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Renormalization in various schemes of nucleon-nucleon chiral EFT

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Summary. — Renormalizability of an effective field theory allows one to perform a systematic expansion of the calculated observable quantities in terms of some small parameter in accordance with a certain power counting. We consider chiral effective field theory in application to the nucleon-nucleon interaction at next-to-leading order in the chiral expansion as an example of the renormalizability of a theory with a nonperturbative leading-order interaction. The requirement of the renormalizability imposes nontrivial constraints on a choice of such interaction. Two different approaches are discussed: the finite- and the infinite-cutoff schemes. Another considered example is the quantum mechanical problem of the inverse-square potential, which is often regarded as a toy model for various physical systems.

1. – Introduction

The effective field theory (EFT) methods in the presence of a nonperturbative leading-order (LO) interaction were first applied to the nucleon-nucleon (NN) and few-nucleon systems by Weinberg [1, 2] in the framework of chiral EFT. Similar methods have turned out to be relevant in studies of molecular states in the heavy-quarkonia sector as well as in the few-nucleon calculations within pionless EFT, see refs. [3-8] for reviews.

In such schemes, the next-to-leading-order (NLO) and higher-order interactions (potentials) can still be treated perturbatively. This is not necessary, but has an advantage of having the theoretical errors for observables coming from a truncation of an EFT expansion under control. Moreover, in some cases including the NN chiral EFT or the 3-nucleon pionless EFT with an infinite (much larger than the EFT breakdown scale Λ_b) cutoff, the nonperturbative treatment of subleading corrections leads to physically unacceptable results [9-13].

In the discussed approaches, the series for the LO amplitude T_0 and for the unrenormalized NLO amplitude T_2 can be schematically represented by

$$T_0 = \sum_{n=0}^{\infty} T_0^{[n]}, \qquad T_0^{[n]} = V_0 (G V_0)^n, \tag{1}$$

$$T_2 = \sum_{m,n=0}^{\infty} T_2^{[m,n]}, \qquad T_2^{[m,n]} = (V_0 G)^m V_2 (G V_0)^n, \tag{2}$$

where G is the free Green's function and $V_0\,(V_2)$ is the LO (NLO) potential.

In the nonperturbative regime for the LO potential, these equations generalize to

$$T_0 = V_0\,(\mathbb{1} - G V_0)^{-1}, \tag{3}$$

$$T_2 = (\mathbb{1} - V_0 G)^{-1}\,V_2\,(\mathbb{1} - G V_0)^{-1}. \tag{4}$$

Here for definiteness, we assign the subscripts 0 and 2 analogously to the LO and NLO interactions in NN chiral EFT, which is a demonstrative example we use in this study. For simplicity, we assume the uncoupled S-wave scattering, so that the Lippmann-Schwinger equation for the LO off-shell amplitude, $T_0 = V_0 + V_0 G T_0$, reads

$$\begin{aligned} T_0(p', p; p_{\rm on}) &= \int \frac{p''^2 \mathrm{d}p''}{(2\pi)^3} V_0(p', p'') G(p''; p_{\rm on}) T_0(p'', p; p_{\rm on}), \\ G(p''; p_{\rm on}) &= \frac{m_N}{p_{\rm on}^2 - p''^2 + i\epsilon}. \end{aligned} \tag{5}$$

The naive dimensional power counting formulated for small (compared to the hard scale Λ_b) particle momenta is often violated by contributions to the amplitude coming from large loop momenta of the order of the cutoff Λ. To restore the power counting, a renormalization procedure is necessary: one absorbs the power counting breaking terms by redefining contact interactions splitting the unrenormalized low-energy constants (LECs) into the renormalized ones and the counter terms. This method works for a large class of interactions provided the cutoff is of the order of the hard scale $\Lambda \sim \Lambda_b$ and the LO interaction is in some sense perturbative [14]. In the case of a nonperturbative LO interaction, to maintain renormalizability of a theory, one has to impose additional constraints on the LO potential [15, 16] when considering calculations beyond leading order. In what follows we consider several applications of those constraints.

