

Measurement of the anomalous spin precession frequency ω_a in the Muon $g - 2$ experiment at Fermilab

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Summary. — The muon magnetic anomaly $a_\mu = (g_\mu - 2)/2$ is a low-energy observable, which can be both measured and computed to high precision, making it a sensitive test of the Standard Model (SM). In April 2021, the E989 Collaboration at Fermilab National Accelerator Laboratory (FNAL) published the first result based on the first year of data taking (Run-1), and in August 2023 a new result was published based on two more years of data taking (Run-2 and Run-3). The new result was in agreement with the first one and with the previous experiment at Brookhaven National Laboratory (BNL), and the combination of these results brought the uncertainty on the experimental measurement of a_μ to the unprecedented value of 0.19 parts per million (ppm). This paper will present details about the improvements and upgrades since the 2021 result, and it will describe the final statistical and systematic sources of uncertainty on ω_a in the 2023 result.

1. – The magnetic moment of the muon

The intrinsic magnetic moment of a charged particle with spin is defined by $\vec{\mu} = g(e/2m)\vec{S}$, where e is the particle charge, m its mass, \vec{S} its spin vector and g the so-called gyromagnetic ratio, a dimensionless parameter. The Dirac equation predicts the value $g = 2$ for charged particles with spin $\frac{1}{2}$, but deviations from 2 arise due to radiative corrections in the Standard Model (SM). We can define the magnetic anomaly of the muon as the fractional difference of g_μ from 2: $a_\mu = (g_\mu - 2)/2$. The contribution to a_μ from the quantum chromodynamics (QCD) sector amounts to ~ 60 parts per million (ppm) and carries the largest uncertainty. The major contribution comes from hadronic vacuum polarization (HVP), where the energy scale is of order of the muon mass, well below the region where QCD can be studied perturbatively: a dispersion relation approach can be used to evaluate the contribution, using the total experimental

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cross-section $\sigma_{tot}(e^+e^- \rightarrow \text{hadrons})$ as an input. Lattice QCD can also be used to determine the HVP contribution to a_μ using an ab-initio calculation. In 2020, the Theory Initiative recommended a value for the theoretical prediction of a_μ in the White Paper in ref. [1], based on the dispersive approach. Figure 1 presents the experimental values of a_μ as measured by BNL E821 [2] and FNAL E989 in 2021 [3] and 2023 [4]. The current discrepancy between the experimental value and the SM calculation from the Muon $g - 2$ Theory Initiative is $a_\mu^{exp} - a_\mu^{SM} = (249 \pm 48) \cdot 10^{-11}$, with a significance of 5.1σ . In recent years, puzzles in the theoretical prediction of a_μ have arisen, which prevent a solid comparison with the experimental value. In 2021, the BMW Collaboration presented a prediction of a_μ^{HVP} with lattice QCD with an uncertainty of 0.8% [5], which was in tension with the dispersion approach. Since then, other groups have been working to confirm or not the BMW result. In 2023, the measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross-section with the CMD-3 detector [6] evaluated a hadronic contribution to the muon anomalous magnetic moment that was significantly larger than the value obtained from previous measurements. The results from BMW and CMD-3 tend to shift the theoretical prediction closer to the experimental value, thus reducing the significance of the discrepancy.

2. – Measurement principle of the $g - 2$ experiment at Fermilab

When a particle with spin, charge e and mass m is placed in a uniform external magnetic field \vec{B} , it will follow a circular path because of the Lorentz force, with cyclotron frequency ω_C . Its spin will also precess around the direction of the magnetic field, with frequency ω_S . We define the anomalous precession frequency ω_a as the difference

$$(1) \quad \vec{\omega}_a \equiv \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right].$$

\vec{E} is the electric field, $\vec{\beta}$ the particle's speed and γ its Lorentz factor. In the Muon $g - 2$ experiment, a spin-polarized beam of positively charged muons is injected into a ~ 7 m radius superconducting storage ring that produces a 1.45 T magnetic field. Electrostatic quadrupole (ESQ) plates provide weak focusing for vertical confinement. The second term in eq. (1) vanishes for muons that travel orthogonally to the magnetic field, $\vec{\beta} \cdot \vec{B} = 0$, and the third term vanishes for muons with the ‘‘magic momentum’’ $p_\mu \simeq 3.1$ GeV/c, so that $\gamma = \sqrt{1 + 1/a_\mu} \simeq 29.3$. In this configuration, the expression for the anomalous precession frequency becomes $\omega_a = a_\mu(e/m)B \simeq 1.43$ rad/ μ s, so a_μ is

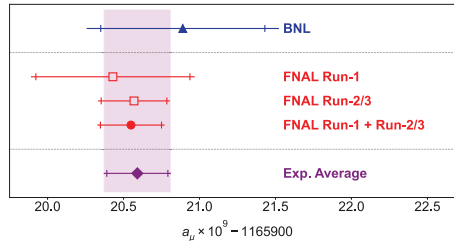


Fig. 1. – Experimental values of a_μ from BNL E821 and FNAL E989, and new experimental average. The inner tick marks indicate the statistical contribution to the total uncertainties [4].

