IL NUOVO CIMENTO **47 C** (2024) 246
DOI 10.1393/ncc/i2024-24246-1

Colloquia: HADRON2023

Data-driven approximations to the Hadronic Light-by-Light scattering contribution to the muon $(g-2)$

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received 21 December 2023

Summary. — We review recent progress on the numerical determination of the Hadronic Light-by-Light contribution to the anomalous magnetic moment of the muon. We advocate for a slight increase of the White Paper number for its Standard Model prediction, to $(102 \pm 17) \times 10^{-11}$, accounting for a revised contribution from axial-vector mesons and short-distance constraints. This $\sim 10\%$ larger result seems to be supported by the most recent lattice QCD evaluations.

1. – Why does it matter?

The Standard Model (SM) uncertainty on the muon $g-2$ $(2a_\mu = g_\mu - 2)$ is dominated by the hadronic vacuum polarization (HVP) piece, amounting to 4.0×10^{-10} (for an overall error of 4.3×10^{-10}) [1](1). This is contributed very mildly by the error of the Hadronic Light-by-Light (HLbL) scattering part, 1.9×10^{-10}, that we will discuss here(2). Clearly, the most urgent thing is to clarify the discrepancy between the data-driven results [1-5] and the competitive lattice QCD evaluation, by the BMW Collaboration [6], of a_μ^{HVP}. To this end, several approaches have been developed, exploiting the so-called windows in Euclidean time [7-17]. Reference [16] (based on the isospin-breaking corrections

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(1) These and the following numbers are quoted —unless otherwise stated— from the White Paper of the Muon $g-2$ Theory Initiative [1] (WP), a collaboration which has been aiming for a community consensus value of the Standard Model prediction of the muon $g-2$, see `https://muon-gm2-theory.illinois.edu/`.
(2) See talks focusing on diverse aspects of the HVP contribution by Matthia Bruno, Christoph Redmer, Francesca de Mori, Álex Miranda, Camilo Rojas and David Díaz Calderón.

computed in ref. [18]), points to nice agreement between data-driven predictions using $\tau^- \to \pi^-\pi^0\nu_\tau$ data [19-22] (instead of $e^+e^- \to \pi^+\pi^-$ measurements) with lattice QCD evaluations. The barely acceptable discrepancy between KLOE [23-27] and BaBar [28,29] $e^+e^- \to \pi^+\pi^-$ data has been aggravated by the new CMD-3 measurement [30], being this puzzle still not understood (see also, *e.g.*, the measurements [31-33]). Amid this conundrum, halving the error of the SM prediction for a_μ^{HLbL} [1] is still necessary, according to the final precision that the Fermilab experiment will achieve measuring a_μ, but not the top priority.

On the contrary, the experimental situation seems crystal-clear: FNAL measurements [34,35] are extremely consistent with the BNL outcome [36] and their joint picture is fully convincing, yielding

$$a_\mu^{\rm Exp} = 116592059(22) \times 10^{-11}. \tag{1}$$

This situation further enhances the pressing need for theoretical progress.

2. – Why such a large error for $a_\mu^{\rm HLbL}$?

The outsider may wonder why the uncertainty of the $a_\mu^{\rm HLbL}$ is $\mathcal{O}(20\%)$, while that of the $a_\mu^{\rm HVP}$ is only $\mathcal{O}(0.6)\%$. This much better precision stems from its calculation via a single dispersive integral that is related to the accurately measured $\sigma(e^+e^- \to \text{hadrons})$ [37] plus a mild contribution from perturbative QCD. On the contrary, a data-driven approach to $a_\mu^{\rm HLbL}$ is very much complicated by the additional loop and multi-scale nature of the problem. Despite enormous advances towards a fully dispersive computation of $a_\mu^{\rm HLbL}$ [38-43], a completely dispersive evaluation is not feasible yet. This framework provided a rationale for the historical arrangement of the main contributions (starting from the dominance of the pseudoscalar-pole cuts [44]) and could in principle be used up to arbitrary complex multiparticle ones.

