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Theoretical modelling of Heavy Ion Double Charge Exchange reactions and calculations for the NUMEN project

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Summary. — Double Charge Exchange nuclear reactions induced by Heavy Ions (HIDCE) are discussed, focusing on the scenario where these processes are described as two-step direct reactions and, in particular, as an incoherent sequence of two single charge exchange reactions (Double Single Charge Exchange, DSCE). HIDCE reactions share fundamental characteristics, such as the spin-isospin transition operator, with the elusive neutrinoless double beta decay. Hence, the study of this kind of reactions could allow to get information on the Nuclear Matrix Element (NME) involved in double beta decay rate. Second order DWBA allows to describe Heavy Ion DSCE reaction amplitude in terms of distortion factors, storing information on initial and final state ion-ion elastic interactions, and nuclear matrix elements. The latter are determined within QRPA theory. The formalism adopted to describe DSCE transition form factors is discussed, focusing on the approximations allowing to get disentangled projectile and target NMEs within DSCE cross section expression. Results are shown for the DCE reactions ^{40}Ca (^{18}O, ^{18}Ne$_{gs}$) ^{40}Ar$_{gs}$, ^{76}Se (^{18}O, ^{18}Ne$_{gs}$) ^{76}Ge$_{gs}$ and ^{76}Ge(^{20}Ne,^{20}O$_{gs}$)^{76}Se$_{gs}$, studied within the NUMEN collaboration with a beam energy of 15.3 AMeV.

1. – Introduction

The term Charge Exchange transition refers to processes inducing a change of the nuclear charge by one or more units, leaving the mass number unchanged. This kind of transitions can proceed via weak interaction (*e.g.*, single and double beta decays) or

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via strong nuclear interaction (Charge Exchange reactions). The latter processes refer to nuclear reactions which can be induced by several kind of projectile particle (*e.g.*, pions, light ions, heavy ions), depending on the physics to probe. Modern high resolution experiments led to a revived interest in Double Charge Exchange reactions induced by Heavy Ions (HIDCE), because they allow to probe a wide range of frontiers physics topics, such as the properties of the drip-line nuclei, the theorized Double Gamow-Teller Giant Resonance (DGTGR) and the elusive neutrinoless double beta decay ($0\nu\beta\beta$).

The latter phenomenon is a weak decay hitherto not observed, which would allow to probe physics beyond the Standard Model. Indeed, the occurrence of this kind of transition would imply total lepton number violation (in particular $\Delta L = 2$), against Standard Model predictions, and that neutrinos are Majorana particles (neutrino and anti-neutrino are the same particle), which in turn could offer an explanation of matter-antimatter asymmetry in the Universe. An accurate estimate of $0\nu\beta\beta$ NMEs is crucial to make a reliable prediction of the decay half-life. Hitherto, different nuclear structure models led to $0\nu\beta\beta$ NMEs values spanning by about a factor of 3 [1]. In this respect, HIDCE reactions could represent an interesting tool to get data-driven information on double beta decay NMEs. The latter topic is supported by the existence of heuristic analogies between $0\nu\beta\beta$ and HIDCE reactions, such as the same kind of spin-isospin operator, describing charge exchange transitions [2-4]. Relying on these analogies, the NUMEN collaboration is looking for constraints on $0\nu\beta\beta$ NMEs, from measurements of HIDCE cross section involving those nuclei which are also candidate to undergo $0\nu\beta\beta$ decay [2]. The measurements have been mainly performed at Istituto Nazionale di Fisica Nucleare - Laboratori Nazionali del Sud (INFN-LNS), in Catania.

The feasibility of this kind of studies is given by the existence of a linear correlation between the NMEs of DGT-DCE reactions and $0\nu\beta\beta$ decay, provided by different nuclear structure models [5,6] and also by the possibility to factorize the dependence on projectile and target NMEs within DCE cross section expression [7, 8].

Two main reaction mechanisms allow to explain HIDCE reactions:

- **"direct" DCE**: the reaction is induced by the exchange of charged mesons, which in turn can be further classified into two more mechanisms:
  - **double single charge exchange (DSCE)**: HIDCE reaction is described as two-step process, in particular, as a sequence of two uncorrelated single charge exchange (SCE) reactions (each one induced by charged-meson exchange);
  - **Majorana-like DCE (MDCE)**: HIDCE reaction is described as an effective one-step process, due to the exchange of neutral mesons between the pair of nucleons involved in the transition inside projectile and/or target nuclei, *i.e.*, assuming the two SCE reactions are correlated [9].
- **"transfer-DCE"** the charge exchange transition is the result of a sequence of multi-nucleon transfer reactions [2, 10].

The former kind of reactions allows to recover analogies between HIDCE reactions and double beta decays.

The present proceeding focuses on HIDCE described by means of the DSCE reaction mechanism, through which it is possible to recover projectile and target DSCE NME pretty close to the NMEs involved in $2\nu\beta\beta$ observables [8]. Moreover, it is interesting to note that the nuclear states involved in the off-shell intermediate channel are the same both in DSCE reactions and $0\nu\beta\beta$ decay, implying possible connections between DSCE and $0\nu\beta\beta$ NMEs.