proportional to ω_a/B . We account for deviations from the ideal case by applying beam dynamics corrections to our measurements. The magnetic field is expressed by means of the Larmor precession frequency of free protons ω_p , measured with Nuclear Magnetic Resonance techniques, via $\hbar\omega_p = 2\mu_p|\vec{B}|$, where μ_p is the proton magnetic moment [7]. In this paper we will focus on the measurement of ω_a , which is based on the arrival time distribution of decay positrons in the high-energy tail of the spectrum, detected by 24 electromagnetic calorimeters placed around the inner radius of the storage ring. Due to parity violation in the muon weak decay, high energy positrons are emitted preferentially in the muon’s spin direction in the center-of-mass frame. As a consequence, in the lab frame the energy spectrum of emitted positrons has a different shape depending on the angle between muon spin and muon momentum, *i.e.*, the anomalous precession phase. If we take the integral of the spectrum above a fixed threshold, which corresponds to counting all positrons above a certain energy, we will find a distribution that is modulated by the ω_a frequency, described—in the ideal case—by eq. (2):

$$(2) \quad N(t) = N_0 e^{-t/\gamma\tau} [1 + A_0 \cos(\omega_a t + \phi_0)],$$

where N_0 is a normalization parameter, A_0 the amplitude of the oscillation, ϕ_0 the initial phase, and $\gamma\tau$ is the muon lifetime in the lab frame. We choose a threshold of 1.7 GeV that minimizes the statistical uncertainty on ω_a with this method. In the so-called “Asymmetry-weighted” method, instead, we lower the threshold to 1 GeV and weight our data with the energy-dependent asymmetry, which is the A_0 parameter in eq. (2) obtained from fits to ~ 10 MeV energy slices, to reduce the statistical uncertainty [8].

3. – ω_a analysis and improvements in Run-2 and Run-3

The electromagnetic calorimeters measure the energies and arrival times of incident positrons. Each segmented calorimeter features a 6×9 array of lead fluoride crystals, in which incoming positrons produce Cherenkov photons. Each crystal is coupled to a silicon photomultiplier, which responds to Cherenkov photons with electrical current; this current is then converted into a voltage signal and recorded for offline analysis. To reconstruct the positron events, we perform a template fit on the waveforms of each crystal, and then apply a clustering algorithm. When two or more positrons hit the same calorimeter very close in time, the reconstruction is not always able to separate the

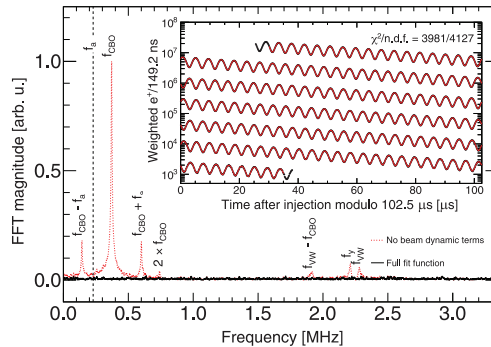


Fig. 2. – FFT of residuals from the ω_a fit (inset plot) in the case of 5-parameter function (dashed red) or complete fit function in Run-3 (solid black) [4].

events and the incident particles are reconstructed as a single hit. These pileup events can be identified, by studying the distribution of clusters, and subtracted. The time distribution of detected positrons above the 1.7 GeV threshold is shown in the inset plot of fig. 2. In principle, we can perform a very simple fit with the 5 parameters of eq. (2). A more complicated function takes into account many beam dynamics effects: for instance, the radial motion of the muons, and the aliased Coherent Betatron Oscillation (CBO) frequency; or the muons that scatter away over time. Figure 2 shows the fit to Run-3 data and the fast fourier transform (FFT) of the residuals: the red dashed curve shows peaks at all the frequencies that are not accounted for in the 5-parameter fit; the solid black curve is the FFT when fitting with the full function, which removes all residual frequencies. There are many sources of systematic uncertainties to ω_a , most of them related to the reconstruction of positron events or our parametrization of beam dynamics effects. In Run-1, the biggest sources of systematic uncertainty on ω_a were CBO and pileup, contributing 38 and 35 parts per billion (ppb), respectively. In Run-2 and Run-3, there were many hardware and software improvements to reduce the systematics uncertainties. For instance, in Run-1 the motion of the muon beam was strongly affected by damaged resistors in the ESQ system, which were repaired before the start of Run-2; in addition, the kicker system was upgraded towards the end of Run-3 to provide a stronger kick to the muon beam and center it. These improvements greatly reduced many systematic effects related to beam dynamics. In Run-2 and Run-3 combined, we collected 4.7 times the number of Run-1 decay positrons, which allowed us to improve our empirical modeling of the CBO, reducing the associated systematic to 21 ppb. On the reconstruction side, we improved our algorithms to better resolve pileup, bringing the associated systematics down to 7 ppb. In addition, a new Asymmetry-weighted ratio method was developed, which consisted in subdividing data into two wiggle plots, weighting positron events by the energy dependent A_0 parameter, shifting the events of one of them in time randomly by \pm half of the anomalous precession period, and taking the ratio between the difference and the sum of the two wiggle plots such that the muon exponential decay was cancelled out. This method preserved statistical power in the ω_a fit, whilst reducing sensitivity to many systematics. With these improvements, in Run-2 and Run-3 the statistical and systematic uncertainties on ω_a were reduced with respect to Run-1, from 434 ppb to 201 ppb and from 56 ppb to 25 ppb, respectively.

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