda_c(-)$, whose mass is approximately 2700 MeV, is expected to be a useful evidence to check our description.

1. – Introduction

Singly heavy baryons (SHBs) composed of a heavy quark and a diquark provide us with useful testing ground to explore the diquark dynamics, since the heavy quark can be regarded as a spectator due to its large mass. In this paper, we review our recent studies on the heavy-quark spin-singlet SHBs, quark contents of which are Qqq and $Qqq\bar{q}q$, *i.e.*, three-quark states and five-quark states, where the Roper-like $\Lambda_c(2765)$ and $\Xi_c(2970)$ are assumed to be the five-quark dominant SHBs [1-3]. In particular, we focus on effects of $U(1)_A$ axial anomaly on the SHBs based on a three-flavor chiral model.

This article is organized as follows: in sect. **2** we introduce our chiral mode describing the three-quark and five-quark SHBs; then, in sect. **3** and sect. **4**, we investigate masses of the SHBs with and without mixings between the three-quark and five-quark states; finally, in sect. **5** we conclude the present work.

2. – Model

In this section, we introduce our chiral model to describe the three-quark and five-quark SHBs.

Toward the model construction, we invent the following four building blocks:

$$B_{R,a} \sim Q^\alpha d^\alpha_{R,a}\ ,\quad B_{L,i} \sim Q^\alpha d^\alpha_{L,i}\ ,\quad B'^{\alpha}_{R,i} \sim Q^\alpha d'_{R,i}\ ,\quad B'_{L,a} \sim Q^\alpha d'^{\alpha}_{L,a}, \tag{1}$$

where $d^{\alpha}_{R,a}$, $d^{\alpha}_{L,a}$, $d'^{\alpha}_{R,a}$ and $d'^{\alpha}_{L,a}$ represent the conventional diquarks and the newly invented *tetra-diquarks* defined by [1,3]

$$
\begin{aligned}
(d_R)^{\alpha}_a &\sim \epsilon_{abc}\epsilon^{\alpha\beta\gamma}(q_R^T)^{\beta}_b C(q_R)^{\gamma}_c \ , \\
(d_L)^{\alpha}_i &\sim \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}(q_L^T)^{\beta}_j C(q_L)^{\gamma}_k \ , \\
(d'_R)^{\alpha}_i &\sim \epsilon_{abc}\epsilon^{\alpha\beta\gamma}(q_R^T)^{\beta}_b C(q_R)^{\gamma}_c [(\bar{q}_L)^{\delta}_i (q_R)^{\delta}_a] \ , \\
(d'_L)^{\alpha}_a &\sim \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}(q_L^T)^{\beta}_j C(q_L)^{\gamma}_k [(\bar{q}_R)^{\delta}_a (q_L)^{\delta}_i] \ ,
\end{aligned}
\tag{2}
$$

respectively. In these equations the subscripts "$a, b, \cdots$" and "$i, j, \cdots$" stand for the left-handed and right-handed chiral indices, respectively, within a three-flavor description, while the superscripts "$\alpha, \beta, \cdots$" indicate the color indices. Thus, in eq. (1), $B_{R,a}$ and $B_{L,i}$ are regarded as the three-quark SHBs, while $B'_{R,i}$ and $B'_{L,a}$ are the five-quark ones. The chiral representation for those diquarks reads

$$
d_R \sim (\mathbf{1}, \bar{\mathbf{3}})_{+2} \ , \quad d_L \sim (\bar{\mathbf{3}}, \mathbf{1})_{-2} \ , \quad d'_R \sim (\bar{\mathbf{3}}, \mathbf{1})_{+4} \ , \quad d'_L \sim (\mathbf{1}, \bar{\mathbf{3}})_{-4}
\tag{3}
$$

where the number attached to the respective bracket, *e.g.*, +2 for d_R, stands for their $U(1)_A$ axial charges. The corresponding SHBs take the identical symmetry properties. We note that the chiral representations carried by d_R and d'_L are the same, likewise, those by d_L and d'_R are the same. That is, those states are distinguished by the $U(1)_A$ axial charges.

