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Toward a theory of neutrino mass and mixing

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Summary. — Among numerous theoretical ideas, approaches, mechanisms, models there are probably few elements which will eventually enter the true theory of neutrino masses and mixing. The task is to identify them. Still something conceptually important can be missed. The problems of construction of the theory are outlined. Perspectives and possible future developments are discussed.

1. – Introduction

The theory of neutrino mass and mixing does not exist yet. Probably there is no sense to talk about such a theory separately, without connection to masses and mixing of other fermions as well as some other phenomena. What we call a theory of neutrino mass and mixing appears presently as a multi-dimensional landscape of approaches, models, schemes and mechanisms [1-4](1). Possible energy scales responsible for the neutrino mass generation spread over 50 orders of magnitude from sub-sub- ($\sim 10^{-20}$) eV up to the Planck mass. Concerning the mixing, the ideas range from nearly exact symmetries to anarchy. One, two or more extra dimensions can be involved. No unique line of developments can be traced; various directions are still open and not much "stuff" is excluded.

Essentially scanning of possibilities was performed in the framework of "QFT plus Flavor symmetries". Classification of the produced theoretical material can be done. Effective field theory was used to describe the possibilities in terms of operators of various dimensions.

Probably correct elements of the theory are already among numerous proposals. The task is then to identify them. And still something important can be missed. Below are several "items" which have a chance to "survive" or to play important role in the identification procedure.

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(1) I apologize for very poor citations: space for complete reference list would be order of magnitude longer than the text of the paper.

2. – Beginning and the end

The theory of mass and mixing may start and end here:

$$(1) \qquad \frac{g_{\alpha\beta}}{\Lambda} l_\alpha^T l_\beta H^T H \to m_{\nu,\alpha\beta} = g_{\alpha\beta}\frac{\langle H\rangle^2}{\Lambda}, \quad \alpha, \beta = e, \mu, \tau,$$

where l_α and H are the lepton and Higgs doublets [5]. For $g_{\alpha\beta} \sim O(1)$, the scale of new physics $\Lambda \sim few \times 10^{14}$ GeV, and Weinberg would say that's it. The $\beta\beta_{0\nu}-$decay should be eventually seen, and probably proton decay will be observed. Dark matter and energy may not be connected to neutrinos. That's it, unless we will discover some new physics at lower energy scales, new particles, lepton number violating processes, detect gravitational waves from the phase transitions associated to the violation of lepton number [6], *etc.* Theory of $g_{\alpha\beta}$ couplings can be elaborated without real tests.

The opposite in many aspects scenario is νMSM, which is SM $+3\nu_R$ plus idea of minimality. The νMSM is essentially the phenomenological model with UV completion at the string-Planck scale such that new physics below M_{Pl} does not exist [7], [8], [9]. The model pretends to explain all the observations: Smallness of neutrino mass is obtained by combination of the low (EW) scale seesaw and smallness of the Dirac Yukawa couplings. The lepton asymmetry of the Universe is generated via oscillations of the RH neutrinos ν_{2R} and ν_{3R} and then converted by sphalerons to the baryon asymmetry. The RH neutrinos ν_{2R} and ν_{3R} may be produced in $B-$decays (Br $\sim 10^{-10}$) and tested at SHiP. Higgs inflation was invented. ν_{1R} being the DM particle is produced via the resonance conversion of active neutrinos in the Early Universe, it decouples from generation of light neutrino mass. The mass spectrum can be supported by symmetry with starting point of the degenerate pair ν_{2R}, ν_{3R} and massless state ν_{1R}. The model is still alive and can be tested. A possibility of Grand Unification is problematic.