3. – Contributions

Amazingly, the whole $a_\mu^{\rm HLbL}$ is basically saturated by the contribution from the lowest-multiplicity cut (even more so because of the approximate cancellations among the other contributions), corresponding to the lightest pseudoscalar (π^0, η, η') poles, yet it could be related to a combined chiral and large-N_C expansion [45]. This can be computed straightforwardly [44] knowing the corresponding pseudoscalar transition form factors (TFFs) as functions of both photons virtuality. See Redmer's talk on the precious experimental input to these (and others required for $a_\mu^{\rm HLbL}$) TFFs. In addition, there are some theoretical properties constraining these TFFs, like the chiral limit, the singly and doubly virtual asymptotic limits predicted by QCD, analyticity and unitarity, etc. The dispersive evaluation [46,47] yields a very precise result for the π^0 contribution,

$$a_\mu^{\pi^0,{\rm HLbL}} = \left(63.0^{+2.7}_{-2.1}\right) \times 10^{-11}, \tag{2}$$

confirming the rational approximants' determination [48],

$$a_\mu^{\pi^0,{\rm HLbL}} = (63.6 \pm 2.7) \times 10^{-11}. \tag{3}$$

These results are also supported by, *e.g.*, Dyson-Schwinger equations evaluations, yielding $a_\mu^{\pi^0,\mathrm{HLbL}} = (62.6 \pm 1.3) \times 10^{-11}$ [49], and $a_\mu^{\pi^0,\mathrm{HLbL}} = (61.4 \pm 2.1) \times 10^{-11}$ [50] and by holographic QCD results [51-53] (see, however, [54]) and chiral Lagrangians including resonances [55, 56]. For the $\eta^{(\prime)}$ contributions there is no dispersive computation yet. The rational approximants' calculation [48, 57-59] yields

$$a_\mu^{\eta,\mathrm{HLbL}} = (16.3 \pm 1.4) \times 10^{-11}, \quad a_\mu^{\eta,\mathrm{HLbL}} = (14.5 \pm 1.9) \times 10^{-11}, \tag{4}$$

which are the reference values for this contribution. Again, they are supported by the different approaches mentioned before where, in particular, Dyson-Schwinger equations results in $a_\mu^{\eta,\mathrm{HLbL}} = (15.8 \pm 1.2) \times 10^{-11}$, $a_\mu^{\eta',\mathrm{HLbL}} = (14.7 \pm 1.9) \times 10^{-11}$ [49] and $a_\mu^{\eta,\mathrm{HLbL}} = (13.3 \pm 0.9) \times 10^{-11}$, $a_\mu^{\eta',\mathrm{HLbL}} = (13.6 \pm 0.8) \times 10^{-11}$ [50], respectively. From the dispersive and rational approximants calculations, the WP quotes

$$a_\mu^{\pi^0+\eta+\eta',\mathrm{HLbL}} = \left(93.8^{+4.0}_{-3.6}\right) \cdot 10^{-11}, \tag{5}$$

still to be considered the data-driven SM prediction for this leading contribution to HLbL, coming from the lightest pseudoscalar poles.

The very well-known pseudoscalar electromagnetic form factors are the key objects to determine their box contributions to a_μ^{HLbL}. The dispersive result for the π case

$$a_\mu^{\pi-box,\mathrm{HLbL}} = -(15.9 \pm 0.2) \times 10^{-11}, \tag{6}$$

was later on confirmed by Dyson-Schwinger evaluations $a_\mu^{\pi-box,\mathrm{HLbL}} = -(15.7 \pm 0.4) \times 10^{-11}$ [49], and $a_\mu^{\pi-box,\mathrm{HLbL}} = -(15.6 \pm 0.2) \times 10^{-11}$ [60]. For the kaon case, the early evaluation of ref. [61], $a_\mu^{K-box,\mathrm{HLbL}} = -(0.46 \pm 0.02) \times 10^{-11}$ was slightly revised within Dyson-Schwinger and then also using a dispersive framework [62], both agreeing on

$$a_\mu^{K-box,\mathrm{HLbL}} = -(0.48 \pm 0.02) \times 10^{-11}. \tag{7}$$

The SM prediction comes from eqs. (6) and (7), still coinciding with the WP number [1]

$$a_\mu^{(\pi/K)-box,\mathrm{HLbL}} = -(16.4 \pm 0.2) \times 10^{-11}. \tag{8}$$

Now we turn to another contribution coming from two-particle cuts, that associated to pseudoscalars rescattering. For the pions case, the dispersive evaluation [42, 43] is quite precise for the contribution associated to the π-pole left-hand cut (LHC),

$$a_{\mu,J=0}^{\pi\pi,\pi-poleLHC} = -(8 \pm 1) \times 10^{-11}, \tag{9}$$

where contributions from D- and higher-orders partial waves were covered by the uncertainty. This agrees with other evaluations [63-66] that include additional scalar contributions, converging to [66]

$$a_\mu^{Scalars} = -(9 \pm 1) \times 10^{-11}, \tag{10}$$

again in accord with the WP [1]. Similarly, the tensors contribution [67]

$$a_\mu^{Tensors} = -(0.9 \pm 0.1) \times 10^{-11} \tag{11}$$

is unchanged with respect to ref. [1].

