IL NUOVO CIMENTO **47 C** (2024) 360
DOI 10.1393/ncc/i2024-24360-0

Colloquia: MAYORANA 2023

Recent results in the theory of lepton number violating processes

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received 6 August 2024

Summary. — In view of the non-observation to-date of the mass mechanism of neutrino-less double beta decay (DBD), we discuss possible alternative scenarios including Majoron emission, sterile neutrinos and short- and long-range non-standard mechanisms.

1. – Introduction

Neutrino-less double beta decay (DBD) has not been observed so far (2023). The mechanism for this decay mostly considered is the so-called "mass mechanism", in which neutrinos have masses m_i and couplings U_{ei} $(i = 1, 2, 3)$. The half-life for this mechanism is given by

$$\left[\tau_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+)\right]^{-1} = G_{0\nu} \left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 \qquad (1)$$

where $G_{0\nu}$ is a phase-space factor (PSF), $M_{0\nu}$ the nuclear matrix element (NME), and $f(m_i, U_{ei})$ a function of masses and couplings. Two mass scenarios have been considered: (i) Emission and re-absorption of a light ($m_{light} << 1$ keV) neutrino, and (ii) emission and re-absorption of a heavy ($m_{heavy} >> 1$ GeV) neutrino, for which the functions f and the neutrino potentials v which appear in the calculation of the NMEs are respectively

$$\begin{aligned} f &= \frac{\langle m_\nu \rangle}{m_e}, & \langle m_\nu \rangle &= \sum_{k=light} (U_{ek})^2 m_k, & v(p) &= \frac{2}{\pi}\frac{1}{p(p+\tilde{A})} \\ f_h &= m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle, & \left\langle m_{\nu_h}^{-1} \right\rangle &= \sum_{k=heavy} (U_{ek})^2 \frac{1}{m_{k_h}}, & v(p) &= \frac{2}{\pi}\frac{1}{m_p m_e}. \end{aligned} \qquad (2)$$

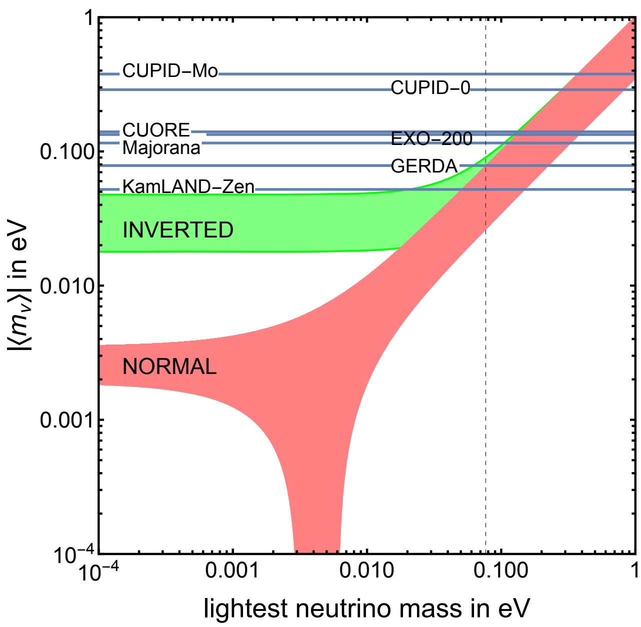

Fig. 1. – Current limits to $\langle m_\nu \rangle$ from CUORE [3], CUPID-0 [4], CUPID-Mo [5], EXO-200 [6], GERDA [7], KamLAND-Zen [8], and Majorana [9], and using IBM-2, Argonne SRC, isospin restored NMEs, and $g_A = 1.269$. The limit from the Planck Collaboration [10] is shown by a vertical line.

Concomitant with neutrino-less DBD there is DBD with the emission of two neutrinos. This process is allowed by the Standard Model. Its half-life can be, to a good approximation, factorized in the form

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu} \left|M_{2\nu}\right|^2 \tag{3}$$

where $G_{2\nu}$ is the phase space factor (PSF) and $M_{2\nu}$ the nuclear matrix element (NME).