The right handed (RH) neutrinos, ν_R, should be present in the SM of particle physics. Why not, if other SM fermions do have RH components? Their existence is justified in plausible extensions of SM: gauged $L-R$, Pati-Salam symmetry, $SO(10)$. The interaction of ν_R with ν_L and Higgs (with coupling g) is not forbidden, and consequently, the Dirac mass term should be generated:

$$(2) \qquad y\bar{l}\nu_R H + h.c. \to y\bar{\nu}_L\nu_R\langle H\rangle + h.c. \to m_D = y\langle H\rangle.$$

If the observed neutrino mass $m_\nu = m_D$, *i.e.*, neutrinos are the Dirac particles, the required coupling $y \simeq 2\cdot 10^{-13}$ is uncomfortably (for us but may not for Nature) small.

The term (2) could be forbidden if ν_R has some quantum number which distinguishes it from other SM particles. In this case more appropriate to call ν_R new neutral lepton not associated to fermionic generations rather than the RH neutrino.

3. – Right handed neutrinos are the key

Existence and properties (interactions) of ν_R are the key points of theory of ν mass and mixing. Various possibilities are related to their nature and scale of masses M_R. If ν_R is a Majorana particle and has large Majorana mass $M_R \gg m_D$, the seesaw type I mechanism is realized [10] [11] [12] [13], which is behind the operator (1) with $\Lambda = M_R \simeq (10^9 - 10^{14})$ GeV. If $M_R \simeq M_{Pl}$, the neutrino mass generated via seesaw, $m_\nu \sim 10^{-5}$ eV [14], is much smaller than the observed one. Some other mechanism should give the main contribution, *e.g.*, the radiative, seesaw type II, environmental mechanisms. Here the seesaw acts as the mechanism of suppression of the Dirac mass term effect. Among radiative mechanisms the scotogenic one [15], [16] with new neutral lepton S and new scalar doublet odd with respect to Z_2 symmetry looks attractive establishing connection to the Dark Matter in the Universe.

The RH neutrinos may have no Majorana mass term but couple to new leptons S with Majorana mass M_S:

$$M_D \bar{\nu}_R S + \frac{1}{2} M_S S S + h.c. \tag{3}$$

If $M_S \ll M_D$, then ν_R and S form pseudo-Dirac neutrino with mass $\approx M_D$. By adding the term (2) to (3), the inverse seesaw can be realized for usual neutrinos [17]. Here the $\nu_R - S$ system plays the role of the right handed neutrino with an effective mass $\Lambda = M_R^{eff} \approx -M_D^2/M_S$. If M_S is very small (*e.g.*, in the keV range) large value of M_R^{eff} can be obtained for M_D at the LHC scale. Now Λ is not a fundamental but a fictitious scale composed of two much smaller scales.

For $M_S \gg M_D$ the Majorana mass of ν_R is generated by higher scale seesaw $M_R = -M_D^2/M_S$. In this way, for active neutrinos the double seesaw is realized. A possibility $M_D \simeq M_{GUT}$, $M_S \simeq M_{Pl}$, which gives $M_R \simeq 10^{14}$ GeV required by the usual seesaw, looks very suggestive.

The Majorana mass of ν_R may have non-trivial dynamical origin, for instance, $M_R = h_\Delta \langle \Delta_R \rangle$, where Δ_R is the $SU(2)_R$ Higgs triplet in the L-R symmetric models or $M_R = h_\phi \langle \sigma \rangle$, where σ is the gauge singlet. It can originate from condensate of new strongly interacting sector.

Singlets S, which couple to ν_R, can interact with new scalar and vector bosons, thus forming whole new sector - the Dark sector of Nature. Then ν_R plays the role of "portal" to this sector.

Not only masses but also mixing can be related to properties of ν_R. Many singlets S organized in special way may exist which can produce large mixing or kind of random mixing patterns of light neutrinos.

Smallness of coupling y in (2) neutrino mass can be related to localization of ν_R is extra dimensions which differs from localization of particles with non-zero EW charges. As a result, the overlap of WF of ν_R and ν_L, and consequently, masses are strongly suppressed. *E.g.*, ν_R can propagate in whole extraD space while ν_L is localized in 3D brane [18]. Or ν_L and ν_R are localized on different branes and overlap of their wave functions is exponentially suppressed [19].

