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Quantum Monte Carlo calculations of magnetic moments for $A \leq 10$ nuclei

G. Chambers-Wall([1]), G. King([1]), A. Gnech([2])([3]), M. Piarulli([1]) and S. Pastore([1])

([1]) *Department of Physics, Washington University in St. Louis - St. Louis, MO USA*
([2]) *European Center for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler - Strada delle Tabarelle, Trento, Italy*
([3]) *INFN-TIFPA Trento Institute of Fundamental Physics and Applications - Trento, Italy*

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Summary. — We report Quantum Monte Carlo calculations of magnetic moments for $A \leq 10$ nuclei using the Norfolk two- and three-nucleon (NV2+3) chiral interactions and one- and two-body electromagnetic currents. We use low-energy constants (LECs) to parameterize these currents up to next-to-next-to-next-to leading order (N3LO). We compare two models using different LECs and regulator parameterizations. The magnetic moment calculations agree with data for both models considered.

1. – Introduction

Current and future experiments involving electroweak phenomena such as long-baseline neutrino oscillations, neutrinoless double beta decay, and precision beta decay measurements will require theoretical input to eventually make determinations of new physics. Studying electromagnetic observables, for which data are abundant and in most cases known with great accuracy, is essential to test nuclear models. Accordingly, determining model dependencies is vital for the progress of fundamental physics.

We calculate magnetic moments of $A \leq 10$ nuclei using Quantum Monte Carlo with Norforlk two- and three-nucleon interactions and electromagnetic currents. We compare two models to determine the dependencies of choice of low energy constants (LECs) and regulators used in the interactions.

2. – Theory

2˙1. *Quantum Monte Carlo.* – Determining the magnetic structure of light nuclei requires the calculation of the many-body ground state. We employ Quantum Monte Carlo (QMC) methods to solve the Schrödinger equation $H\Psi(J^\pi;T,T_z) = E\Psi(J^\pi;T,T_z)$, where $\Psi(J^\pi;T,T_z)$ is the nuclear wavefunction with spin-partiy J^π and isospin quantum numbers T and T_z. We perform Variational Monte Carlo (VMC), which utilizes the variational principle to optimize a wavefunction by minimizing the expectation value

$$E_V = \frac{\langle\Psi_V|\, H\,|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \geq E_0, \quad (1)$$

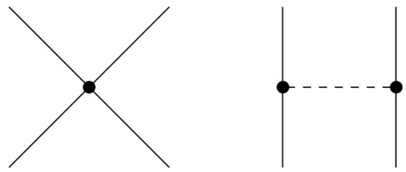

Fig. 1. – Leading order NN interaction diagrams in χEFT. Solid lines are nucleons and the dashed line is a pion.

where E_0 is the true ground state energy. The variational ansatz used for the trial wavefunction is

$$|\Psi_V\rangle = \mathcal{S}\prod_{i<j}\left[1 + U_{ij} + \sum_{i<j\neq k}\tilde{U}_{ijk}^{TNI}\right]|\Psi_J\rangle \tag{2}$$

where $\mathcal{S}$ is the symmetrization operator, U_{ij} and $\tilde{U}_{ijk}^{TNI}$ are two- and three-body correlation operators, and Ψ_J is a Jastrow-like wavefunction. The correlation operators reflect short-distance interactions, and Ψ_J reflects the longer-range cluster structure of the nucleus. The variational parameters are embedded in these correlation operators.

2˙2. *Norfolk interaction.* – The Hamiltonian in (1) consists of non-relativistic nucleon kinetic energies and two- and three-nucleon interaction terms. These interactions are derived from chiral effective field theory (χEFT), a low energy representation of QCD constrained by chiral symmetry. We use the Norfolk two- and three-body interactions (NV2+3), which have nucleons, pions, and Δ-resonances as degrees of freedom [1-4].

The long- and medium-range forces are described by pion exchange mechanisms. Short-ranged forces are given by contact terms, which rely on LECs that capture the physics at this scale. Figure 1 shows these contributions for leading order (LO) nucleon-nucleon (NN) interactions.

In this work, we compare two Norfolk models: Ia* and IIb*. Model I (II) uses LECs determined by fitting NN scattering data up to 125 (200) MeV. Additionally, short- and long-range regulators are introduced to the interaction, each of which requires a choice of parameter, denoted R_s and R_L. For model a (b), the choice of these parameters is [R_L, R_s] = [1.2 fm, 0.8 fm] ([R_L, R_s] = [1.0 fm, 0.7 fm]). The star in both models denotes the fitting to triton ground state energy and Gamow-Teller matrix element for β-decay [3].