All recent calculations have made use of PSF given in [1]. Several methods have been used to evaluate the NMEs, including QRPA (Quasi-particle Random Phase Approximation), ISM (Interacting Shell Model), IBM-2 (Interacting Boson Model), DFT (Density Functional Theory), and others. Results are very sensitive to the value of the axial vector coupling constant, g_A, and to the nuclear model. A summary of calculations with IBM-2 NME and $g_A = 1.269$ is given in fig. 1. Three possible scenarios for g_A are [2]: (i) free value, $g_A = 1.269$; (ii) quark value, $g_A = 1$ and (iii) maximal quenching $g_A = 1.269A^{-0.18}$. If g_A is renormalized to $\sim 1 - 0.5$, all estimates for half-lives should be increased by a factor of $\sim 4 - 34$, making it very difficult to reach in the foreseable future even the inverted region of fig. 1. Possibilities to escape this negative conclusion are: (1) Neutrino masses are degenerate and large. This possibility is in tension with the cosmological bound on the sum of neutrino masses [10], $\sum_i m_i \leq 0.230$ eV, shown in fig. 1 by a vertical line. (2) Other scenarios must be considered, such as (2.1) Majoron emission and (2.2) sterile neutrinos. (3) Other mechanisms must be considered, such as non-standard short- and long-range mechanisms.

2. – Other scenarios

2·1. *Majoron emission.* – The process $0\nu\beta\beta M$ decay $(A,Z) \rightarrow (A,Z+2)+2e^- + m\chi_0$ in which m scalar particles are emitted was first suggested in [11], and subsequently by many other authors. The half-life for this process is

$$\left[\tau_{1/2}^{0\nu M}\right]^{-1} = G_{m\chi_0 n}^{(0)} \left|\langle g_{\chi_{ee}}^{M}\rangle\right|^{2m} g_A^4 \left|M_{0\nu M}^{(m,n)}\right| \tag{4}$$

where $G_{m\chi_0 n}^{(0)}$ is the PSF, $g_{\chi_{ee}}^{M}$ the coupling constant of the Majoron with the neutrino and $M_{0\nu M}^{(m,n)}$ the NME. Here n is the so-called spectral index and $m = 1, 2$ is the number of emitted Majorons. We have recently calculated PSFs and NMEs for this process [12,13]. Limits on half-lives can be set by high-statistics measurements of $2\nu\beta\beta$ decay, for which one can extract limits on coupling constants. A summary is given in table I of [13].

2·2. *Sterile neutrinos.* – A scenario currently being discussed is the mixing of additional "sterile" neutrinos, that is neutrinos with non-standard couplings, suggested first by Pontecorvo [14]. PSFs for this process are the same as in the standard mass scenario. NMEs for sterile neutrinos of arbitrary mass, m_N, and coupling, U_{eN}, can be calculated [15] by using a transition operator as in mass scenario, but with

$$f = \frac{m_N}{m_e}, \qquad v(p) = \frac{2}{\pi}\frac{1}{\sqrt{p^2+m_N^2}\sqrt{p^2+m_N^2}+\tilde{A}} \tag{5}$$

when the mass m_N is intermediate between light and heavy, and, especially, when it is of order of magnitude of the Fermi momentum p_F, the factorization of eq. (1) cannot be used, and physics beyond the standard model is entangled with nuclear physics. However, as shown in [16], an approximate expression can be derived, leading to the half-life

$$\left[\tau_{1/2}^{0\nu I}\right]^{-1} = G_{0\nu} g_A^4 \left|M^{(0\nu_h)}\right|^2 m_p \left|\sum_N (U_{eN})^2 \frac{m_N}{\langle p^2\rangle + M_N^2}\right|^2 \tag{6}$$

with $\langle p^2\rangle = (M^{(0\nu_h)}/M^{(0\nu)}) m_p m_e$ where $M^{(0\nu)}$ and $M^{(0\nu_h)}$ are the matrix elements calculated in the limits $m_N \rightarrow 0$ and $m_N \rightarrow \infty$, respectively. Several types of sterile neutrinos have been suggested, heavy sterile neutrinos with masses $m_N >> 1$ eV and light sterile neutrinos with masses $m_N \simeq 1$ eV. For ligth sterile neutrinos, eq. (6) simplifies to