The RH neutrino masses may be the origins of the EW scale ("Neutrino option") [20]: Both the Higgs mass term and quartic coupling (absent at tree level) are generated by neutrino ($\nu_L - \nu_R$) loops. This requires $M_R = (10^7 - 10^9)$ GeV, and $y = (10^{-6} - 10^{-4.5})$.

4. – Environmental mass: VEV *versus* EV

The neutrino mass may have an environmental origins being related to the Dark matter or/and Dark energy [21] in the Universe, as well as to new physics at very low energy scales. Indeed, the oscillation results can be explained by any term in the Hamiltonian of evolution with $1/E$ dependence. This can be potential produced by particles of background if these particles and mediators of interactions are sufficiently light.

In the standard model neutrino mass is of vacuum origins generated by neutrino interaction with Higgs field in its lowest energy state - VEV. If the vacuum mass is suppressed by Planck scale seesaw, the dominant contribution may come from the neutrino interactions (coupling g) in the background composed of scalars particles ϕ. At high number density, n_ϕ, the background can be treated as classical field - the expectation value of the field operator in the coherent state of scalar background: $\langle\phi\rangle \simeq \sqrt{n_\phi/m_\phi}$, where m_ϕ is the mass of scalar [22]. Then the effective neutrino mass equals $m_\nu^b = g\langle\phi\rangle \propto g\sqrt{n_\phi/m_\phi}$. Oscillations of ultra-relativistic neutrinos are determined by masses squared: $(m_\nu^b)^2/2E = g^2 n_\phi/(2Em_\phi)$. The same result can be obtained considering refraction: neutrino elastic forward scattering on scalar bosons ϕ [23] [24] [25].

Locally, the difference between the vacuum and refraction masses (for scalar DM) is not so significant: In one case the mass is due to VEV, in another one – due to EV (Expectation Value). In contrast to the vacuum mass the refraction mass being proportional to n_ϕ depends on space-time coordinates, as well as on energy. Additional time dependence, $\cos m_\phi t$, appears for non-relativistic coherent state of ϕ. In model [25] the values of parameters are $m_\phi < 10^{-10}$ eV, $g < 10^{-10}$ and $m_\chi < 10^{-4}$ eV (mass of mediator).

It is not completely clear if refraction mass can explain oscillation results without contradicting other, in particular, cosmological observations. It does not provide (add) any new insight into mixing and mass spectrum. But it is important to search for the experimental consequences of refraction mass: Discovery of space - time dependence of the oscillation parameters will shed light not only on nature of neutrino mass but also Dark Matter.

Yet another interesting possibility exists: neutrino condensate $\Phi_{\alpha\beta} = \langle\nu_\alpha^T\nu_\beta\rangle$ can be formed due to non-perturbative gravitational interactions (gravitational θ-term) in analogy with the quark condensate [26]. Neutrino mass generated by this condensate equals $m_{\alpha\beta} \simeq \Phi_{\alpha\beta}$.

5. – Mass, mixing and symmetries

Ideas about the mass-mixing connection range from strict relationships to complete decoupling. Neutrinos have the weakest mass hierarchy (if any) among fermions. For normal ordering one finds $m_2/m_3 \geq \sqrt{\Delta m_{21}^2/\Delta m_{31}^2} \simeq 0.17$. Large lepton mixing can be due to this weak hierarchy, via the Gatto-Sartori-Tonin type relation: $\theta \simeq \sqrt{m_2/m_3}$ [27]. It can be realized even better if neutrino spectrum is non-hierarchical. As a general guideline, mixing is related to different mass hierarchies of the upper and down fermions, while difference of quark and lepton mixings is related to smallness of neutrino mass.