From the chiral representation (3) with definition of the SHB fields (1), one can construct a chiral model for the SHBs interacting with a light-meson nonet $\Sigma = S + iP$ whose chiral representation is $\Sigma \sim (\mathbf{3}, \bar{\mathbf{3}})_{-2}$. Our Lagrangian is based on the following counting scheme: first we include all possible terms, which are invariant under both $U(1)_A$ axial and $SU(3)_L \times SU(3)_R$ chiral transformations, and next, we additionally incorporate contributions which only violate $U(1)_A$ axial symmetry to take into account the anomalous contributions with the minimal number of $\Sigma^{(\dagger)}$. Then, our chiral Lagrangian within the heavy-baryon effective theory is constructed as [3]

$$
\mathcal{L}_{\rm SHB} = \mathcal{L}_{3q} + \mathcal{L}_{5q} + \mathcal{L}_{\rm mix}
\tag{4}
$$

where

$$
\begin{aligned}
\mathcal{L}_{3q} = & \sum_{\chi=L,R} (\bar{B}_\chi iv\cdot\partial B_\chi - \mu_1 \bar{B}_\chi B_\chi) - \frac{\mu_3}{f_\pi^2}\left[\bar{B}_L(\Sigma\Sigma^\dagger)^T B_L + \bar{B}_R(\Sigma^\dagger\Sigma)^T B_R\right] \\
& - \frac{g_1}{2f_\pi}\left(\epsilon_{ijk}\epsilon_{abc}\bar{B}_{L,k}\Sigma_{ia}\Sigma_{jb}B_{R,c} + \text{h.c.}\right) - g'_1(\bar{B}_L\Sigma^* B_R + \text{h.c.})
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\mathcal{L}_{5q} = & \sum_{\chi=L,R} (\bar{B}'_\chi iv\cdot\partial B'_\chi - \mu_2 \bar{B}'_\chi B'_\chi) - \frac{\mu_4}{f_\pi^2}\left[\bar{B}'_R(\Sigma\Sigma^\dagger)^T B'_R + \bar{B}'_L(\Sigma^\dagger\Sigma)^T B'_L\right] \\
& - \frac{g_2}{6f_\pi^3}\left[(\epsilon_{abc}\epsilon_{ijk}\Sigma^\dagger_{ci}\Sigma^\dagger_{bj}\Sigma^\dagger_{ak})(\bar{B}'_R\Sigma^* B'_L) + \text{h.c.}\right] \\
& - \frac{g_3}{2f_\pi^3}\left(\epsilon_{abc}\epsilon_{ijk}\bar{B}'_{R,l}\Sigma^\dagger_{cl}\Sigma^\dagger_{bi}\Sigma^\dagger_{aj}\Sigma^\dagger_{dk}B'_{L,d} + \text{h.c.}\right) + g'_2\left(\bar{B}'_R\Sigma^* B'_L + \bar{B}'_L\Sigma^T B'_R\right)
\end{aligned}
\tag{6}
$$

and

$$\mathcal{L}_{\rm mix} = -\,\mu_1'(\bar{B}_R B_L' + \bar{B}_L' B_R + \bar{B}_L B_R' + \bar{B}_R' B_L) \\ -\; g_4(\bar{B}_R'\Sigma^* B_R + \bar{B}_L \Sigma^* B_L' + {\rm h.c.}) \tag{7}$$

with v^μ and f_π being a velocity of the SHBs and a pion decay constant. In this Lagrangian, $\mathcal{L}_{3q}$ describes interactions among the three-quark SHBs and light-meson nonet. Similarly, $\mathcal{L}_{5q}$ describes those among the five-quark SHBs and the nonet. The last piece, $\mathcal{L}_{\rm mix}$, is responsible for interplay between the three-quark and five-quark SHBs mediated by the light mesons. We note that only g_1', g_2' and μ_1' terms correspond to the anomalous contributions. Meanwhile, all the remaining terms proportional to μ_1, μ_2, μ_3, μ_4, g_1, g_2, g_3 and g_4 are $U(1)_A$ invariant although some of them are of fourth order of $\Sigma^{(\dagger)}$.

