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Inquiring golden ratio by electric nets and elastic springs

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Summary. — An interdisciplinary teaching activity in the Physics laboratory is described here using the hands-on methodology to experiment with high school students an interdisciplinary learning crunch focused on the Fibonacci sequence. Students were involved in the equivalent resistance calculation of an infinite two-dimensional electrical circuit. In the laboratory, they built a system of resistances and, analogically, a system of springs, and verified the same symmetric properties by rediscovering the gold number. The students developed content knowledge, critical thinking, collaboration, creativity, and communication skills. The hands-on experience increased students' motivation and participation, making them more involved in the educational process.

1. – Introduction

Interdisciplinarity plays an important role in various fields for the following reasons: a) the first step is to get students to reveal their abilities and then guide them to define their place in society, b) it is also necessary for students to learn before acquiring any particular body of knowledge and c) lastly and more generally, it is important to allow students to find themselves in the present-day world to understand and criticize the flood of information they are deluged with daily [1]. In interdisciplinary tasks, students' attitude to generalize disciplinary knowledge to new application contexts is encouraged naturally. The European Union recently published recommendations to integrate all scientific disciplines with their applications in technology and engineering and with artistic expressions (STEAM). Benefits would be, for example, the positive effects of art in interaction with different disciplines, including Mathematics and Physics, from the affective and motivational point of view. Moreover, it fosters the development of creativity and critical thinking. Here, we focused on integrating Mathematics, Arts and Physics in high school [2]. Students were involved in building a system of electrical resistances of the same value and a system of springs using springs with the same constant. They verified the same symmetric properties in both systems, so re-discovering the gold number. Starting from the physics laboratory, learning by doing aids and tools, students were asked to "reinvent" mathematical concepts [3] about the golden ratio [4]. The Fibonacci sequence that appears in the result of the equivalent resistance and the equivalent spring constant was the leitmotiv to extend the work to several disciplines, including Art, proposing an interdisciplinary learning unit to the students. Our research question is: how can the Physics laboratory activities help cross and overcome several disciplines' boundaries?

2. – Theoretical framework

In this paper, we start from a theoretical framework born in the Mathematics education, called the KOM framework, that operates with eight distinct yet mutually related mathematical competencies: mathematical thinking, problem handling, modeling, reasoning, representation, symbols and formalism, communication, aids and tools. Each of these competencies consists of a producing side and an analytical side. The aids and tools competency "consists of, on the one hand, having knowledge of the existence and properties of the diverse forms of relevant tools used in mathematics and having an insight into their possibilities and limitations in different sorts of contexts and, on the other hand, being able to use such aids reflectively" [5]. These competencies are distinct but not disjoint in that each competency has a well-defined identity, but they cross like the petals of a flower (fig. 1).

In this work, the physics laboratory is the tool to achieve the aids and tools competence. So, the KOM framework is used as an interdisciplinary framework. The crossing between the "petals" represents the contact between the different disciplines.

3. – Methods and findings

The Project involved about 60 students aged 14–15 years in the South of Italy. A qualitative analysis has been conducted considering what mathematical and science skills students have acquired. The activities have been divided into two steps, each activity within a specific field of Physics: electricity and mechanics. Electric resistences and springs have been used to realize some aids and tools which are the boundary objects between disciplines.

Step 1: Students were involved in the equivalent resistance calculation of an infinite two-dimensional electrical circuit. Secondly, in the Physics laboratory, they built a system of resistances using a breadboard, electrical resistances of the same value, jumps and an ohmmeter.

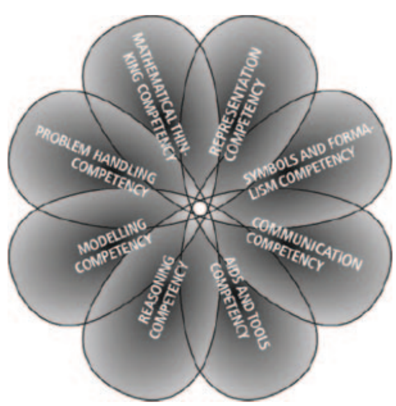

Fig. 1. – The mathematical competency flower [5].

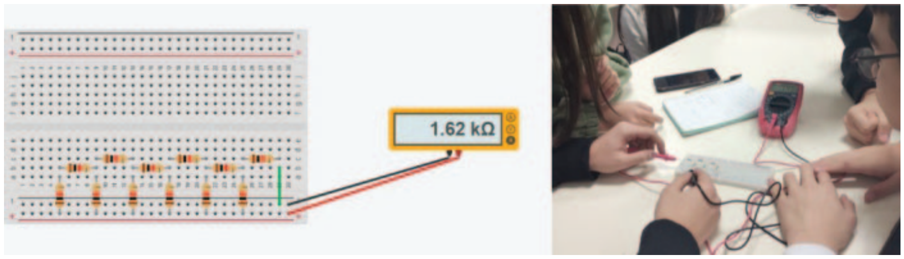

Fig. 2. – Students build a network of resistances using a breadboard.