2˙3. *Magnetic moment.* – The magnetic form factor can be expressed in an expansion of magnetic multipole operators M_L and total angular momentum of the nucleus J as a sum over odd angular momentum L:

$$F_M^2(q) = \frac{1}{2J+1}\sum_{L=1}^{\infty}|\langle J||M_L||J\rangle|^2. \tag{3}$$

The magnetic moment can then be extracted in the low momentum transfer limit ($q \to 0$) of this form factor. The current $\boldsymbol{j}(\boldsymbol{q})$ can be expressed as a multipole expansion of M_L [6]. In practice for QMC, the matrix element of the current is evaluated with respect to a specific state ($M_J = J$) with a choice of direction of $\boldsymbol{q}$ to isolate the matrix

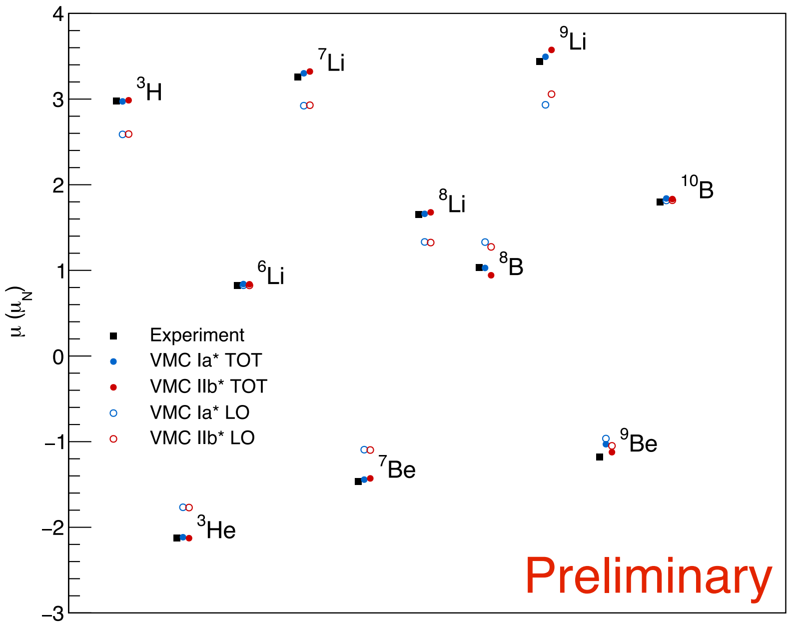

Fig. 2. – VMC calculations of magnetic moments for $A \leq 10$ comparing N2+3-Ia* (blue), N2+3-IIb* (red), and experiment (black). Open circles represent only LO contributions while closed circles show the total (TOT) N3LO contribution that includes many-nucleon electromagnetic currents [7, 8].

elements of each M_L, which can then be related to the reduced matrix elements of eq. (3) via the Wigner-Eckart theorem. Since the higher-order multipoles are negligible in the small q limit, the magnetic moment takes a concise form,

$$\mu = -i\frac{2m_N}{q} \langle JJ| \, j_y(q\hat{\boldsymbol{x}}) \, |JJ\rangle \tag{4}$$

with a choice of coordinates imposed where $\hat{\boldsymbol{z}}$ is the spin-quantization axis of the nucleus. We calculated this matrix element at values of small q and fit a polynomial to extract the magnetic moment.

3. – Results and conclusions

We calculated magnetic moments in light nuclei using VMC and compared two NV2+3 models. Figure 2 shows these results with model Ia* (blue), IIb* (red), and experiment (black). The open circles denote only leading order contributions in the electromagnetic currents, and closed circles denote all contributions up to N3LO. For all nuclei considered in this work, the magnetic moment calculations at N3LO show good agreement with data. There is virtually no model dependencies for magnetic moments of the nuclei considered except for ^{8}B, ^{9}Li, and ^{9}Be, which still have $< 5\%$ difference between models at both LO and N3LO.

Future work includes performing Green's Function Monte Carlo (GFMC) to further improve the VMC results. GFMC involves rewriting the Schrödinger equation in imaginary time, which gives solutions that can then be taken to large imaginary time to project out the true ground state. These calculations should further improve the ground state wavefunction and magnetic moment result. We will also perform these QMC calculations on additional light nuclei ^{9}C and ^{9}B.

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