3. – Masses of the pure three-quark and five-quark SHBs

In this section, we examine the $U(1)_A$ axial anomaly effects on masses of the pure three-quark SHBs and five-quark SHBs.

First, we investigate the mass spectrum of the pure three-quark SHBs. The parity eigenstates, *i.e.*, the mass eigenstates of them are defined by $B_{\pm,i} = (B_{R,i} \mp B_{L,i})/\sqrt{2}$, with the diagonal parts of left- and right-handed indices: $i = a$, where the subscript "$\pm$" stands for the parity eigenvalues. Here, the SHBs with $i = 1, 2$ and $i = 3$ represent $\Xi_c^{[\mathbf{3}]} \sim cus\,(cds)$ and $\Lambda_c^{[\mathbf{3}]} \sim cud$ for the charm sector, where the superscript "$[\mathbf{3}]$" is attached to emphasize that those states are three-quark SHBs. Thus, the mass formulas for $B_{\pm,i}$ are read off by quadratic terms of B_R and B_L in $\mathcal{L}_{3q}$ in eq. (5), which yields [3]

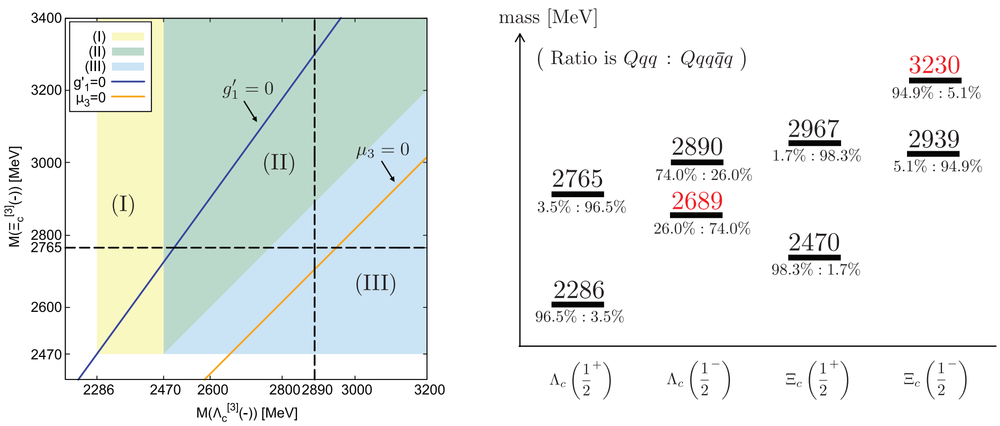
$$\begin{aligned} M[\Lambda_c^{[\mathbf{3}]}(\pm)] =& m_B + \mu_1 + \mu_3 \mp f_\pi(g_1 + Ag_1') \\ M[\Xi_c^{[\mathbf{3}]}(\pm)] =& m_B + \mu_1 + A^2_{\mu_3} \mp f_\pi(Ag_1 + g_1'). \end{aligned} \tag{8}$$

In this equation the sign "$\pm$" again indicates the parity eigenvalue. An arbitrary constant m_B is additionally included to defined the masses to incorporate the universal "heavy mass". Besides, in deriving the mass formulas (8), the chiral-symmetry-breaking effects are taken into account by replacing the light meson nonet Σ by its vacuum expectation value: $\langle\Sigma\rangle = f_\pi {\rm diag}(1, 1, A)$, where $f_\pi = 93\,{\rm MeV}$ and $A = (2f_K - f_\pi)/f_\pi = 1.38$.

