

## Exotic states of fully heavy hadrons

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**Summary.** — QCD supports the existence of hadrons with a structure richer than quark-antiquark mesons and three-quark baryons that are conventionally referred to as exotic. Many candidates for such states have been discovered experimentally in the spectrum of heavy quarks, with their minimal quark content being four-quark: two heavy plus two light quarks. In addition, recent results of the LHCb Collaboration on the double- $J/\psi$  production near the threshold hint at the existence of fully charmed tetraquark states. A coupled-channel analysis of the LHCb data and a possible theoretical interpretation of the near-threshold exotic state predicted by this analysis are discussed. In the hadronic molecule interpretation, the strength of the interaction in the double- $J/\psi$  system mediated by soft-gluon exchanges is proportional to the chromopolarisability of the  $J/\psi$ . The same low-energy parameter evaluated for a fully heavy baryon appears several times larger than that for the heavy quarkonium composed of the heavy quarks of the same flavour. Thus a theoretical interpretation of the LHCb observation results in the prediction of a possible existence of di-baryon molecules formed by fully heavy baryons.

### 1. – Double-charmonium production in proton-proton collisions

The double- $J/\psi$  production spectrum in proton-proton collisions measured by the LHCb Collaboration revealed a nontrivial energy dependence of the line shape in the energy range from 6.2 GeV (the double- $J/\psi$  threshold) to approximately 7.2 GeV [1]. In particular, a narrow resonance-like structure at around 6.9 GeV and a broad structure just above double- $J/\psi$  threshold were reported, with the statistically significant (above  $5\sigma$ ) deviation from nonresonant double- $J/\psi$  production. The data provided by the CMS [2] and ATLAS [3] Collaborations demonstrate a similar pattern. These results attracted a lot of attention of the community and, in particular, many attempts were made to theoretically understand and describe the above narrow structure that received the name  $X(6900)$ .

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## 2. – Coupled-channel data analysis

The data on the double- $J/\psi$  production provided by LHCb calls for a coupled-channel interpretation. The key idea of the approach employed in [4] is to resort to a minimal model consistent with the data and able to extract the poles responsible for the visible structures. To this end many channels with the thresholds lying in the considered energy range were disregarded as i) not  $S$ -wave ones ( $J/\psi h_c$ ,  $\psi(2S)h_c$ ,  $\chi_{c0}\chi_{c1}$ ), ii) inconsistent with light-hadron exchanges ( $\chi_{c0}\chi_{c0}$ ,  $\chi_{c1}\chi_{c1}$ ), iii) forbidden by heavy-quark spin symmetry (HQSS) for the transition from the initial double- $J/\psi$  production channel ( $h_c h_c$ ). Thus two versions of the model were considered: a two-channel model defined by the set  $\{J/\psi J/\psi, J/\psi\psi(2S)\}$  and a three-channel model including the channels  $\{J/\psi J/\psi, J/\psi\psi(2S), J/\psi\psi(3770)\}$ . It was found that indeed the rises and dips in the measured line shape can be understood as a result of an interplay of these channels. After the parameters of the amplitude (7 parameters for the two-channel model and 8 parameters for the three-channel model) were fixed from a fit to the data, poles of the amplitude on the complex energy plane were found. It was concluded that the LHCb data could not unambiguously fix the position of the pole near 6.9 GeV (a combined fit to the LHCb [1], CMS [2], and ATLAS [3] data might potentially improve on this pole extraction). In the meantime, a robust prediction of both versions of the coupled-channel model employed in [4] is the existence of a pole in the close vicinity of the double- $J/\psi$  threshold. It was named  $X(6200)$  and its possible nature was the subject of further theoretical investigations. Notice that the existence of a near-threshold pole in the amplitude reveals itself in a steep rise of the line shape right above the threshold. It is, therefore, instructive to notice that both CMS and ATLAS Collaborations reported an improvement in the description of their data if an additional Breit-Wigner resonance was artificially positioned very near the double- $J/\psi$  threshold. Then it is legitimate to conjecture that this additional resonance mimics the effect of the  $X(6200)$  pole.