As shown in fig. 2, starting from a single cell of equivalent resistance $2R$, adding to this a second elementary cell, we obtain an equivalent resistance equal to $5/3R$. Adding cells to the starting one, we obtain the series of equivalent resistances between A and B, for each new added cell: $13/8R$, $34/21R$, $89/55R$ and so on. If we now consider the succession consisting of the numerator and denominator elements of these fractions, the numerator takes the even place and the denominator the odd place: $1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$ which we recognize as a Fibonacci succession whose generic term is $a_{2n+1} = a_{2n} + a_{2n-1}$. By indicating by n the number of cells, the equivalent resistance of a network with n cells will be

$$R_{eq} = \frac{a_{2n}}{a_{2n-1}}.$$

For $n \to \infty$ this ratio goes to the gold number $1.618\ldots$, therefore, the equivalent resistance of an infinite one-dimensional network of resistors is $R_{eq} = 1.618\ldots$

Step 2: As an analogy, it is possible to compute the equivalent elasticity constant of a series of springs (see fig. 3), taking into account that two resistors in series behave like two springs in parallel and vice versa. So, we have $2K$, $5/3K$, $13/8K\ldots$ In this step, they built a system of springs with the geometry in fig. 3, using springs with the same spring constant, a dynamometer, and some mechanical rods (fig. 4). Also in this system, the students experimentally re-discovered the Fibonacci sequence, thus approximating the gold number, considering a system of springs analog to the system of resistances.

4. – Discussion and conclusions

Through this activity we sought to remove traditional borders between individual disciplines and between areas within the same discipline, inviting students to innovate and invent while promoting problem solving and critical thinking skills. Interdisciplinary

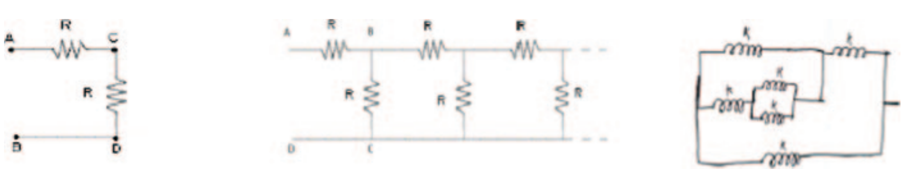

Fig. 3. – Single-cell of an infinitive network of resistance, some cells of an infinitive network of springs.

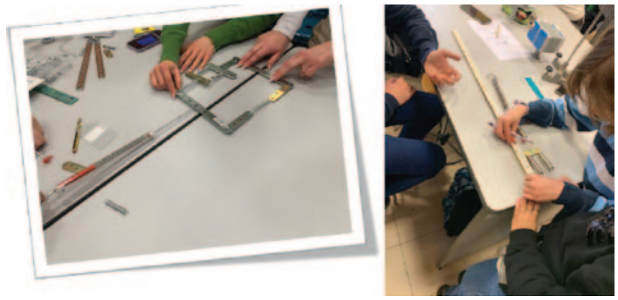

Fig. 4. – Students build a system of springs and measure the equivalent elasticity constant.

education sought to provide students with opportunities to remove artificial borders between disciplines so that students can better understand the connected nature of knowledge using critical skills. We deduced that students learn Physics more quickly, are more motivated to ask questions and seek resources from different disciplines if they are involved in a real challenging problem. The following points seem to emerge from the activities carried out, which seem to answer the initial research question:

1) Mathematics and Physics can model the real world. Students are presented with a problem or challenge that makes the learning experience more meaningful

2) Students are encouraged to ask questions and ask themselves questions, to connect by recognizing the different disciplines as tools for taking a global view of reality

3) The use of aids and tools taken from physics also help develop the other mathematical skills described in the Niss framework

4) From an epistemological point of view, there is a fundamental difference between Mathematics and the other domains of scientific knowledge: mathematical objects, in contrast to scientific phenomena, are never accessible by perception or instruments. The laboratory allows us direct access to the mathematical object.

Furthermore, by working on concrete projects, students acquire autonomy and responsibility, develop skills, apply knowledge and learn meaningfully. Finally, the boundaries between the disciplines are crossed and overcome, integrating theoretical and practical knowledge.

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