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On the multipole giant resonances

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Summary. — In this work, we study isoscalar and isovector monopole, dipole, quadrupole, and octupole giant resonances in the ^{90}Zr, ^{92}Zr, ^{94}Zr, and ^{96}Zr nuclei. The nuclear theory framework used is the quasiparticle random-phase approximation (QRPA). The calculations were done in large no-core single-particle bases using a Bonn-A meson-exchange-based effective interaction. The isoscalar and isovector strength functions were extracted.

1. – Introduction

Tests of nuclear models are essential in order to produce reliable calculations about interesting nuclear processes and properties of nuclei. For example, computation of the nuclear matrix elements of the neutrinoless double-beta ($0\nu\beta\beta$) decay involves various nuclear models, which still need more testing using complementary physical processes, like electromagnetic processes and β decays, among others [1]. One particular family of nuclear models are the quasiparticle-based models, in particular the quasiparticle random-phase approximation (QRPA) [2]. The QRPA is a relative of its proton-neutron version, the proton-neutron QRPA, used heavily in the $0\nu\beta\beta$ calculations [1, 3]. QRPA and pnQRPA are based on the same quasiparticle mean field [2] and reliable probes of it are needed.

One good probe of QRPA-based nuclear models are the studies of isoscalar and isovector spin-multipole resonances. One such study was previously done by us in [4], where spin-dipole and spin-quadrupole strength functions and giant resonances were investigated in the mother and daughter nuclei of several $0\nu\beta\beta$-decay triplets. These studies were complementary to those of [5] where spin-multipole giant resonances of the intermediate nuclei of these triplets were probed by pnQRPA calculations. In this work we study the isovector and isoscalar multipole transitions from the ground states to various excited states in the ^{90}Zr, ^{92}Zr, ^{94}Zr, and ^{96}Zr nuclei. These particular nuclei are relevant for this study since some of these nuclei double beta decay (^{94,96}Zr) and the strength functions in the ^{90,92,94}Zr nuclei have been studied experimentally by Bonasera *et al.* [6].

In our study we concentrate on the monopole, dipole, quadrupole and octupole strength functions and giant resonances.

2. – Theory

The used nuclear model is the quasiparticle random-phase approximation (QRPA). One can read more of this theoretical framework from [2]. The main idea is that protons are paired up with protons and neutrons with neutrons, correspondingly, making up bosonic quasiparticle pairs. A QRPA excitation operator creates the excitations in the form

$$Q^{\dagger}_{\omega} = \sum_{a\leq b}[X^{\omega}_{ab}A^{\dagger}_{ab}(JM) + Y^{\omega}_{ab}\tilde{A}_{ab}(JM)]\,, \tag{1}$$

where $A^{\dagger}$ and $\tilde{A}$ are the quasiparticle-pair creation and annihilation operators, and X and Y are the amplitudes that describe the probability for a pair excitation ($A^{\dagger}$) and annihilation ($\tilde{A}$) to happen. The transition matrix element needs a transition operator, which in the isovector case is of the form

$$\mathcal{O}^{v}_{L,JM} = i^{L} f_L(r)[Y_L \mathbf{1}]_{JM} t_0\,, \tag{2}$$

where $f_L(r)$ is the radial part, Y_L is a spherical harmonic, $\mathbf{1}$ is the unity operator, and t_0 is the z component of the isospin operator. For the isoscalar operator $\mathcal{O}^s$, the only difference is that the t_0 is not included. The transition operator included the spin operator in our previous calculations [4]. Here it has to be noted that the isoscalar giant dipole excitation is split into $1\hbar\omega$ and $3\hbar\omega$ components and the $3\hbar\omega$ component can be viewed as a non-isotropic compression mode [7]. We have removed the spurious center-of-mass components by adopting the radial dependence $f^s_1(r) = r^3 - (5/3)\langle r^2\rangle r$, $\langle r^2\rangle$ being the second radial moment, as advocated, *e.g.*, in [6].

The transition matrix element $(\omega\|\mathcal{O}^{s,v}_{L,J}\|\mathrm{QRPA})$ can now be constructed from the excitation (1) and the operator (2), and the correlated QRPA ground state. The transition probability, or the strength, can then be calculated by taking the square of the transition matrix element.

