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Gamow-Teller decays: Probing nuclear structure and weak interactions

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Summary. — We describe the Gamow-Teller decays for nuclear systems outside the ^{40}Ca and ^{56}Ni closed cores in the framework of the realistic shell model, starting from a nuclear Hamiltonian and electroweak currents as consistently obtained by means of chiral perturbation theory. The effective shell-model Hamiltonians and decay operators are derived using many-body perturbation theory, allowing the role of both electroweak currents and many-body correlations to be taken into account as the origin of the problem of the quenching of the axial coupling constant g_A.

1. – Introduction

It is well known that the quenching of spin-isospin matrix elements is a quite general phenomenon in nuclear physics. It means that the observed transition rates of Gamow-Teller (GT) decays, such as β-decays, electron capture, double-beta decays, are hindered by the calculated values obtained using different nuclear structure models.

This effect is usually treated by introducing an effective axial coupling constant g_A^{eff} which is obtained empirically by quenching the bare g_A with a factor $q < 1$ to reproduce the data [1-5].

The phenomenon of the quenching of the spin-isopsin matrix elements has been extensively studied since the 80s of the last century, but in recent years there has been a renewed interest in this subject because of its possible implications in the neutrinoless double-β decay ($0\nu\beta\beta$). As it can be seen from eq. 1, the inverse of the half-life of this decay is proportional - assuming light-neutrino exchange scenario - through the phase-space factor $G^{0\nu}$ and the squared neutrino effective mass $\frac{\langle m_\nu \rangle}{m_e}$ to the squared nuclear matrix element $M^{0\nu}$, connecting the structure of the parent and grand-daughter nuclei,

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and thus to the fourth power of the axial coupling constant

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \left|\frac{\langle m_\nu\rangle}{m_e}\right|^2 \quad . \tag{1}$$

It is clear that the understanding of the microscopic mechanisms that govern the quenching of spin-isopsin matrix elements is crucial to obtain reliable nuclear matrix elements involved in $0\nu\beta\beta$-decay.

Actually, the need to quench g_A in nuclear structure calculations can be traced back to two main sources, both of which arise from the fact that some degrees of freedom are neglected in the description of the electroweak decay. On the one hand, all nuclear models, with the exception of *ab initio* approaches, are based on a truncation of the full Hilbert space of the configurations and thus provide a description of the wave-function in a reduced model space. This means that one has to consider effective Hamiltonians and operators to effectively account for the many-body configurations that are not explicitly considered in the model [6]. On the other hand, nucleons are not structureless particles and their consituent quarks degrees of freedom must be taken into account. This can be done considering processes where the weak probe prompts a meson to be exchanged between two nucleons giving rise to two-body meson exchange electroweak currents [7].

Recently, we have reported in ref. [8] a study of the derivation of the effective shell-model (SM) GT decay operator Θ_{eff}, considering the effects of the two-body electroweak currents to determine their relative weight with respect to the many-body renormalization. More precisely, starting from chiral perturbation theory (ChPT), both for the nuclear Hamiltonian [9,10] as well as for the expansion of the electroweak currents which account for the composite structure of the nucleons [11-13], effective SM operators and Hamiltonians have been consistently constructed using many-body perturbation theory and used to calculate the nuclear matrix elements of GT transitions. In this proceeding, we will discuss some of the results obtained in this work.

The paper is organized as follows. Section **2** briefly describes the theoretical framework of the work. We introduce the chiral nuclear Hamiltonian and electroweak currents utilized in this study, providing details on the perturbative approach employed to derive the effective SM Hamiltonians and decay operators. The results of the shell-model calculations are reported in sect. **3** and compared with the available experimental data, while finally some concluding remarks are reported in sect. **4**.

2. – Outline of calculations

2·1. *The chiral nuclear Hamiltonian and electroweak currents.* – Chiral effective field theory provides a valuable tool to treat hadronic interactions in the low-energy regime of nuclear systems in a systematic and model-independent approach [9, 10]. In present work, we consider a nuclear Hamiltonian based on ChPT [9, 10], which consists of a high-precision two-nucleon (2N) potential derived at next-to-next-to-next-to-leading order (N^3LO) [14], and a three-nucleon (3N) component at N^2LO in ChPT [15].