$$\left[\tau_{1/2}^{0\nu_N}\right]^{-1} = G_{0\nu} g_A^4 \left(\frac{\langle m_{N,light}\rangle}{m_e}\right)^2 \left|M^{(0\nu)}\right|^2 \tag{7}$$

and $\langle m_{N,light}\rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + \sum_i U_{ei}^2 m_i$. A particularly interesting case is that of a 4th neutrino with mass $m_4 = 1$eV and coupling $|U_{e4}|^2 = 0.03$ [17]. The presence of sterile neutrinos changes completely the picture for neutrinoless DBD as shown in fig. 2. With sterile neutrinos and $g_A = 1.269$, the inverted hierarchy is reachable in the near future.

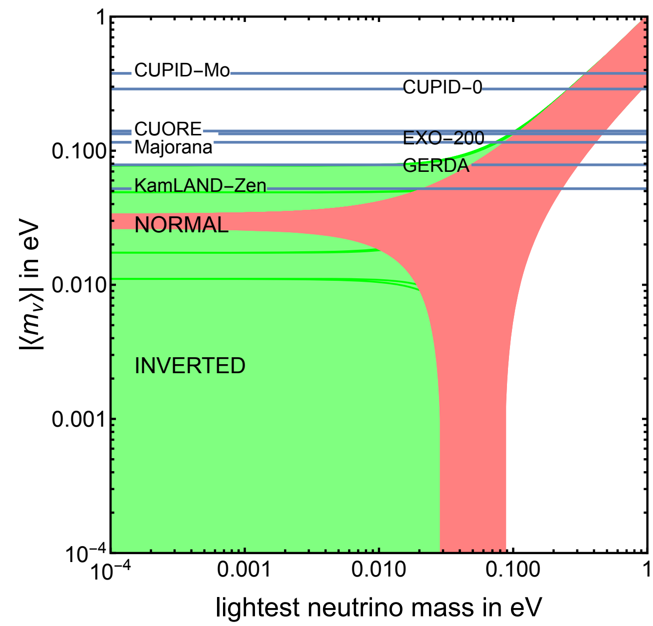

Fig. 2. – (Color online). Current limits for $\langle m_N, light\rangle$ from CUORE [3], CUPID-0 [4], CUPID-Mo [5], EXO-200 [6], GERDA [7], KamLAND-Zen [8], and Majorana [9], and using IBM-2, Argonne SRC, isospin restored NMEs, and gA = 1.269. The figure is in logarithmic scale. Red shows the normal hierarchy and green the inverted hierarchy. In this figure the scenario suggested in [17], relevant to LSND and reactor anomaly, is considered.

3. – Non-standard mechanisms

An exhaustive study of all possible non-standard mechanisms has been recently done.

3·1. *Short-range mechanisms.* – The Lagrangian for these mechanisms is [18]

$$\mathcal{L} = \frac{G_F^2 \cos^2\theta_c}{2m_p}\left[\varepsilon_1 JJj + \varepsilon_2 J^{\mu\nu}J_{\mu\nu}j + \varepsilon_3 J^\mu J_\mu j + \varepsilon_4 J^\mu J_{\mu\nu} j^\nu + \varepsilon_5 J^\mu J j_\mu\right] + H.c., \tag{8}$$

