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Dissipative dynamics in the cavity-mediated energy transfer process between ultrastrong coupled devices

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Summary. — We analyze the coherent energy transfer process between a quantum charger and a quantum battery mediated by a photonic cavity. The analysis is brought up in the ultrastrong coupling regime, which allows faster transfer performances compared to the limiting weak-coupling scenario. In particular, exploiting higher coupling constant and more and more photons inside the cavity leads to improved energy transfer performances. Moreover, we consider dissipative effects due to interactions with environments and we show that in the ultrastrong regime, thanks to the mediator, it is still possible to achieve great transfer performances.

1. – Introduction

Quantum batteries (QBs) have gained more and more interest since their first appearance in 2013 when R. Alicki and M. Fannes introduced the first time the theoretical concept [1]. In particular, the possibility of achieving better storage and transfer of the energy compared to the classical batteries, thanks to quantum effects [2]. This have led to several theoretical realistic models and possible experimental implementations [3], based on simple quantum systems, mostly collections of two-level systems (TLSs), also known as qubits [4]. This simple system allows to indentify the QB as empty when the system is in the ground state and as full when the system is in the excited state. Different scenarios have been considered to charge the QB, *i.e.*, allowing transitions between the empty and full QB. Most of them have been based on the well known platforms already used for quantum computations, such as artificial atoms [5] and circuit quantum electrodynamics [6, 7]. In this direction first experimental works have appeared in the last two years [8, 9]. In this work we consider such systems and analyze the energy transfer performances [10], by going in the so called ultrastrong coupling (USC) regime, where a faster transfer from one qubit to the other can be achieved [11]. In particular, we are considering a cavity as the mediator between a quantum charger and a QB and we want to prove that by going in the USC regime and adding more and more photons it is possible to obtain improved energy transfer performances compared to the usually addressed weak-coupling regime. Moreover, we consider dissipative effects due to interactions with the environment, in the conventional Caldeira-Leggett picture [12, 13], where one bath is coupled to the cavity and the other one to the QB, to prove the stability of the model.

2. – Model and figure of merit

In this work, we analyze the energy transfer between a quantum charger (C) and a QB (B), modeled as TLSs, coupled by means of the photons in the cavity, which play the role of a mediator (M), in presence of dissipation described by the conventional Caldeira-Leggett picture [12, 13]. The total Hamiltonian can be written as

$$H_{\rm tot}(t) = H(t) + H_{\rm R1} + H_{\rm R2} + H_{\rm RI1} + H_{\rm RI2}. \tag{1}$$

Here, the first term $H(t)$ represents the Hamiltonian of the closed system, and it reads (hereafter we set $\hbar = 1$)

$$H(t) = \frac{\omega_{\rm C}}{2}\sigma_z^{\rm C} + \frac{\omega_{\rm B}}{2}\sigma_z^{\rm B} + \omega_{\rm M} a^\dagger a + g f(t)(a^\dagger + a)(\sigma_x^{\rm C} + \sigma_x^{\rm B}), \tag{2}$$

where $\omega_{\rm C,B}$ are the energy gaps between $|0_{\rm C,B}\rangle$ and $|1_{\rm C,B}\rangle$ and $\sigma_{x,z}^{\rm C,B}$ are the Pauli matrices along the $\hat{x}, \hat{z}$ directions. Moreover, $\omega_{\rm M}$ is the frequency of the photons and a ($a^\dagger$) is the annihilation (creation) operator of the photons. g represents the interaction strength, modulated in time by the switch on and off function $f(t) = \theta(t) - \theta(t-\tau)$, where $\theta(t)$ is the Heaviside function and τ is the time for which the coupling is turned on.

The baths Hamiltonians and the baths interaction Hamiltonians are written in terms of bosonic creation (annihilation) operators $b_j^{\dagger(i)}$ ($b_j^{(i)}$) as

$$\begin{aligned} H_{{\rm R}i} &= \sum_j \Omega_j^{(i)} b_j^{\dagger(i)} b_j^{(i)}, \quad H_{\rm RI1} = \sigma_x^{\rm B} \sum_j \lambda_j (b_j^{\dagger(1)} + b_j^{(1)}) \\ H_{\rm RI2} &= (a^\dagger + a) \sum_j \kappa_j (b_j^{\dagger(2)} + b_j^{(2)}), \end{aligned} \tag{3}$$

where $\Omega_j^{(i)}$ are the harmonic oscillators frequencies and $i = 1, 2$ indicates the two different baths. The spectral properties of them are characterized by the spectral functions

$$J_1(\omega) = \sum_j \lambda_j^2 \delta(\omega - \Omega_j^{(1)}) \qquad J_2(\omega) = \sum_j \kappa_j^2 \delta(\omega - \Omega_j^{(2)}). \tag{4}$$

These equations can be written in the continuum limit and, assuming Ohmic dissipation, they become [13] ($i = 1, 2$) $J_i(\omega) = \alpha_i \omega e^{-\frac{\omega}{\omega_{\rm cut}}}$. Here, α_1 and α_2 are the dissipation strength and $\omega_{\rm cut}$ is the cut-off frequency of the baths.