Decoupling of mass from mixing looks very counterintuitive and non-trivial. The approximate decoupling can be achieved since certain relations (*e.g.*, equalities) between elements of the mass matrix give mixing independently on the size of elements. This can be a consequence of certain symmetry. In turn, the ratios of mass terms from different relations fix ratios of mass eigenvalues. In the first approximation the lepton mixing is described by the TBM matrix [28] with deviations of the order of Cabibbo angle - "Cabibbo haze" [29]. This can be accidental and even consistent with anarchy [30]. In turn, the "haze" can be random or organized as in the quark - lepton complementarity approach. Alternative point of view is that the TBM pattern is non-accidental and originates from certain broken symmetry. Residual symmetry approach was elaborated to realize the latter.

The approach is based on (i) decoupling of masses and mixing, and (ii) different intrinsic symmetries of the $\nu-$ and $l-$ mass matrices. Indeed, TBM can not be related to mass ratios, and therefore implies decoupling of masses and mixing. The mixing appears as a result of different ways of the flavor symmetry breaking by flavon fields in the neutrino and charged lepton (Yukawa) sectors. For this “sequestering” of the corresponding flavons required. In turn, this difference can be related to nature of neutrino and charge leptons masses: Majorana *versus* Dirac, and to different flavor symmetry charges (representations) of the RH neutrinos and charged leptons. The flavor symmetry is broken down to residual symmetries (different for ν and l) which can coincide with intrinsic symmetries of the corresponding mass matrices. One can proceed in the opposite way and reconstruct the flavor symmetry(ies) from the intrinsic symmetries and the TBM mixing. The fact, that rather simple symmetries like S_4 are obtained [31], indicates that something substantial can be in this approach.

Realization of the residual symmetries program in specific gauge models turns out to be complicated and not convincing with *ad hoc* introduced structures, large number of parameters, *etc.* The dilemma is “wrong prediction versus no predictions”. The flavor charge assignment (set of free discrete parameters) has no clear logic, some low dimensional representation are absent (the missing representations problem). Mass hierarchy arises from a kind of Froggatt-Nielsen mechanism [32]: Hierarchy of mass terms is due to high dimensional non-renormalizable operators with products of different numbers of flavon fields. Discrete symmetries provide restricted possibilities to explain also masses and usually lead to degenerate or partially degenerate spectra.

Generic problem is that masses are functions of Yukawa couplings and VEV’s of flavon fields $\langle\phi_\alpha\rangle$:

$$m = F(Y, \langle\phi_\alpha\rangle), \tag{4}$$

which follow from independent sectors of theory: Y - from the Yukawa sector, while $\langle\phi_\alpha\rangle$ from the scalar potential. Here F is determined by mechanism of mass generation. Potentially, supersymmetry can establish connections since both sectors originate from the superpotential. To get TBM, parameters of these two sectors should be correlated, which requires auxiliary symmetries, additional fields, *etc.* Modular symmetry as the flavor symmetry was expected to resolve the problem of the “traditional” symmetries discussed above [1], [2], [33]. It is motivated by string theory, and therefore unavoidable if we believe in strings. The symmetry is related to compactification of extra dimensions. Primary it is realized on the moduli fields τ which describe geometry of the compactified space. New elements in the model building are (i) transformations: appearance of the weight factor in transformations; (ii) Yukawa couplings: the couplings are modular forms - non-linear functions of moduli fields which compose multiplets and transform under representation of finite symmetry group Γ_N. Invariance condition for weights gives additional restrictions, forbids some mass terms, leads to texture zeros.

Models based on modular symmetries are not motivated by TBM, so the approximate TBM appears here accidental. The goal was to reduce number of parameters, make theory more predictive, and also connect masses and mixing. For fixed level N, which fixes the discrete symmetry group, the fit parameters are the VEV of moduli fields (continuous complex number), weights of matter multiplets and Yukawas, the overall couplings at the invariant interaction terms. The models allow to reproduce the observed mixing angles and mass splits and to predict the absolute values of masses, and CP-phases. Typically weak hierarchy and often quasi-degenerate mass spectrum are predicted. For some specific points of moduli space the hierarchical mass spectrum can appear, and it seems, $\tau \simeq i$ plays special role. Here model building is reduced to symmetry building to match the data.