2. – DSCE formalism

The cross section describing DSCE reactions is treated within the second order DWBA, which allows to express the DSCE transition matrix element (TME) as the convolution of the two SCE TMEs and the Green function G_γ, dealing with the propagation of the two nuclei created by the first SCE process.

In order to get a separation of projectile and target DSCE NME, within DSCE cross section expression, it is first of all necessary to work within a representation scheme focusing on the two-step evolution of each interacting nucleus (s-channel representation) [11], instead of the standard description of a sequence of two SCE processes (t-channel representation) [8]. A simple rotation in angular momentum space allows to switch to the new (s-channel) representation scheme. In a similar fashion of [8], the propagator and the nuclear states populated within the intermediated reaction channel are still treated within the Closure approximation and the bi-orthogonality property of the distorted waves, involved in the intermediate channel, is used to disentangle distortion factors from nuclear structure terms.

The above prescriptions, together with the unitary transformation in angular momentum space above discussed, allow to get the following two-step TME

$$\mathcal{M}^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \overline{G}_\gamma \int d^3\frac{\xi}{2}\int d^3\eta\, \rho_{1P}(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2})\rho^*_{2P}(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2})\rho_{1T}(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2})\rho^*_{2T}(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}) \\ V^{(SCE)}_{NN}(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2})V^{(SCE)}_{NN}(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2})N_{\alpha\beta}(\boldsymbol{\eta}) \tag{1}$$

where η and ξ represent the sum and the difference of the linear momenta transfers involved in the two incoherent SCE reactions, respectively; $\overline{G}_\gamma$ is the propagator, which is assumed to be constant; $N_{\alpha\beta}(\boldsymbol{\eta})$ is the DSCE distortion factor, $V^{(SCE)}_{NN}$ is the Fourier transform of nucleon-nucleon (NN) interaction potential and the four ρ_{ij}, $(i=1,2\,,\quad j=P,T)$ are the Fourier transforms of target (T pedice) and projectile (P pedice) one-body transition densities, accounting for first (i=1) and second (i=2) SCE transition. It is straightforward noting that in eq. (1) projectile and target transition densities (*i.e.*, DSCE NMEs) appear still entangled. In order to disentangle these transition densities two approximations are adopted:

1) average-ρ approximation

$$\rho_{1P}(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2})\rho^*_{2P}(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}) \to \frac{1}{V_\xi}\int d^3\frac{\xi}{2}\quad \rho_{1P}(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2})\rho^*_{2P}(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}) \\ = \frac{(2\pi)^3}{V_\xi}\int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}}\rho_{1P}(\mathbf{r})\rho^*_{2P}(\mathbf{r}) \equiv \rho^{(Av)}_P(\boldsymbol{\eta}) \tag{2}$$

implying the replacement of the product of the Fourier transform of first and second step SCE one-body transition densities (OBTDs) with their average over ξ; V_ξ is a normaization volume, allowing to recover the correct dimensions of the two-body transition densities (2BTDs).

2) collinear approximation

$$\rho_{1P}(\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{2})\rho^*_{2P}(\frac{\boldsymbol{\eta}-\boldsymbol{\xi}}{2}) \to \rho_{1P}(\frac{\boldsymbol{\eta}}{2})\rho^*_{2P}(\frac{\boldsymbol{\eta}}{2}) \equiv \rho^{(Col)}_P(\boldsymbol{\eta}) \tag{3}$$

where only the contribution from $\xi = 0$, is retained both in first- and second-step SCE OBTDs, *i.e.*, the same momenta transfers are considered in the two SCE reactions.

Both these approximations, applied to eq. (1), lead to the following expression of the DSCE NN interaction potential

$$V_{NN}^{DSCE}(\boldsymbol{\eta}) \equiv (2\pi)^3 \int d^3r \, |V_{NN}^{(SCE)}(\mathbf{r})|^2 e^{i\boldsymbol{\eta}\cdot\mathbf{r}} \quad . \tag{4}$$

Hence, the DSCE TME of eq. (1) becomes

$$\mathcal{M}_{\alpha\beta}^{(2)} = \overline{G}_\gamma \int d^3\eta \, \rho_P^{(x)}(\boldsymbol{\eta}) \rho_T^{(x)}(\boldsymbol{\eta}) V_{NN}^{DSCE}(\boldsymbol{\eta}) N_{\alpha\beta}(\boldsymbol{\eta}) \tag{5}$$

where $x = Av$ or Col depending on which of the two approximations above discussed is used. It is interesting to note that the expression found in eq. (5) is very close to that of a one-step TME. This kind of TME can be factorised for small momentum transfer values [7], this in turn implying that DSCE cross section can be factorized as follows

$$\frac{d\sigma}{d\Omega} \simeq_{\eta \ll 1} |\rho_P^{(x)}(\mathbf{k}_{\alpha\beta})|^2 |\rho_T^{(x)}(\mathbf{k}_{\alpha\beta})|^2 |V_{NN}^{DSCE}(\mathbf{k}_{\alpha\beta})|^2 |\tilde{n}_{\alpha\beta}(\mathbf{k}_{\alpha\beta})|^2 \tag{6}$$

where $\tilde{n}_{\alpha\beta}$ represents the DSCE distortion factor properly integrated in momentum space (further details are illustrated in [7]).