We now comment on the total initial state, assuming that, at time $t = 0$, the system and the baths are described by the factorized total density matrix $\rho_{\rm tot}(0) = \rho(0) \otimes \rho_{\rm R1}(0) \otimes \rho_{\rm R2}(0)$. In this paper, the initial states of the qubits and cavity, at $t = 0$, will be $|\psi(0)\rangle = |1_{\rm C}, 0_{\rm B}, n\rangle$, where n is the number of photons inside the cavity at the initial time. The corresponding density matrix will be $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$. Moreover, the reservoirs are at thermal equilibrium with density matrices given by $\rho_{\rm Ri}(0) = \frac{e^{-\beta H_{\rm Ri}}}{{\rm Tr}\{e^{-\beta H_{\rm Ri}}\}}$, with $\beta = 1/(k_B T)$ the inverse temperature. To solve the complete dynamics associated to the Hamiltonian in eq. (1) we will apply the routinely used Lindblad equation [14]. For a complete analysis of this argument see ref. [15]. Notice that we have used the PYTHON toolbox QuTiP [16] to solve the dynamics of the system.

Before concluding this section, we briefly recall the definitions of the quantities of interest to characterize the energy transfer performances of the device. The energy transferred from the quantum charger to the QB can be written as $E_{\rm B}(t) \equiv {\rm Tr}_{\rm S}\{\rho(t)H_{\rm B}\} - {\rm Tr}_{\rm S}\{\rho(0)H_{\rm B}\}$, where $H_{\rm B} = \frac{\omega_{\rm B}}{2}\sigma_z^{\rm B}$ is the QB Hamiltonian, S stands for the trace over the system, $\rho(0)$ the initial density matrix of the system and $\rho(t)$ is its time evolved according to the Lindblad equation. Since in realistic situations it is important to transfer as much energy as possible from the quantum charger to the QB in the shortest time, it is also useful to define $E_{\rm B,max} \equiv E_{\rm B}(t_{\rm B,max})$, which corresponds to the maximum of the stored energy in the QB, occurring at the transfer time $t_{\rm B,max}$.

3. – Results

In the following we present the main results, starting by considering the case where no dissipation is present ($\alpha_1 = \alpha_2 = 0$). We will confront the results in the weak coupling regime at the illustrative value $g = 0.01\omega_{\rm B}$ and we will compare them with the one in the USC regime at the representative value $g = 0.1\omega_{\rm B}$. In particular, we will present an analysis varying the number of photons in the cavity at the initial time of the system. For sake of clarity, the results will be reported in the resonant regime $\omega_{\rm C} = \omega_{\rm M} = \omega_{\rm B}$, but similar results can be obtained for the off-resonant regime [10]. Notice that from now on we will indicate the closed system results with the apex 0.

In fig. 1 we report $E_{\rm B,max}$ and $t_{\rm B,max}$ as function of n. As we can see in panel (a), the energy transferred to the QB is generally higher in the weak coupling regime, being $\sim \omega_{\rm B}$ for all the regime, while in the USC the interaction with more photons tends to have detrimental effects.

Conversely, the time required to transfer the energy from the charger to the QB is sensibly faster in the USC regime [see fig. 1(b)]. This is quite interesting, since the advantage is visible even at small values of n, where $E_{\rm B,max}$ is more than 90% and comparable with the one in the weak coupling regime. Other interesting results in the USC regime can be seen in presence of dissipation. In particular, assuming the baths at the same temperature $\beta\omega_{\rm B} = 10$ and two dissipative strengths in the regime of validity of the Lindblad equation, *i.e.*, $0 \leq \alpha_{1,2} \lesssim 0.1$, we can observe in fig. 2, that the USC regime is less impacted by dissipative effects. In fact, the weak coupling regime without