As regards the one- and two-body components of the electroweak axial currents $\mathbf{J}_A$, whose explicit expressions can be found in ref. [8], they are derived through a chiral expansion up to N^3LO, and the low-energy constants (LECs) appearing in their expressions are consistent with those of the nuclear potential we are starting from. The details on the derivation of the axial currents within the chiral effective theory can be found in ref. [13].

2˙2. *The effective shell-model Hamiltonian and operators.* – The effective SM Hamiltonian $H_{\rm eff}$ - the single-particle energies (SPEs) and the two-body matrix elements of the residual interaction (TBMEs) - must account for the degrees of freedom that are not explicitly included in the reduced model space, which in our case is spanned by the proton/neutron $0f1p$ orbitals (^{40}Ca core), or by the proton/neutron $0f_{5/2}1p0g_{9/2}$ orbitals (^{56}Ni core). In the present work, $H_{\rm eff}$ has been derived starting from the chiral potential with 2NF and 3NF components that has been described in sect. **2**˙1 within the time-dependent perturbation theory, namely by expressing $H_{\rm eff}$ through the Kuo-Lee-Ratcliff folded-diagram expansion in terms of the $\hat{Q}$-box vertex function [16-18]. The details about the derivation of $H_{\rm eff}$ are given in ref. [8], and their SPEs and TBMEs are reported in the Supplemental Material of refs. [19] and [8], for the ^{40}Ca and ^{56}Ni cores, respectively.

From the diagonalization of the $H_{\rm eff}$ we obtain the projections of the true nuclear wave functions onto the chosen model space P, then we have to renormalize each transition/decay operator Θ to take into account the neglected degrees of freedom of the Q space.

The approach adopted to derive the effective SM transition/decay operators $\Theta_{\rm eff}$ is consistent with that used to construct $H_{\rm eff}$, since it is based on the perturbative expansion of a vertex function $\hat{\Theta}$ box, analogous to the derivation of $H_{\rm eff}$ in terms of the $\hat{Q}$ box. The details of such a procedure, first introduced by Suzuki and Okamoto [20], can be found in ref. [18].

3. – Results

In this section, we present and discuss some of the results obtained in the study reported in ref. [8]. More precisely, we focus on the properties related to the GT decay (the GT$^-$ strength distributions and the $M^{2\nu}$s) of some nuclei that are candidates for $0\nu\beta\beta$ decay, namely ^{48}Ca, ^{76}Ge, and ^{82}Se.

In order to disentangle the role played by the electroweak two-body currents and by the Q-space configurations in the renormalization of the GT-decay operator, we will show results obtained considering

a) the bare $\mathbf{J}_A$ at LO in ChPT, namely the usual spin-isospin dependent GT operator $g_A\boldsymbol{\sigma}\cdot\boldsymbol{\tau}$;

b) the effective $\mathbf{J}_A$ at LO in ChPT, that accounts for the contributions of configurations outside the model space (see sect. **2**˙2);

c) the effective $\mathbf{J}_A$ at N^{3}LO in ChPT, a SM operator that owns both one- and two-body components;

and compare them with the experimental data.

The GT strength distributions can be calculated using the definition:

$$B({\rm GT}) = \frac{\left|\langle\Phi_f||\mathbf{J}_A||\Phi_i\rangle\right|^2}{2J_i+1} \ , \tag{2}$$

where indices i, f refer to the parent and daughter nuclei, respectively. Their values can be compared with the data that are obtained from charge-exchange reactions, following

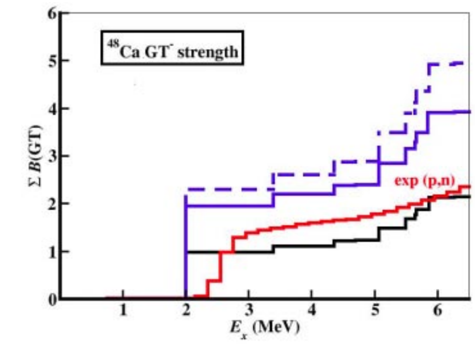

Fig. 1. – Running sums of the ^{48}Ca B(GT) strengths as a function of the excitation energy E_x up to 6.5 MeV (see text for details). Data from ref. [21]

the standard approach in the distorted-wave Born approximation (DWBA) (see, for instance, refs. [22, 23]).