The factorized expression in eq. (6) is a first fundamental step towards the possibility of inferring data-driven information on DCE NMEs.

3. – Results

The approximations adopted within s-channel representation are tested by comparing the corresponding results with those obtained within t-channel representation. Calculations are shown for the following DCE nuclear reactions (studied within the NUMEN collaboration): ^{40}Ca(^{18}O, ^{18}Ne$_{gs}$)^{40}Ar$_{gs}$, ^{76}Se(^{18}O, ^{18}Ne$_{gs}$)^{76}Ge$_{gs}$ and ^{76}Ge(^{20}Ne, ^{20}O$_{gs}$)^{76}Se$_{gs}$ at 15.3 AMeV beam energy. Figure 1 illustrates that for the three systems studied, average-ρ approximation allows to reproduce the diffraction pattern of t-channel angular distribution at small scattering angles, whereas collinear approximation leads to a diffraction pattern resembling that of t-channel calculations over a larger angular range. It is also interesting to note that the order of magnitude of angular distributions within collinear approximation is very different from that of t-channel results (very small values of N_C scaling factor in fig. 1), while average-ρ angular distributions show less discrepant values with respect to t-channel ones. In particular, average-ρ approximation and t-channel calculations give the same order of magnitude for the lightest nuclear system studied (see fig. 1), while for the heavier systems studied, t-channel order of magnitude is recovered only if transitions with multipolarities $J_\gamma < 5$ are taken into account in the sum over intermediate channel states. The present results reveal that further checks on the approximations made within s-channel framework are necessary to solve the discrepancies discussed.

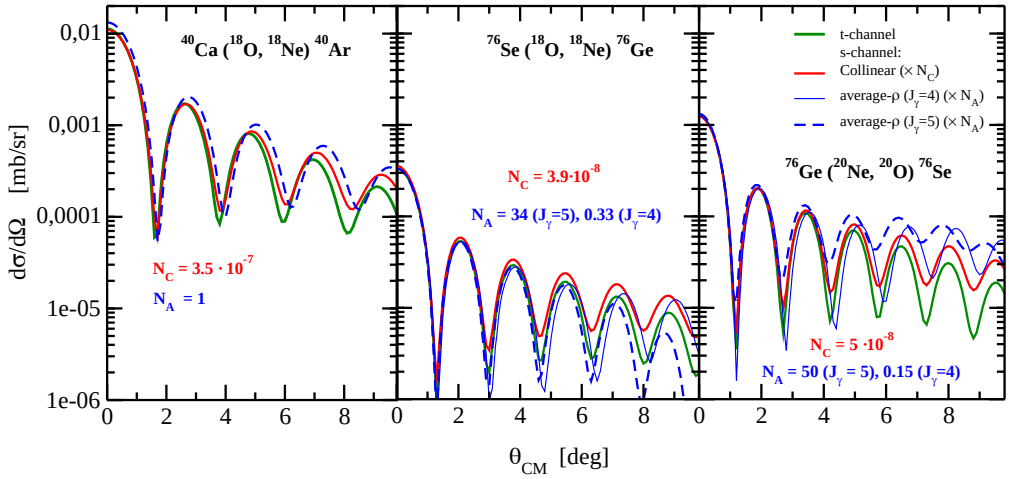

Fig. 1. – Comparison among t-channel and the two kind of s-channel calculations of DSCE cross sections for three nuclear reactions at 15 AMeV beam energy.

However, the similar diffraction patterns of t-channel and s-channel calculations (at least at small scattering angles), together with their order of magnitude reveal the feasibility of the s-channel approach here discussed, using average-ρ approximation for defining projectile and target 2BTDs.

In order to reach a complete understanding of DCE data, it is mandatory to consider a coherent sum of the contribution from all the possible reaction mechanisms, described in the introductory section, *i.e.*, multi-nucleon transfer feeding DCE, the direct two-step reaction mechanism (*i.e.*, the DSCE described here) and the Majorana-like mechanism.

4. – Conclusions

HIDCE reactions are described as two-step processes within second order DWBA. A proper recoupling of all angular momenta allows to derive simple disentangled and expressions of projectile and target 2-body transition densities, together with DSCE NN interaction potential.

The adopted formalism provides a direct relation between the DSCE cross section and the disentangled product of projectile and target NMEs. This result turns out to be useful for directly extracting information on double beta decay-like NMEs, once the contributions to the DCE cross section from all the possible reaction mechanisms are known and coherently added to the present calculations.

Further improvements on the approximations discussed are in progress, together with a better description of experimental energy spectra. Moreover, improvements on the nuclear structure inputs used, including the test of nuclear deformation effects, and check on the effects of different nuclear structure models are ongoing.

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