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Simulations of RF waves propagation in hot magnetized (^{2}H) plasma in DTT Scenario

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Summary. — An analytical model and numerical simulations of the propagation and absorption of radio-frequency waves in hot magnetized (^{2}H) plasma are presented. First, an investigation of the impact of thermal effects on the dispersion relation of the waves has been conducted by expanding the hot plasma dielectric tensor to the first order in temperature. Second, starting with the realistic Ion Cyclotron Heating (ICH) antenna model designed for the Divertor Tokamak Test project, a simple 1D (1-dimensional) model of the wave propagation has been developed. The antenna's spectra have been extracted and analyzed using CST®, and the 1D wave propagation has been performed through MATLAB®.

1. – Introduction

The ion cyclotron resonance heating is a powerful technique used to heat plasma in fusion devices. It works by leading to resonant absorbtion of high-frequency (range) electromagnetic waves, launched by radianting structure (antennas), in the plasma ions, which results in the transfer of energy to the ions. Within the context of thermonuclear fusion research, the issue of coupling electromagnetic power to a tokamak plasma via a radio frequency antenna has been studied in the recent past. By defining appropriate boundary conditions, the task is to simultaneously account for a modelling of the propagative mode inside the plasma and for a thorough geometrical description of the antenna. Brambilla has adopted a sophisticated and efficient method to address these issues in the numerical codes FELICE and TORIC [1, 2]. Nevertheless, the description

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of the antenna in both codes is not as precise as it is in TOPICA [3] or COMSOL Multiphysics®. In this study, our goal is to develop a simple description of the electromagnetic waves propagation in plasma for magnetically confined fusion . This approach intentionally ignores the slow wave effect and relies on the cold plasma approximation (it is assumed that the kinetic velocity of particles in plasma is much smaller than the phase velocity of waves), considering a hot description as a perturbation. The theory of magnetized cold plasma provides a good description of wave propagation and identifies regions of non-propagation and resonance. Through this approach, it is possible to explain a wide range of phenomena of interest. However, this approximation has limitations in accurately reproducing the damping of waves, a typically nonlinear process that manifests as a first-order correction to the cold plasma approximation. To account for collisionless damping of waves, it is necessary to consider the finite temperature of the plasma (hot plasma) and adopt a kinetic treatment of the waves. The electromagnetic (EM) model has been then coupled to an advanced model of the antenna in the Divertor Tokamak Test (DTT) scenario.

2. – 1D Wave propagation model

In the scenario of a cold plasma (where thermal effects are not considered), the stationary wave equation arising from the Maxwell-Vlasov model can be expressed as an exact differential equation

$$\nabla \times \nabla \times \vec{E}(\vec{r}) - \frac{\omega^2}{c^2}\overline{\overline{\epsilon}}^H(\vec{r}) \cdot \vec{E}(\vec{r}) = 0, \tag{1}$$

where ω is the antenna frequency, c is the light speed and $\overline{\overline{\epsilon}}^H(\vec{r}) = (\overline{\overline{I}} + \frac{4\pi i}{\omega}\overline{\overline{\sigma}}^H(\vec{r}))$ is the Hermitian dielectric tensor as function of the radial variable $\vec{r}$. Generally, for a homogeneous, stationary, and unbounded thermal plasma, the dielectric tensor can be derived from the Maxwell-Vlasov system, which, in this case, reduces to a system of algebraic equations (dispersion relation, refer to [4]). The dielectric tensor is a complex tensor and its Hermitian (H) component describes propagation characteristics in the complex domain of frequency and wave-vector, while the anti-Hermitian (A) part accounts for plasma absorption of power carried by the wave. We have considered a slab plasma in Cartesian geometry (see fig. 1(a)), where the kinetic profiles (ionic and electronic density, ionic and electronic temperature, magnetic field) depend solely on the slab coordinate (indicated as $\hat{x} = r/a$ along x, where "a" is the minor radius of the plasma). Meanwhile, in the y (vertical) and z (parallel to the external magnetic field) directions, the plasma is homogeneous and unbounded. To derive valuable insights from eq. (1) and demonstrate the solution in a plasma slab, aligning the field with specific boundary conditions set by the antenna, we focus on eq. (1) under the conditions of a cold plasma and a single propagative mode (specifically, the fast mode). Additionally, we treat the damping coefficient (anti-Hermitian part of the dielectric tensor) as a perturbation of the Hermitian part. This approach yields the following second-order differential equation for the $\tilde{E}_y$ component of the electric field:

$$-\frac{\partial^2 \tilde{E}_y}{\partial x^2} + \left[k_z^2 - \frac{\omega^2}{c^2}(\epsilon_{yy}^H + i\epsilon_{yy}^A) + \frac{\omega^4}{c^4}\frac{(\epsilon_{xy}^H + i\epsilon_{xy}^A)^2}{k_z^2 - \frac{\omega^2}{c^2}(\epsilon_{xx}^H + i\epsilon_{xx}^A)}\right]\tilde{E}_y = 0. \tag{2}$$

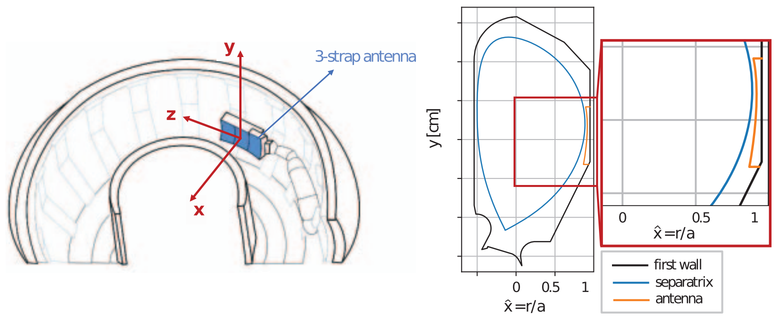

Fig. 1. – (a) Reference frame adopted for propagation model; (b) reference frame of the parametric study. Antenna placed at $\hat{x} = 1$ and plasma center at $\hat{x} = 0$.

2·1. *Parametric studies in Ideal Tokamak Example.* – The eq. (2) highlights which elements of the tensor matter for the 1D case of interest. Parametric studies have been conducted on ϵ_{xx}, ϵ_{xy} and ϵ_{yy} with respect to the spatial coordinate $\hat{x}$ and the parallel wavenumber $n_z = \frac{k_z c}{\omega}$ ($n_z = [1, 10]$; see [5]). The parametric studies have been carried out within a DTT scenario [6] (Major radius $R_0 = 219$ cm, minor radius $a = 75$ cm, aspect ratio $\epsilon = 2.92$). The main goal of the parametric studies has been to compare three dielectric tensor forms: COLD approximation (see [7], chapt. 5 par. 17), Finite Larmor Radius (FLR) approximation (see [7], chapt. 6 par. 26), and the complete tensor version, referred to as FULL (see [7], chapt. 4 par. 14). For this purpose, it has been analyzed a deuterium plasma (^2H) with the following density, temperature, and magnetic field profiles [8-10]:

- Magnetic field profile $B(x) = \frac{B_0 R_0}{R} = \frac{B_0}{(1+\epsilon\hat{x})}$ ($B_0 = 6 \times 10^4$ G);
- Electronic and ionic density profile $n_e(\hat{x}) = n_i(\hat{x}) = n_{e0}(1-(\hat{x})^2)^{0.5}$ ($n_{e0} = 2 \times 10^{14}$ cm^{-3});
- Electronic and ionic temperature profile $T_e(\hat{x}) = T_i(\hat{x}) = T_{e0}(1-(\hat{x})^2)^2$ ($T_{e0} = 10$ keV).