2. – Renormalization in the nucleon-nucleon sector

The renormalized expression for the NLO NN amplitude, $\mathbb{R}(T_2)$, is obtained by adding the relevant counter term δV to V_2,

$$\mathbb{R}(T_2) = (\mathbb{1} - V_0 G)^{-1}\,(V_2 + \delta V)\,(\mathbb{1} - G V_0)^{-1}. \tag{6}$$

Below, we consider the typical case of two NLO contact terms, momentum independent and quadratic in momentum, determined by the LECs $C_0^{\rm NLO}$ and $C_2^{\rm NLO}$, so that the contact part of the NLO amplitude is given by [16]

$$T_{\rm ct}(p_{\rm on}) = C_0^{\rm NLO}\psi_\Lambda(p_{\rm on})^2 + C_2^{\rm NLO} 2\psi_\Lambda(p_{\rm on})\psi'_\Lambda(p_{\rm on}), \tag{7}$$

with

$$\begin{aligned} \psi_\Lambda(p_{\rm on}) &= F_\Lambda(p_{\rm on}^2) + \int \frac{p^2{\rm d}p}{(2\pi)^3} G(p;p_{\rm on}) F_\Lambda(p^2) T_0(p,p_{\rm on};p_{\rm on}), \\ \psi'_\Lambda(p_{\rm on}) &= p_{\rm on}^2 F_\Lambda(p_{\rm on}^2) + \int \frac{p^2{\rm d}p}{(2\pi)^3} p^2 G(p;p_{\rm on}) F_\Lambda(p^2) T_0(p,p_{\rm on};p_{\rm on}), \end{aligned} \tag{8}$$

where $F_\Lambda(p^2)$ is a regulator with the cutoff Λ assumed here to be the same as in the LO potential (in general, this is not necessary).

One can choose the renormalization conditions to be determined by the amplitude at two on-shell momenta p_0 and p_1. These conditions become inconsistent when the quantity $\zeta_\Lambda(p_0,p_1)$ vanishes,

$$\zeta_\Lambda(p_0,p_1) = 0, \tag{9}$$

where

$$\zeta_\Lambda(p_0,p_1) = \frac{e^{-i\delta^{(0)}(p_1)}}{\psi_\Lambda(p_0)} \begin{vmatrix} \psi_\Lambda(p_0) & \psi'_\Lambda(p_0) \\ \psi_\Lambda(p_1) & \psi'_\Lambda(p_1) \end{vmatrix}, \tag{10}$$

with the phase determined by the LO phase shift $\delta^{(0)}$.

Analyzing various realistic NN EFT interactions, it was found that for the cutoff values $\Lambda \lesssim \Lambda_b$, the condition in eq. (9) is never fulfilled, which guarantees renormalizability of such theories.

On the other hand, if one sends the cutoff to infinity, as is done, *e.g.*, in ref. [17], an infinite number of "exceptional" cutoffs appear. For such cutoffs, eq. (9) holds, and in their neighborhoods, the theory becomes nonrenormalizable, which imposes limitations on the whole scheme [16].

3. – Example of the inverse-square potential

To demonstrate common features of renormalization in the nonperturbative regime, one often utilizes the inverse-square–potential example. The ultraviolet behavior of this interaction is similar to the one that can appear, *e.g.*, in the three-body systems with short-range forces, see refs. [18-21] for calculations beyond leading order.

The most interesting is the case of a singular attractive LO potential. In ref. [22], the model with the long-range LO potential proportional to $1/r^2$ and the long-range NLO potential proportional to $1/r^4$ was introduced. For the regulator, a sharp momentum cutoff was implemented. In ref. [16], it was shown that the "exceptional" cutoffs in such a model do not destroy renormalizability because the zeros of $\zeta_\Lambda(p_0,p_1)$ factorize and coincide with the zeros of the vertex function,

$$\tilde{\psi}_\Lambda(p_0) = e^{-i\delta^{\rm LO}(p_{\rm on})}\psi_\Lambda(p_0). \tag{11}$$

However, such factorization is a unique feature of this model. To demonstrate this, we performed various minor modifications of the scheme. In particular, we have modified the regulator introducing a smooth cutoff or a combination of the smooth and sharp cutoffs. We have also tried to slightly change the potential without modifying its short-range behavior. In all such cases, the above factorization is absent and the "exceptional" cutoffs again destroy renormalizability. This fact should be kept in mind when using the inverse-square potential as a toy model for realistic interactions.

To summarize, we have discussed the renormalization of various effective field theories with nonperturbative LO interactions. We have emphasized that it is important to verify the fulfillment of the renormalizability constraints when going beyond leading-order calculations.

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