where we have deleted the chirality indices, and the related half-life

$$\begin{aligned}\left[\tau_{1/2}^{0\nu}\right]^{-1} =& G_{11+}^{(0)}\left|\sum_{I=1}^{3}\varepsilon_I^L M_I + \varepsilon_\nu M_\nu\right|^2 + G_{11+}^{(0)}\left|\sum_{I=1}^{3}\varepsilon_I^R M_I\right|^2 + G_{66}^{(0)}\left|\sum_{I=4}^{5}\varepsilon_I M_I\right|^2\\ &+ G_{11-}^{(0)} \times 2\,\mathrm{Re}\left[\left(\sum_{I=1}^{3}\varepsilon_I^L M_I + \varepsilon_\nu M_\nu\right)\left(\sum_{I=1}^{3}\varepsilon_I^R M_I\right)^*\right]\\ &+ G_{16}^{(0)} \times 2\,\mathrm{Re}\left[\left(\sum_{I=1}^{3}\varepsilon_I^L M_I - \sum_{I=1}^{3}\varepsilon_I^R M_I + \varepsilon_\nu M_\nu\right)\left(\sum_{I=4}^{5}\varepsilon_I M_I\right)^*\right],\end{aligned} \tag{9}$$

where the Gs are PSFs and M_Is the NMEs. Upper limits on the effective mass M_ν and on short-range couplings $\varepsilon_i (i = 1, 2, 3, 4, 5)$ have been obtained [18]. It has been found that it is possible to distinguish the mass mechanism from mechanisms 4 and 5 by a measurement of the angular distribution of emitted electrons, although this measurement is very difficult since it will require the observation of many events. Mechanisms 1-3 are instead indistinguishible from the mass mechanism.

3˙2. *Long-range mechanisms*. – The Lagrangian for these mechanisms is [21]

$$\mathcal{L}_{long} = \frac{G_F \cos\theta_c}{\sqrt{2}} \left[J^{\dagger}_{V-A,\mu} j^{\mu}_{V-A} + {\sum_{\alpha,\beta}}' \varepsilon_{\alpha,\beta} J^{\dagger}_{\alpha} j_{\beta} + H.c. \right], \quad \alpha, \beta = S \pm P, V \pm A, T \pm T_5. \tag{10}$$

where the prime over the summation denotes the fact that we have included the $V-A$ term separately. Each model of long-range $0\nu\beta\beta$ decay is defined by 12 coefficients $\varepsilon^{V\mp A}_{V\mp A}, \varepsilon^{S\mp P}_{S\mp P}, \varepsilon^{T\mp T_5}_{T\mp T_5}$. Two models have received considerable attention, $L-R$ models and $SUSY$ models.

3˙2.1. $L-R$ models. $L-R$ models, first discussed in [19,20], are characterized by three parameters, $\varepsilon^{V-A}_{V+A}, \varepsilon^{V+A}_{V\mp A}$. We have recently calculated the NME within the framework of IBM-2 [21]. The parameters ε can be written in terms of three other parameters, κ, λ, η, as $\varepsilon^{V-A}_{V+A,i} = \kappa U_{ei}, \varepsilon^{V+A}_{V+A,i} = \lambda U_{ei}, \varepsilon^{V+A}_{V-A,i} = \eta U_{ei}$. The parameter κ can be put to zero, since it appears in combination with the standard mass mechanism. The results depend then on only two parameters, $\langle\lambda\rangle = \lambda \sum_i U_{ei} V_{ei}$, and $\langle\eta\rangle = \eta \sum_i U_{ei} V_{ei}$. The half-life, $\tau^{0\nu}_{1/2}$, and angular correlation coefficient K are given by

$$\left[\tau^{0\nu}_{1/2}(0^+ \to 0^+)\right]^{-1} = A^{(0)}, \qquad K = \frac{A^{(1)}}{A^{(0)}} \tag{11}$$

with

$$\begin{aligned} A^{(i)} =& C^{(i)}_{mm} \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 + C^{(i)}_{\lambda\lambda} \langle\lambda\rangle^2 + C^{(i)}_{\eta\eta} \langle\eta\rangle^2 + 2C^{(i)}_{m\lambda} \frac{\langle m_\nu \rangle}{m_e} \langle\lambda\rangle \\ & + 2C^{(i)}_{m\eta} \frac{\langle m_\nu \rangle}{m_e} \langle\eta\rangle + 2C^{(i)}_{\lambda\eta} \langle\lambda\rangle \langle\eta\rangle, \end{aligned} \tag{12}$$

and $i = 0, 1$. Limits on the parameters $\langle\lambda\rangle$ and $\langle\eta\rangle$ have been obtained [21].