There is certain similarity of the residual ("traditional") symmetry and modular symmetry approaches. Indeed, the considered finite modular symmetry subgroups are isomorphic to the groups A_4, S_4, A_5 ... (determined by the level N) used in the residual symmetry approach. Yukawas are modular forms $Y(\tau) = [Y_1(\tau),\ Y_2(\tau),\ Y_3(\tau)...]$, while in usual approach the effective Yukawa couplings depend on VEV's, $\langle\phi\rangle = [\langle\phi_1\rangle, \langle\phi_2\rangle, \langle\phi_3\rangle...]$, or products of VEV's of flavon fields $Y^{eff} = y(\Pi_i\langle\phi_i\rangle/\Lambda^n)$. But one can establish the correspondence:

$$Y(\tau) \leftrightarrow \frac{1}{\Lambda}\langle\phi\rangle. \tag{5}$$

Yukawa couplings with different weights can be used, which can be constructed as products of modular forms of lower weight: *e.g.*, $Y^{2n} \sim (Y^n)^2$. Then the correspondence is

$$Y^{2n} \leftrightarrow \frac{1}{\Lambda^2}(\langle\phi\rangle)^2, \quad etc.$$

For $\langle\phi\rangle \ll \Lambda$ the Froggatt-Nielsen mechanism is realized which means that higher weight modular forms can produce hierarchy of masses. If

$$\langle\phi_1\rangle : \langle\phi_2\rangle : \langle\phi_3\rangle... = Y_1(\tau) : Y_2(\tau) : Y_3(\tau)...,$$

the "traditional" flavon approach can reproduce results of the modular symmetry approach. The advantage of modular symmetries is that components of $Y-$mutiplets, $Y_i(\tau)$, are fixed by group parameters: level N, and weight k, as well as τ. In contrast, flavon VEV's depend on parameters of potential and (in most of the cases) are not controlled by the flavor symmetry.

In the minimal model only one moduli and therefore only one continuous complex number (moduli VEV) is involved; the rest is determined by structure of symmetry: level, representations, weights and still arbitrary constants in front of different terms. However, minimal versions of models do not work well. Additional freedom should be introduced by using two or more moduli, flavons, etc.

Furthermore, complete and consistent top-down construction based on heterotic string theory compactified on orbifold leads to the "eclectic" flavor symmetries [33]. They include simultaneously and in non-trivially unified way symmetries $G_{traditional}$ and $G_{modular}$ as well as discrete $R-$symmetry, G_R, and CP-symmetry. The Kähler potential (kinetic terms) introduces additional parameters and freedom. Applications of the modular symmetries is still in explorative phase.

6. – Mixing from the Darkness

Theory of neutrino mass and mixing should be constructed together with theory quark masses and mixing. GUT in some version should exist: nothing better than GUT was proposed for BSM physics. No theory of quark masses and mixing exists in spite of the fact that information in the quark sector is complete. What one can expect for leptons? More modest task is to understand the difference of quark and lepton mixings.

The mixing patterns of quarks and leptons are strongly different but still can be related. The 1-2 and 2-3 quark and lepton mixing angles sum up approximately to the maximal mixing angle $\pi/4$. This Quark-Lepton complementarity (QLC) [34] can be formalized by the product

$$U_{PMNS} \approx V_D^\dagger U_X, \tag{6}$$

where $V_D \simeq V_{CKM}$ and $U_X \simeq U_{TBM}, U_{BM}$. Equation (6) implies that V_D and V_{CKM} emerge from a common sector of "the CKM physics", again implying q-l symmetry, or unification and GUT. In turn, U_X follows from the Dark sector coupled to usual neutrinos via the ν_R portal. The relation (6) led to prediction $\sin\theta_{13} \approx \sqrt{1/2}\sin\theta_C$ [35] [34] as well as to prediction for the Dirac CP phase provided that V_D is the only source of CP violation. The dark sector at the String - Planck scale is responsible for large lepton mixing and smallness of neutrino mass via the double seesaw.