When the ground-state SHBs are regarded as the three-quark SHBs, $M\big[\Lambda_c^{[\mathbf{3}]}(+)\big] = 2286\,{\rm MeV}$ and $M\big[\Xi_c^{[\mathbf{3}]}(+)\big] = 2470\,{\rm MeV}$ are satisfied, then two free parameters are left. When taking $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big]$ and $M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ to be free, one can draw mass hierarchies in the $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big]$ - $M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ plane as in fig. 1(a) with $g_1' = 0$ exhibited by the blue line. In this figure, the three colored regions represent the following possible mass orderings:

$$\begin{aligned} &{\rm (I)}\; M[\Lambda_c^{[\mathbf{3}]}(+)] < M[\Lambda_c^{[\mathbf{3}]}(-)] < M[\Xi_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(-)]\;, \\ &{\rm (II)}\; M[\Lambda_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(+)] < M[\Lambda_c^{[\mathbf{3}]}(-)] < M[\Xi_c^{[\mathbf{3}]}(-)]\;, \\ &{\rm (III)}\; M[\Lambda_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(+)] < M[\Xi_c^{[\mathbf{3}]}(-)] < M[\Lambda_c^{[\mathbf{3}]}(-)]\;. \end{aligned} \tag{9}$$

The regions (I) and (II) indicate that the negative-parity SHBs satisfy the normal mass hierarchy $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big] < M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ as naively expected from their quark contents. On the other hand, in the region (III) those masses read $M\big[\Lambda_c^{[\mathbf{3}]}(-)\big] > M\big[\Xi_c^{[\mathbf{3}]}(-)\big]$ despite



(a) The three mass hierarchies in Eq. (9).   (b) The mass spectrum with the mixing effects.

Fig. 1. – (Color online) (a) The mass orderings for the pure three-quark SHBs listed in eq. (9), where the normal ((I) and (II)) and inverse (III) mass hierarchies are explicitly shown. (b) The mass spectrum in the presence of the mixing between three-quark and five-quark SHBs. The figures are taken from ref. [3]. Copyright 2023, American Physical Society.

the quark contents. For this reason we call this ordering the *inverse mass hierarchy*. Since the blue line in fig. 1(a) indicates the masses with $g_1' = 0$ where the $U(1)_A$ axial anomaly effects are absent. Thus, we can conclude that the inverse mass hierarchy is induced by the $U(1)_A$ anomaly. This is the main finding of influence of the anomaly on mass spectrum of the pure three-quark SHBs.

Next, as for the pure five-quark SHBs, from $\mathcal{L}_{5q}$ in (6) the mass eigenvalues read [3]

$$
\begin{aligned}
M[\Lambda_c^{[\mathbf{5}]}(\pm)] &= m_B + \mu_2 + A^2\mu_4 \pm A f_\pi \left[A(g_2+g_3) + g_2'\right] \ , \\
M[\Xi_c^{[\mathbf{5}]}(\pm)] &= m_B + \mu_2 + \mu_4 \pm f_\pi \left[A(g_2+g_3) + g_2'\right] \ ,
\end{aligned}
\tag{10}
$$

where $\Xi_c^{[\mathbf{5}]} \sim csdu\bar{u}\,(csud\bar{d})$ and $\Lambda_c^{[\mathbf{5}]} \sim cuds\bar{s}$ and the other notations follow eq. (8). In this case, the common piece of $h \equiv A(g_2+g_3) + g_2'$ appears in $M[\Lambda_c^{[\mathbf{5}]}(\pm)]$ and $M[\Xi_c^{[\mathbf{5}]}(\pm)]$, and thus, the anomalous contributions from g_2' effectively disappear for the mass formulas. Therefore, in contrast to the pure three-quark SHBs, for the pure five-quark SHBs the $U(1)_A$ axial anomaly has no influence on the mass spectrum.

4. – Masses in the presence of the three-quark and five-quark SHBs

In this section, we present a typical predictions of the mass spectrum with mixing effects triggered by $\mathcal{L}_{\rm mix}$ in eq. (7).