Figure 2 presents an example of a parametric study conducted on ϵ_{yy}. Specifically, fig. 2 illustrates the Hermitian and anti-Hermitian components for the FULL, FLR, and COLD approximations as a function of $\hat{x}$, with a fixed value of $n_z = 8$. As illustrated in fig. 2, the anti-Hermitian components of both the FULL and FLR trends coincide, differing from the COLD approximation in the region near the center of the plasma. In contrast, the region near the antenna ($\hat{x} = 1$) could be effectively described by the latter approximation. Regarding the behavior of the Hermitian components of FULL, FLR, and COLD, it is noticeable that near the antenna ($\hat{x} = 1$), they are almost identical, which can be attributed to the low temperatures, where corrections become negligible. In contrast, at a distance from the antenna ($\hat{x} = 0$), the trends no longer coincide. This parametric study aims to emphasize the effectiveness of the COLD approximation in the regions near the antenna ($\hat{x} = 1$). Nevertheless, when moving towards internal regions ($\hat{x} < 1$), it becomes necessary to shift to a FLR, or preferably, a FULL description of the electromagnetic tensor for a more precise representation of the propagation and absorption phenomena of waves in plasma.

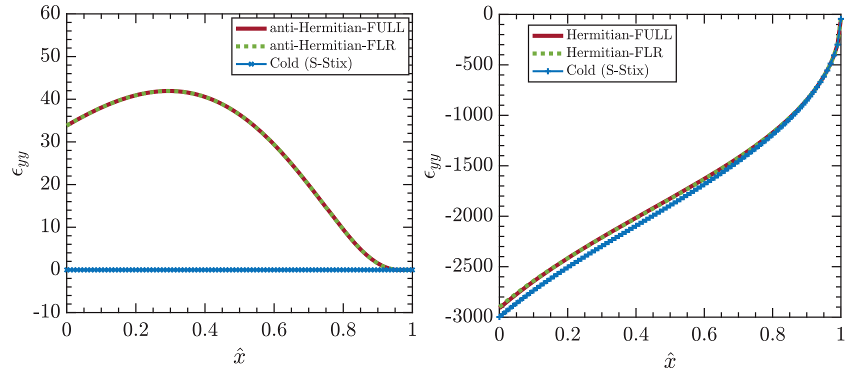

Fig. 2. – Anti-Hermitian (left) and Hermitian (right) parts of $\epsilon_{yy}(\hat{x})$ for $n_z = 8$. Several components of ϵ_{yy} are reported: FULL version, FLR approximation, COLD approximation (S-Stix notation [11]).

2·2. *The 3-strap antenna field extraction and k-spectra analysis*. – To solve eq. (2) and isolate the terms associated with $\tilde{E}_y$, the fields obtained from the antenna have been employed. Field extraction has been conducted through CST® by employing a flat geometry of the 3-strap antenna. The fields have been extracted at a frequency of 60 MHz along specific directions, which are useful for subsequent 1D analysis of wave propagation. In particular, the extracted field component is $E_y(z)$. Fourier transforms have been carried out using MATLAB's Fast Fourier Transform (FFT) algorithm. Figure 3 shows the real and the imaginary parts of E_y extracted along z-axis (2 cm far from the Faraday Screen along x-axis) and their Fourier transforms ($\tilde{E}_y$).

2·3. *Solution of the 1D waves's propagation model: the homogeneous case*. – Equation (2) can be rewritten in the form

$$\frac{\partial^2 \tilde{E}_y(n_z,\hat{x})}{\partial \hat{x}^2} + \delta_0^2[\hat{F}(n_z,\hat{x}) + i\hat{G}(n_z,\hat{x})]\tilde{E}_y(n_z,\hat{x}) = 0, \tag{3}$$

where $\delta_0^2 = (\frac{a\omega}{c})^2$. Equation (3) consists of a coefficient (inside square brackets) that is composed of a real part and an imaginary part. If we guess that $\hat{F}(n_z,\hat{x})$ and $\hat{G}(n_z,\hat{x})$ are constant in space (thus implying that density, temperature and magnetic field profiles are constant in the region closest to the antenna and depend only on n_z), the eq. (3) can be solved analytically. Operating the coordinate change $\xi = 1 - \hat{x}$, the solution is