3˙3. *SUSY models*. – SUSY models were investigated in ref. [22]. In this case, there are four parameters, $\varepsilon^{V-A}_{V-A}, \varepsilon^{S+P}_{S+P}, \varepsilon^{S-P}_{S+P}, \varepsilon^{T-T_5}_{T+T_5}$, but the parameter ε^{V-A}_{V-A} can be put to zero since it appears in combination with the mass term. The remaining three parameters can be written as $\langle\vartheta\rangle = \vartheta \sum_i U_{ei} V'_{ei}$, $\langle\tau\rangle = \tau \sum_i U_{ei} V'_{ei}$, $\langle\varphi\rangle = \varphi \sum_i U_{ei} V''_{ei}$, where the prime and double prime denote the fact that the V_{ei} may not be the same as those of the mass mechanism. The half-life and angular correlations are given by eq. (11) and

$$\begin{aligned} A^{(i)} =& C^{(i)}_{mm} \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 + C^{(i)}_{\vartheta\vartheta} \langle\vartheta\rangle^2 + C^{(i)}_{\tau\tau} \langle\tau\rangle^2 + C^{(i)}_{\varphi\varphi} \langle\varphi\rangle^2 \\ & + 2C^{(i)}_{\vartheta\tau} \langle\vartheta\rangle \langle\tau\rangle + 2C^{(i)}_{\vartheta\varphi} \langle\vartheta\rangle \langle\varphi\rangle + 2C^{(i)}_{\tau\varphi} \langle\tau\rangle \langle\varphi\rangle \\ & + 2C^{(i)}_{m\vartheta} \frac{\langle m_\nu \rangle}{m_e} \langle\vartheta\rangle + 2C^{(i)}_{m\tau} \frac{\langle m_\nu \rangle}{m_e} \langle\tau\rangle + 2C^{(i)}_{m\varphi} \frac{\langle m_\nu \rangle}{m_e} \langle\varphi\rangle. \end{aligned} \tag{13}$$

Limits on the parameters $\langle\vartheta\rangle$, $\langle\tau\rangle$ and $\langle\varphi\rangle$ have been obtained [21].

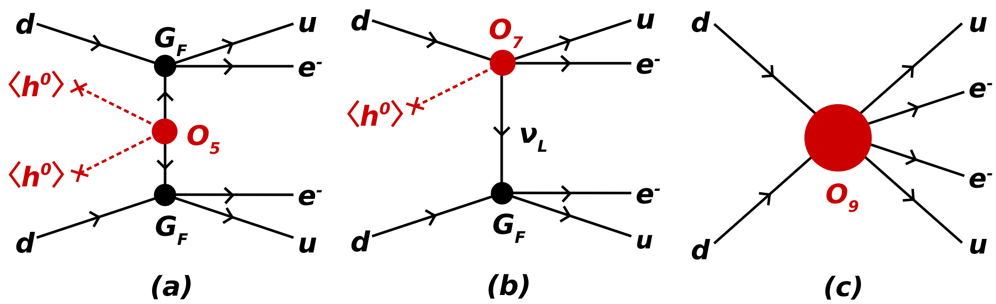

Fig. 3. – Contributions to $0\nu\beta\beta$ decay from effective higher-dimensional LNV operators: (a) 5-dim Weinberg operator (standard mass mechanism), (b) 7-dim operator leading to long–range contribution, (c) 9-dim operator leading to short–range contribution. Adapted from [18].

4. – Conclusions

In conclusion, a complete study of all possible mechanisms of neutrinoless double beta decay up to dimension 9 (dim-9) is now available. These mechanisms are summarized in fig. 3. In this figure, dim-5 is the standard mass mechanism, dim-7 is the non-standard long-range mechanism and dim-9 is the non-standard short-range mechanism. In addition, also the possible emission of scalar particles (Majorons) and the possible existence of sterile neutrinos has been considered.

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The work of JK was supported by the Academy of Finland (Grant Nos. 314733, 320062, and 345869) and NEPTUN (Project no. CF 264/29.11.2022 of the EU call PNRR-III-C9-2022-I8).

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