When the mixing is present, in general, the mass eigenstates read, *e.g.*,

$$
\begin{pmatrix} \Lambda_c^L(\pm) \\ \Lambda_c^H(\pm) \end{pmatrix} = \begin{pmatrix} \cos\theta_{\Lambda_c(\pm)} & \sin\theta_{\Lambda_c(\pm)} \\ -\sin\theta_{\Lambda_c(\pm)} & \cos\theta_{\Lambda_c(\pm)} \end{pmatrix} \begin{pmatrix} \Lambda_c^{[\mathbf{3}]}(\pm) \\ \Lambda_c^{[\mathbf{5}]}(\pm) \end{pmatrix} \ ,
\tag{11}
$$

for the Λ_c sector with $\theta_{\Lambda_c(\pm)}$ being a mixing angle. Similar equation follows for the Ξ_c sector. The superscript "L/H" represents the eigenstate whose mass is lower/higher, and those mass eigenvalues $M[\Lambda_c^{L/H}(\pm)]$ and $M[\Xi_c^{L/H}(\pm)]$ are evaluated by diagonalizing the corresponding mass matrix from eqs. (5), (6) and (7). In the following analysis, we will assume the $U(1)_A$ anomaly effect is absent for a transparent demonstration with the mixing: $g_1' = g_2' = \mu_1' = 0$. Hence, there remain seven free parameters to be fixed: μ_1, μ_2, μ_3, μ_4, g_1, $h = A(g_2 + g_3)$ and g_4.

For positive-parity states, denote the ground-state and Roper-like SHBs as Λ_c^L (Ξ_c^L) and Λ_c^H (Ξ_c^H), respectively. Then the following four inputs are employed: $M[\Lambda_c^L(+)] = 2286\,\text{MeV}$, $M[\Xi_c^L(+)] = 2470\,\text{MeV}$, $M[\Lambda_c^H(+)] = 2765$ MeV and $M[\Xi_c^L(+)] = 2967\,\text{MeV}$. For the negative-parity SHBs, first, we employ a quark-model prediction for $\Lambda_c(-)$ provided in ref. [4] as another input: $M[\Lambda_c^L(-)] = 2890\,\text{MeV}$. Next, the experimentally observed $\Xi_c(2930)$ would be regarded as $\Xi_c^L(-)$, hence, the last input can be $M[\Xi_c^L(-)] = 2939\,\text{MeV}$. Here, we have attached "L" for both the inputs $\Lambda_c(-)$ and $\Xi_c(-)$ since, as shown below, those states will be found to be the lower mass-eigenvalue states.

From the above six inputs, only one free parameter is left. By fixing this last parameter at which $\theta_{\Xi_c(-)}$ becomes the largest value allowed by the decay width of $\Xi_c(2930)$, one can obtain the mass spectrum in the presence of the mixing as fig. 1(b). In this figure the mass values indicated in black and red correspond to the inputs and outputs, respectively. Also, the ratios shown below the mass values represent $Qqq : Qqq\bar{q}q$ for the respective states. Then, from this figure one can see that the ground-state (Roper-like) negative-parity SHBs are dominated by the five-quark (three-quark) components, while the positive-parity SHBs exhibit the opposite tendency due to the assumption. One notable prediction is the presence of five-quark dominant $\Lambda_c(-)$ whose mass is 2689 MeV. This SHB decays only through the heavy-quark spin-symmetry-breaking processes, and the resultant width is of order a few MeV. Meanwhile, the three-quark dominant $\Xi_c(-)$ whose mass is 3230 MeV has a catastrophically large decay width. We note that, even when the $U(1)_A$ axial anomaly effects are present, our main prediction of $\Lambda_c(-)$, whose mass is approximately 2700 MeV, does not change as shown in ref. [3].

5. – Conclusions

In this paper, we have unveiled effects of the $U(1)_A$ axial anomaly on the three-quark and five-quark SHBs based on a chiral model, and presented a prediction of the mass spectrum of those SHBs. The predicted five-quark dominant $\Lambda_c(-)$, whose mass is approximately 2700 MeV, is expected to be a useful prove to check our description.

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