$$\tilde{E}_y(\xi, n_z) = C_1 e^{i\sqrt{b^{complex}(n_z)}\xi} + C_2 e^{-i\sqrt{b^{complex}(n_z)}\xi}, \tag{4}$$

where $b^{complex}(n_z) = n_{x0}^2(n_z)[1 + i\pi^{1/2}\hat{\omega}_{ce}^2 \frac{\hat{v}_{the}}{n_z\hat{\Omega}_{ce}^2} e^{-(\frac{1}{n_z v_{the}})^2}] = n_{x0}^2(n_z)(\alpha + i\beta)$ and $n_{x0}^2(n_z) = -(n_z^2 - S) + \frac{D^2}{n_z^2 - S}$. S and D (see [7], chapt. 5 par. 17) and the thermal velocity (v_{the}) have been calculated for density, temperature and magnetic field at the plasma center. Considering $\beta \ll \alpha$ and by imposing the boundary condition at the antenna (vacuum)-plasma interface (see eq. (5)),

$$\tilde{E}_y(\xi = 0, n_z) = \tilde{E}_y^A(n_z), \tag{5}$$

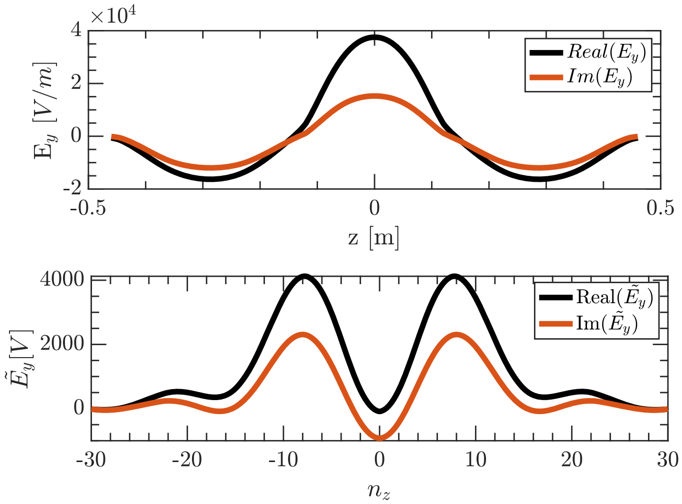

Fig. 3. – (Top) $E_y(z)$ field, (Bottom) Fourier Trasform of $E_y(z)$ as a function of n_z.

the solution of eq. (3) could by written in terms of progressive and regressive waves with damping terms (see eq. (6))

$$\tilde{E}_y(\xi, n_z) = C_1 e^{i\sqrt{\alpha}\xi} e^{-\frac{\beta}{2\sqrt{\alpha}}\xi} + C_2 e^{-i\sqrt{\alpha}\xi} e^{\frac{\beta}{2\sqrt{\alpha}}\xi}. \tag{6}$$

In fig. 4, a plot of the propagating field E_y in the $\hat{z}$-ξ plane(where $\hat{z} = z/a$) is shown. The electric field is plotted in the real space, as a function of the variable ξ. It is clearly visible the propagation of the field towards the center of the Tokamak, with maximum intensity starting from the antenna, and a progressive decrease of the electric field amplitude.

2·4. *Final consideration on 1D homogeneous case.* – The conducted study aimed to comprehend the fundamental characteristics of damping through a simplified analysis. The considered forms of the $\hat{F}$ and $\hat{G}$ functions are simplifications, as we expect the combinations of magnetic field, temperature, and density profiles to be neither linear nor constant. By assuming $\hat{F}$ and $\hat{G}$ to be constant in space, we obtained the simplest possible solution for the propagation case. However, by introducing spatial dependence and considering a linear trend, we obtained Airy-like solutions, the results of which will be presented in future publications.