## 3. – $X(6200)$ as a compact fully charmed tetraquark

In [5] the QCD string model was employed to evaluate the mass of the lowest fully charmed tetraquark state to be 6.2 GeV, that is, it was found to be consistent with the position of the near-threshold pole in the double- $J/\psi$  production amplitude extracted in [4]. The compact fully charmed tetraquarks with a similar mass were also predicted in [6-9]. Therefore, the existence of a compact state pole near the double- $J/\psi$  threshold is supported by various model calculations. However, a relevant worry is related with the results reported in [10], where it is found that, for a purely Coulombic tetraquark formed by two heavy quarks and two light antiquarks, stability of the system can only be reached if the ratio of the light-quark mass to the heavy-quark mass does not exceed some critical value estimated to be around 0.15 for the number of colours  $N_c = 3$ . This observation could preclude the existence of stable fully heavy compact tetraquarks, however it remains to be seen if this result applies to fully charmed systems which are known to be sensitive to the confining potential and as such cannot qualify as purely Coulombic.

## 4. – $X(6200)$ as a di- $J/\psi$ molecule

In [11] the conjecture of a possible di- $J/\psi$  molecule assignment for the  $X(6200)$  was put forward and investigated in detail. The electromagnetic interaction between neutral

composite particles is mediated by two photons and described by the van der Waals force. If the interval between the emission of the two photons is much larger than their travel time between the neutral particles, the corresponding potential is known as the London one. On the contrary, if the two photons are emitted almost simultaneously compared with their travel time, the interaction is described by the Casimir-Polder potential. A similar treatment applies to the interaction between two colourless hadrons mediated by the exchange of soft gluons. It can be described employing the QCD multipole expansion [12, 13]. The theoretical foundations of such soft-gluon-driven interactions in heavy-quark systems were discussed in [14]. The emitted soft gluons are nonperturbative and hadronise to light-meson pairs, so that the long-distance part of this van der Waals potential of the Casimir-Polder type is dominated by pairs of pions [15, 16]. The key parameter defining the strength of this interaction is the chromopolarisability given by the matrix element (see, for example, the derivation provided in [17, 18])

$$(1) \quad \beta'_{\psi\psi} = \frac{1}{9} \left\langle \psi' \left| \mathbf{r} \frac{1}{\hat{H}_O - M} \mathbf{r} \right| \psi \right\rangle$$

between the initial and final heavy-hadron states. Here  $\hat{H}_O$  is the Hamiltonian of the hadron in the colour octet state excited by a single gluon emission. For a heavy quarkonium treated as a purely Coulombic system the chromopolarisability is evaluated to be  $0.93/(\alpha_s^4 m_Q^3)$ , with  $\alpha_s$  and  $m_Q$  for the strong coupling constant and the heavy-quark mass, respectively [17, 18]. Because of large uncertainties in the determination of the latter parameters, the numerical result for the chromopolarisability of  $J/\psi$  is also very uncertain,

$$(2) \quad \beta_{J/\psi J/\psi} = \frac{0.93}{\alpha_s^4 m_c^3} = 19_{-14}^{+15} \text{ GeV}^{-3}.$$

It should also be taken into account that treating  $J/\psi$  as a purely Coulombic system allows only for order-of-magnitude estimates since the corrections due to the confining interaction appear to be of the same order of magnitude as the central value. Nevertheless, the obtained estimate for  $\beta_{J/\psi J/\psi}$  can be compared with the numerical value of the transition chromopolarisability  $|\beta_{\psi(2S)J/\psi}| \approx 1.81 \text{ GeV}^{-3}$  extracted directly from the data on the dipion transition  $\psi(2S) \rightarrow \pi\pi J/\psi$  [11]. Then one arrives at the ratio

$$(3) \quad \xi = \left| \frac{\beta_{J/\psi J/\psi}}{\beta_{\psi(2S)J/\psi}} \right| \simeq 3 \div 19.$$

Alternative estimates of this ratio made in [11] also provide the values ranging from several units to about a dozen, with  $\xi = 3$  being the most conservative one. The interaction potential in the di- $J/\psi$  system is proportional to  $\xi^2$  and can be constructed using a dispersive technique —see [11] for further details of the formalism. The resulting potential is shown in fig. 1. It includes the contributions from the two-pion and two-kaon exchanges between the two  $J/\psi$ 's. Calculations using this potential performed in [11] demonstrate that the existence of a molecular pole near the double- $J/\psi$  threshold is indeed consistent with our knowledge on hadron-hadron interactions.