This framework opens another possibility to realize flavor symmetries. It is much easier to introduce these symmetries in the Dark (SM singlet) sector, S, and transfer the information about mixing to the visible sector via common basis fixing symmetry, *e.g.*, $Z_2 \times Z_2$ [36]. A possible setup is $SO(10)$ with ν_R portal to Dark sector [37] (see fig. 1). The flavor symmetry is broken spontaneously or explicitly down to basis fixing symmetry in the portal and visible sectors. Symmetry breaking effects are small being suppressed by ratio of the GUT and Planck scales at least. Still theory of CKM physics should be developed.

What is the nature of Dark sector: is this the "parallel world", or it is completely asymmetric to the visible sector? Is this sector above GUT energy scale and should be unified with GUT before unification with gravity? In fact, string theory supports existence of the Dark sector. Similar construction can be realized for low scale (10 - 100 TeV) Dark sector using inverse seesaw, *e.g.*, in the framework of the L-R symmetric models.

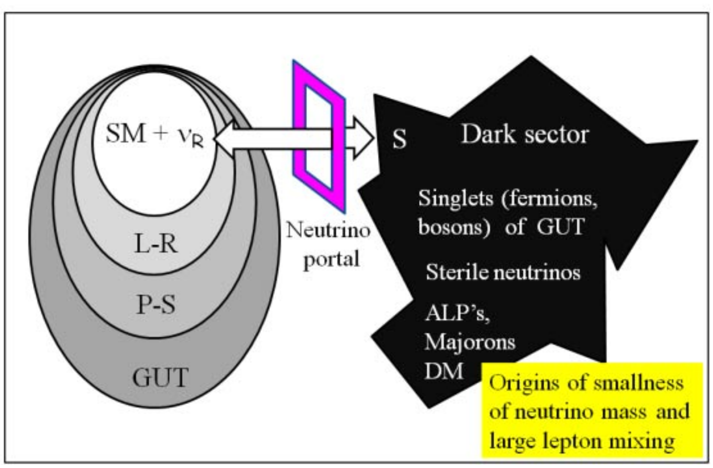

Fig. 1. – Possible set-up for the neutrino mixing from the Dark sector.

7. – Sterile pollution

The eV-scale sterile neutrinos ν_S mixed with active neutrinos may exist, as indicated by some experiments [38]. For $m_4 = (1-3)$ eV and $\sin^2 2\theta_{eS} \approx 4(m_{eS}/m_S)^2 \simeq 0.02-0.1$, the corrections to the mass matrix of active neutrinos after decoupling of sterile neutrino,

$$\delta m_{ee} \approx \sin^2 \theta_{eS} m_4 \sim (0.005 - 0.07) \text{ eV},$$

are at the level of the largest elements (≈ 0.03 eV) of the 3ν- mass matrix. That is, the effect of ν_S is not a small perturbation of the 3ν-picture and possible flavor symmetries. Oscillation data require cancellations and fine tuning in the full mass matrix, and therefore ν_S should be included into theory and symmetry constructions from the beginning. At the same time, mixing with ν_S allows to enhance lepton mixing, to explain the difference of the quark and lepton mixings, and even generate the TBM mixing, if m_{eS}, $m_{\mu S}$, $m_{\tau S}$ have certain relations (symmetry) [39].

8. – Problems and perspectives

Where are the origins of the problems of building the theory? Are we mislead by the data: is our interpretation of the oscillation data correct? First step is to play with observations and to search for regularities. There are many observables and some qualitative regularities. Among those are the $b-\tau$ unification, unification of couplings of the third generation, *etc.* But there is no exact and simple relations between observables which would allow to reduce substantially number of free parameters and give certain hints of the underlying theory. There is nothing like the Balmer series in atomic physics. Maybe the situation is closer to nuclear physics and nuclear spectra. Probably one should use artificial intelligence to search for regularities or conclude about their absence.