3. – Conclusion

A one-dimensional model for the propagation and absorption of electromagnetic waves in plasma has enabled us to explore the interaction between the electromagnetic field emitted by the RF antenna and the plasma. The antenna's radiated field has been examined and computed using the commercial software CST®, which considers the antenna's detailed geometry. A basic absorption model (considering a hot description as a perturbation) has been incorporated into the formulation of the cold wave equation. This solution is based on a comparative evaluation of the dielectric tensor, carried out

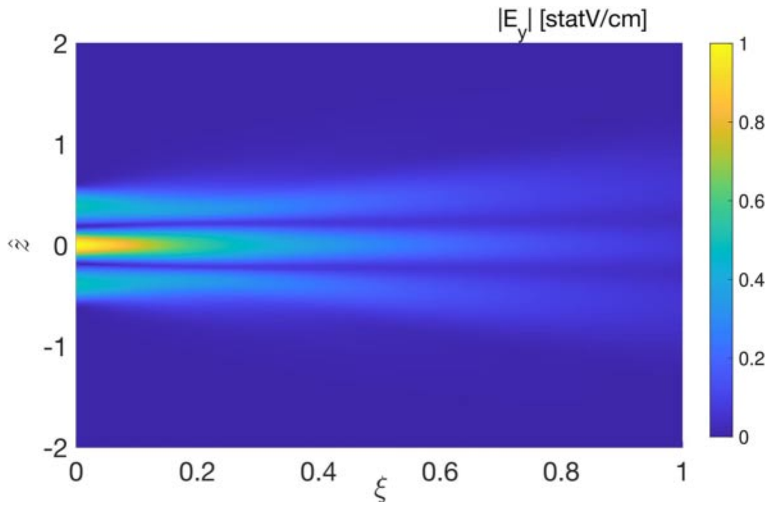

Fig. 4. – 2D plot of electric field E_y propagated by the antenna into the plasma, where $\hat{z}$ represents the spatial coordinate normalized with respect to the minor radius of the antenna a. Antenna placed at $\xi = 0$ and plasma center at $\xi = 1$.

using MATLAB®. The analysis has been conceived to explore antenna and plasma scenarios pertinent to DTT, beginning with an ideal case involving a pure Deuterium (^2H) plasma. This study has enabled us to obtain an example of wave propagation and absorption in plasma through a simplified approach in a "Homogeneous" case. From the perspective of wave propagation, this approach may not be as sophisticated as FELICE or TORIC. However, unlike the latter, it employs an advanced and realistic antenna version. This investigation yields valuable insights into the physics of coupling, propagation, and absorption through a simplified computational approach. It serves as an initial phase before undertaking a more in-depth investigation using advanced numerical tools (such as COMSOL Multiphysics®) in future research and in a more realistic plasma scenario (for example D-He or D-H).

REFERENCES

[1] Brambilla M. *et al.*, *Nucl. Fusion*, **28** (1988) 1813.
[2] Brambilla M., *Plasma Phys. Control. Fusion*, **41** (1999) 1.
[3] Lancellotti V. *et al.*, *Nucl. Fusion*, **46** (2006) S476.
[4] Brambilla M. *et al.*, *A model to evaluate coupling of ion Bernstein waves to tokamak plasmas*, presented at *Europhysics Confenence Abstracts*, Vol. **15C** (EPS) 1991.
[5] Ceccuzzi S. *et al.*, *AIP Conf. Proc.*, **2984** (2023) 030015.
[6] Ambrosino R. *et al.*, *Fusion Eng. Des.*, **167** (2021) 112330.
[7] Brambilla M., *Kinetic theory of plasma waves: homogeneous plasmas*, Vol. **96** (Oxford University Press) 1998.
[8] Casiraghi I. *et al.*, *Nucl. Fusion*, **61** (2021) 116068.
[9] Cardinali A. *et al.*, *J. Phys.: Conf. Ser.*, **2397** (2022) 012017.
[10] Cardinali A. *et al.*, *Plasma Phys. Control. Fusion*, **62** (2020) 044001.
[11] Stix T. H., *Waves in Plasmas* (Springer Science & Business Media) 1992.