In figs. 1 and 2 the calculated running sums of the GT$^-$ strengths (ΣB(GT)) for ^{48}Ca, ^{76}Ge, and ^{82}Se are shown as a function of the excitation energy, and compared with the data (red line).

The results obtained with the bare operator are drawn with a blue dashed line, while those obtained employing the effective GT operator at LO and at N^{3}LO of the chiral perturbative expansion of $\mathbf{J}_A$ are plotted with a solid blue and black line, respectively.

From the inspection of both figs. 1 and 2, it can be seen that, especially for higher energies, only the inclusion of both contributions to the renormalization of the GT operator due to the ChPT expansion of $\mathbf{J}_A$ and to the derivation of a SM effective decay operator can provide a quite good reproduction of the observed GT-strength distributions.

In table I we report the observed [24] and calculated values (in g_A^2 units) of the $M^{2\nu}$s for the $2\nu\beta\beta$ decay of ^{48}Ca, ^{76}Ge and ^{82}Se.

The table shows that the agreement between the experimental $0_1^+ \to 0_1^+$ $M^{2\nu}$s with those calculated by employing SM effective operators (c), is very satisfactory, thus supporting the crucial role of the two-body electroweak currents.

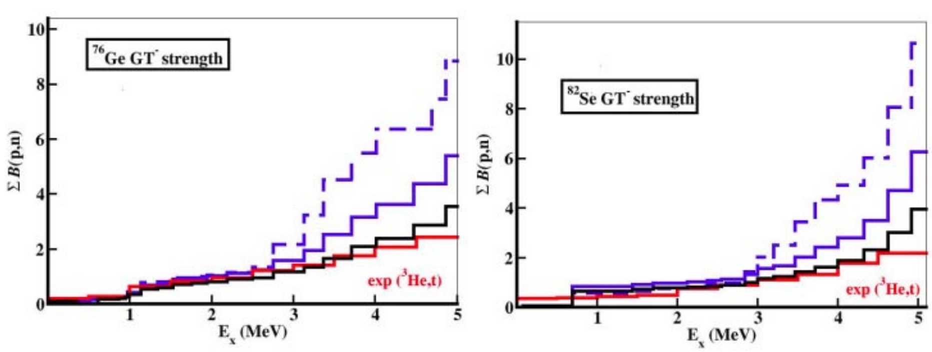

Fig. 2. – Running sums of the ^{76}Ge and ^{82}Se B(GT) strengths as a function of the excitation energy E_x up to 5 MeV (see text for details). Data from refs. [25, 26]

TABLE I. – *Experimental* [24] *and calculated* $M^{2\nu}$*s (in* MeV^{-1}*) for* ^{48}Ca, ^{76}Ge *and* ^{82}Se $2\nu\beta\beta$ *decay. (a), (b), (c) stand for the results obtained using the bare* $\mathbf{J}_A$ *at LO in ChPT, the effective* $\mathbf{J}_A$ *at LO in ChPT, and the effective* $\mathbf{J}_A$ *at* N^3LO *in ChPT, respectively (see text for details).*

Decay	$J_i^\pi \to J_f^\pi$	(a)	(b)	(c)	Exp
^{48}Ca→^{48}Ti	$0_1^+ \to 0_1^+$	0.057	0.048	0.019	0.042 ± 0.004
^{76}Ge→^{76}Se	$0_1^+ \to 0_1^+$	0.211	0.153	0.118	0.129 ± 0.004
^{82}Se→^{82}Kr	$0_1^+ \to 0_1^+$	0.173	0.123	0.095	0.103 ± 0.001

4. – Conclusions

In this proceeding, we have reported on some selected results obtained in a broader study of the effect of two-body chiral electroweak currents on the perturbative renormalisation of the of the shell-model GT-decay effective operator [8].

The comparison of the results with the experiment for a number of observables related to the GT decay shows that the chiral expansion of the electroweak currents and the many-body renormalisations, both of which take into account the neglected degrees of freedom that are not explicitly included in the model, are equally effective in reproducing the data. These results, together with the good reproduction of the low-energy spectroscopic properties of the parent and granddaughter nuclei, as described in ref. [8], should support the confidence to extend our approach in the future to the calculation of the nuclear matrix elements $M^{0\nu}$s for the $0\nu\beta\beta$-decay.

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