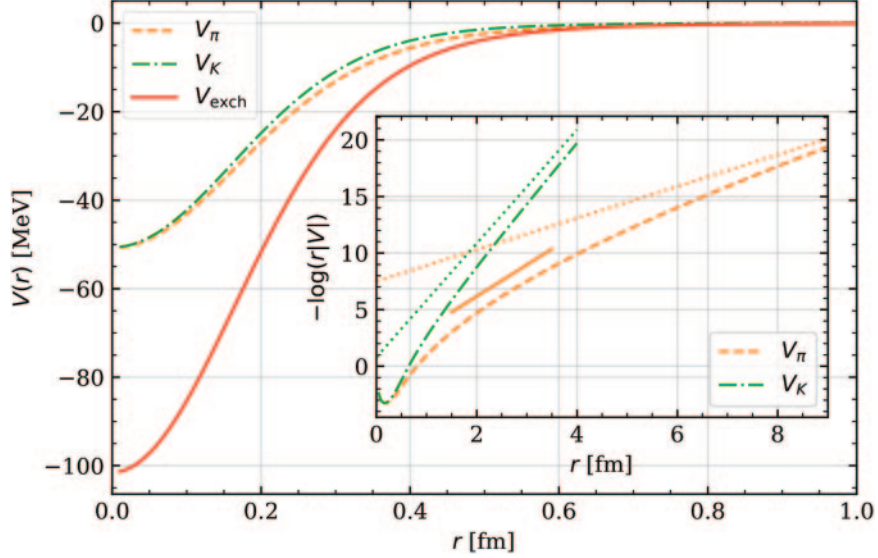


Fig. 1. – The form of the interaction potential in the di- $J/\psi$  system stipulated by the soft-gluon exchanges hadronised at large distances in the form of two-pion and two-kaon exchanges. The inlay zooms the large-separation region and demonstrates the asymptotic behaviour of the different contributions. Adapted from [18].

## 5. – Di-heavy-baryon molecules

In [18] chromopolarisability of a fully heavy baryon  $\Omega = QQQ$ , treated as a purely Coulombic system, was evaluated as

$$(4) \quad \beta_\Omega = \frac{C_\Omega}{\alpha_s^4 m_Q^3},$$

with  $C_\Omega \approx 2.4 \approx 2.6C_\psi$ , where  $C_\psi = 0.93$  is quoted in (2). Corrections to this result due to the confining interaction can be assessed as

$$(5) \quad \frac{\delta\beta}{\beta} \sim \frac{\Lambda_{\text{QCD}}^2}{\alpha_s^3 m_Q^2},$$

with  $\Lambda_{\text{QCD}}$  for the typical low-energy QCD scale, that leads to the corrections of the order 10% in case of  $Q = b$  ( $\Omega_{bbb}$  baryon) and of the order 100% for  $Q = c$  ( $\Omega_{ccc}$  baryon). In other words, similarly to the case of a heavy quarkonium, formula (4) gives a rather accurate estimate for a fully bottomed baryon and provides an order-of-magnitude estimate for a fully charmed baryon. Nevertheless, the results of [18] allow one to conclude that the interaction between two heavy baryons several times exceeds that between two heavy quarkonia. Therefore, if the  $X(6200)$  is indeed a di- $J/\psi$  molecule, then di- $\Omega_{ccc}$  and di- $\Omega_{bbb}$  molecules are also very likely to exist. This conclusion complies well with the results of the recent lattice calculations that indeed indicate that both double- $\Omega_{ccc}$  and double- $\Omega_{bbb}$  systems may be bound. In particular, the binding energy of the double- $\Omega_{ccc}$  system in the  $^1S_0$  channel is computed by the HAL QCD Collaboration

to be  $-5.68_{-0.90}^{+1.28}$  MeV (with the electric Coulomb interaction ignored) [19] while the double- $\Omega_{bbb}$  system was found to be deeply bound in the  $^1S_0$  channel, with a binding energy of  $-89_{-12}^{+16}$  MeV [20].

## 6. – Conclusions

Discovery of the  $X(3872)$  by Belle in 2003 started a new era in hadronic physics of heavy quarks. The recent studies of double- $J/\psi$  production in proton-proton collisions at LHC opened a new chapter in this book. The data collected by the LHCb Collaboration were analysed in a minimal but realistic coupled-channel scheme. The resulting amplitude is unitary and respects an approximate but accurate heavy-quark spin symmetry. The performed theoretical analysis is consistent with the existence of the fully charmed tetraquark  $X(6900)$  and, in addition, predicts the existence of a new state named  $X(6200)$  at the  $J/\psi J/\psi$  threshold. A compact tetraquark assignment for this state is not excluded but calls for additional studies. The conjecture of the molecular nature of the  $X(6200)$  is found to be plausible. Furthermore, the existence of this near-threshold state in the double- $J/\psi$  channel may also imply the existence of doubly heavy-baryon molecules.

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