It seems, "One step - single framework" constructions do not work. Two or more different and independent contributions to mass matrices can be present. And even two contributions with exact symmetries can produce random effect in the sum, especially if the number of degrees of freedom is small. This can be tested by certain mathematical tools. Recall that the proton mass gets contribution from QCD, electromagnetic interactions and quark masses. Smallness of neutrino mass means that independent contributions from all energy scales up to the Planck scale become relevant. Important task is then to "clean up the data from pollution" of non-leading effects. As it was marked, mixing with sterile neutrinos can modify substantially the mass matrix, and consequently, the mixing of active neutrinos. Probably we should be satisfied with some qualitative relations and regularities, and ignore the "haze" of unexplained corrections.

What are perspectives in the field? Which progress the results of forthcoming experiments can provide? That includes establishing mass hierarchy, determination of CP phase, high precision measurements of known oscillation parameters (with reservation that our theoretical predictions have not achieved high accuracy) [40]. In fact, we already explored in advance possible impact of different outcomes of future experiments on theory. In particular, the analysis of present situation was performed in two modes (of possible mass ordering): normal and inverted. Some models may be excluded and parameter space - restricted. Determination of mass ordering will certainly help. Ideas behind the ordering range from fundamental principles and symmetries, to accidental: selection of values of parameters. Establishing NH will testify for the see-saw, quark-lepton similarity or symmetry, unification. Inverted ordering means strong degeneracy of two heavy states, and consequently, a symmetry. It indicates the structure Pseudo-Dirac pair plus very light Majorana (or Weyl) neutrino. The ν_1 and ν_2 states form quasi-degenerate structure imposed by flavor symmetries, *e.g.*, broken $L_e - L_\mu - L_\tau$. Some special values of the CP phase like $0, \pi, \pm\pi/2$ are very suggestive.

Knowledge of the overall scale of neutrino masses, $m_\nu \simeq \sqrt{\Delta m_{31}^2} = 0.045$ eV, does not give any immediate insight into theory being related to some free parameters of specific models. But it is crucial for determination of type of mass spectrum: hierarchical, non-hierarchical, quasi-degenerate, and this is, indeed, very important for theory. Quasi-degenerate spectrum is already disfavored and further progress is expected.

There is strong dependence of structure of mass matrix on the unknown Majorana phases. In turn, different patterns of mixing require different underlying symmetries. But it will be very difficult to determine the phases.

What else should be measured and searched to achieve the progress? Certainly we need to

- further strengthen the bounds on mixing with steriles, thus removing possible "pollution";

- tests of nature of neutrino mass: searches for energy and space-time dependencies of the oscillation parameters.

As far as theory developments are concerned, we should proceed further with exploration of flavor symmetries (other realizations, modifications of symmetry, other applications). Studies of modular symmetries in whole string framework may reveal for simple versions the required corrections, *e.g.*, from Kähler potential. Or maybe modular-like symmetries without strings can be elaborated. If this does not work some attempts can be made to go beyond common framework based on QFT - flavor symmetries - symmetry breaking. May be more can be learned from Swampland? Anyway the line of research GUT - Planck looks very appealing.

Formaly, mathematical structures we use do not map onto the data in a convincing way. With large number of parameters almost any formalism can be used to describe the data. Large number of parameters in the first step of developments should not discourage, provided that something new - new particles, interactions, dynamics are predicted and can be tested.

Model building should be computerized. Once principles and framework are determined the programs should be developed to produce viable (consistent with observations) models instead of writing hundreds of separate papers on models which differ by level N, weight prescriptions for the matter fields and Yukawas, number of moduli fields.

As an alternative, low and very low scale physics responsible for neutrino masses looks interesting. Neutrinos are in between the two "deserts": the high energy and low energy ones, and can provide the key progress in probes of both.

Connection with other phenomena can be decisive. Any further discovery will affect the field and some findings may lead to breakthrough in our understanding. Actually, almost any BSM phenomenon: new particles, sterile neutrinos, Dark mater, proton decay, *etc.*, may shed some light on origins of neutrino mass and mixing.

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