The part which has been evolving less trivially since 2020 corresponds to the axial-vector contributions, which should be regarded together with the remaining perturbative QCD constraints.

Melnikov and Vainshtein [68] put forward that pseudoscalar poles alone cannot satisfy short-distance QCD restrictions and emphasized the importance of axial vectors to fulfil this requirement. Modern studies coincide in smaller values for these contributions than initially advocated.

Reference [69] clarified ambiguities about bases arising because of axials off-shellness and, together with ref. [70], emphasized the relationship between short-distance, axial anomaly constraints, and the axial contributions (with possible relevant role of pseudoscalar resonances, see also [50]), a hot topic since then. References [40,69,71] gave rise to the WP number [1]

$$a_\mu^{Axials} = (6 \pm 6) \times 10^{-11}. \tag{12}$$

This was accompanied by the estimation of the contribution from light-quark loops and remaining QCD short-distance constraints (SDCs) [72-74]

$$a_\mu^{u/d/s-loops+SDCs} = (15 \pm 10) \times 10^{-11}. \tag{13}$$

Given their correlation, these two contributions were combined with errors added linearly (uncertainties are combined quadratically, unless otherwise stated) to [1]

$$a_\mu^{axials+SDCs} = (21 \pm 16) \times 10^{-11} . \tag{14}$$

Finally, the c-quark contribution (with uncertainty to be added linearly to eq. (14)) is [50,72-75]

$$a_\mu^{c-loop} = (3 \pm 1) \times 10^{-11}. \tag{15}$$

The leading-order a_μ^{HLbL} contributions is obtained from eqs. (5), (8), (10), (11), eqs. (14), and (15), yielding

$$a_\mu^{\mathrm{HLbL},LO} = (92 \pm 19) \times 10^{-11}. \tag{16}$$

Progress since the WP on axials and/or SDCs has improved the understanding of the regime where all photon virtualities are large, and when one of them is much smaller than the other two [50, 70, 76-88]. However, different model calculations considering axial-vector mesons and SDCs [51-53,70,83,89] suggest a shift in the central value around

$$a_\mu^{axials+SDCs} = (31 \pm 10) \times 10^{-11}, \tag{17}$$

larger than previously estimated, (14), but compatible within errors. Using eq. (17), the overall contribution would then be

$$a_\mu^{\mathrm{HLbL},LO} = (102 \pm 17) \times 10^{-11}, \tag{18}$$

which is closer to the latest lattice QCD evaluations by the Mainz [90] ($(109.6 \pm 15.9) \times 10^{-11}$) and RBC/UKQCD [91] ($(124.7 \pm 14.9) \times 10^{-11}$) Collaborations (to be compared to $(78.7 \pm 35.4) \times 10^{-11}$ [92] by RBC, used in the WP). At NLO [93] the central value and its uncertainty are increased by only $(2 \pm 1) \times 10^{-11}$.

These observations evince that a better understanding of the role of axial-vector mesons and the intermediate energy region is an important step towards a more precise and reliable estimate for the HLbL contribution. Progress in this direction continues [70, 76-88].

4. – Conclusions

- The WP number, $a_\mu^{\mathrm{HLbL},LO} = (92 \pm 19) \times 10^{-11}$ [1], still stands as the data-driven SM prediction for $a_\mu^{\mathrm{HLbL},LO}$.
- The dominant uncertainty comes from short-distance + axial contributions (correlated uncertainties), with improved understanding since the WP, where work still needs to be done. This may shift the SM prediction slightly, to $a_\mu^{\mathrm{HLbL},LO} = (102 \pm 17) \times 10^{-11}$.
- Measurement of di-photon resonance couplings (particularly for axials) would be very helpful.
- Lattice QCD has just reached a comparable uncertainty to the data-driven determinations of this piece, thereby reducing the uncertainty through their combination to $\leq 10 \times 10^{-11}$, in agreement with the sought accuracy by the time of the final publication of the a_μ measurement by the FNAL experiment. So the ball is on HVP's court.

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The authors acknowledge the organizers of this excellent conference. PR was supported by Conacyt and Cinvestav. We acknowledge Pablo Sánchez-Puertas for nice collaborations and his insightful comments on this manuscript.

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