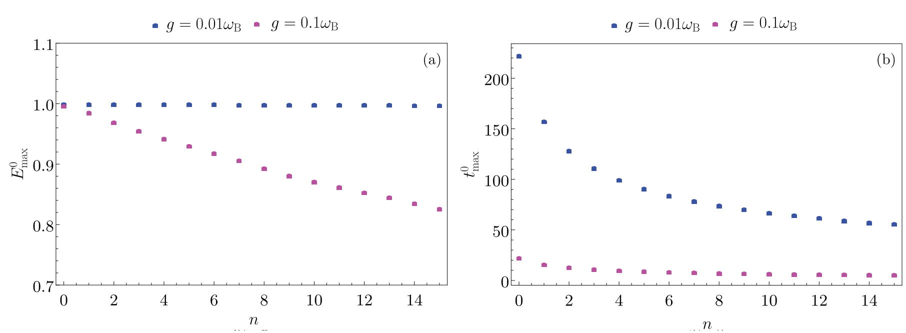

Fig. 1. – Behaviour of $E^0_{\rm B,max}$ in units of $\omega_{\rm B}$ (a) and behaviour of $t^0_{\rm B,max}$ in units of $1/\omega_{\rm B}$ (b) as function of n. The blue squares represent the weak coupling regime at $g = 0.01\omega_{\rm B}$ and the magenta squares represent the USC regime at $g = 0.1\omega_{\rm B}$. Other parameters are $\omega_{\rm C} = \omega_{\rm M} = \omega_{\rm B}$, $\tau = t^0_{\rm B,max}$ and $\alpha_1 = \alpha_2 = 0$.

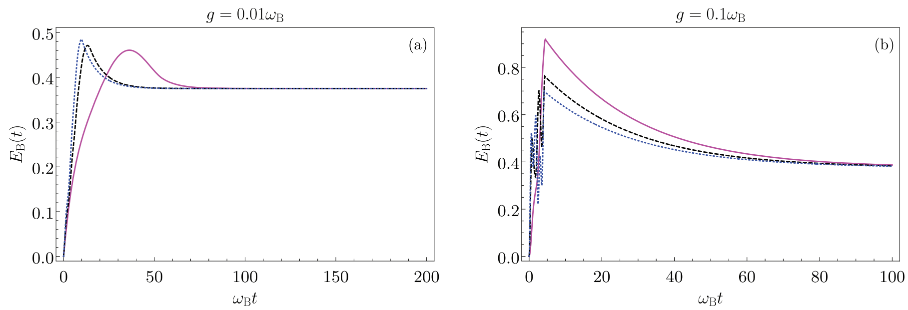

Fig. 2. – Behaviour of $E_{\rm B}(t)$ in units of $\omega_{\rm B}$ as function of $\omega_{\rm B}t$ for the weak coupling regime at $g = 0.01\omega_{\rm B}$ (a) and for the USC regime at $g = 0.1\omega_{\rm B}$ (b). We consider different values of photons inside the cavity: $n = 0$ (full magenta curves), $n = 8$ (dashed black curves) and $n = 15$ (dotted blue curve). Other parameters are $\omega_{\rm C} = \omega_{\rm M} = \omega_{\rm B}$, $\tau = t^0_{\rm B,max}$, $\alpha_1 = 0.03$ and $\alpha_2 = 0.01$.

dissipation lead to an almost complete energy transfer for the considered n. Here, instead, we observe only values $\sim 50\%$, improving when more photons are inside the cavity (see caption for values). The USC regime, allows better performances, but, as observed without dissipation, the presence of more photons inside the cavity leads to detrimental effects.

4. – Conclusions

In this work we have discussed the possibility of modeling the energy transfer performances between a quantum charger and a quantum battery mediated by a photonic cavity. In particular, the ultrastrong coupling regime has been investigated and compared to the usually addressed weak-coupling regime. The first regime allows to improve the transfer times performances, while the second one allows to obtain a better transferred energy increasing the number of photons inside the cavity. Moreover, by taking into account dissipative effects, the ultrastrong coupling regime proves to be more stable and allows to still obtain great transfer performances.

REFERENCES

[1] Alicki R. and Fannes M., *Phys. Rev. E*, **87** (2013) 042123.
[2] Campaioli F. *et al.*, *Thermodynamics in the Quantum Regime* (Springer, Berlin) 2018.
[3] Campaioli F. *et al.*, arXiv:2308.02277 (2023).
[4] Andolina G. M. *et al.*, *Phys. Rev. B*, **98** (2018) 205423.
[5] Le T. P. *et al.*, *Phys. Rev. A*, **97** (2018) 022106.
[6] Ferraro D. *et al.*, *Phys. Rev. Lett.*, **120** (2018) 117702.
[7] Delmonte A. *et al.*, *Entropy*, **23** (2021) 612.
[8] Quach J. Q. *et al.*, *Sci. Adv.*, **8** (2022) eabk3160.
[9] Gemme G. *et al.*, *Batteries*, **8** (2022) 43.
[10] Crescente A. *et al.*, *Phys. Rev. Res.*, **4** (2022) 033216.
[11] Felicetti S. *et al.*, *Phys. Rev. A*, **89** (2014) 013853.
[12] Caldeira A. O. and Leggett A. J., *Phys. A*, **121** (1983) 587.
[13] Leggett A. J. *et al.*, *Rev. Mod. Phys.*, **59** (1987) 1.
[14] Lindblad G., *Commun. Math. Phys.*, **48** (1975) 147.
[15] Crescente A. *et al.*, arXiv:2312.12034 (2023).
[16] Johansson J. R. *et al.*, *Comput. Phys. Commun.*, **184** (2013) 1234.