3. – Results and conclusions

We present here the strength functions of the zirconium isotopes ^{90}Zr, ^{92}Zr, ^{94}Zr, and ^{96}Zr for monopole, dipole, quadrupole, and octupole giant resonances. The strength functions are shown in fig. 1.

In calculating the isoscalar transition strengths, the lowest calculated energies of each multipolarity are fitted to the lowest experimentally measured excitation energy of the same multipolarity, if possible. For the isovector excitations, the Hamiltonian parameters were selected so that the average energy of the excitation would match the experimentally measured average energy. The experimental values for the average energies are taken from [6]. For the isovector excitations, experimental data were available only for the dipole excitation. This is why the same Hamiltonian parameters for each multipole excitation were used. For the nucleus ^{96}Zr, experimental data were not available, so

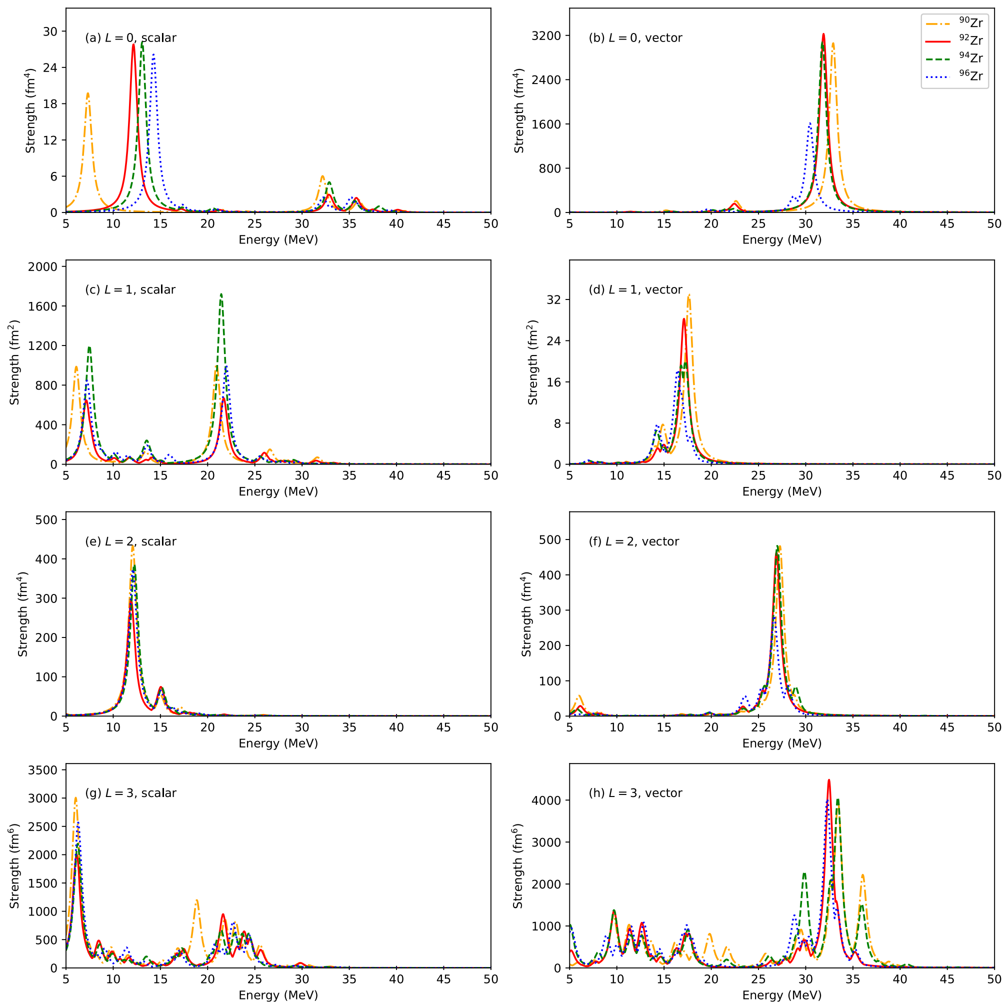

Fig. 1. – Multipole strength functions in the zirconium isotopes discussed in this work. Panels on the left, (a), (c), (e), and (g), present the isoscalar strengths, and the panels on the right, (b), (d), (f), and (h), show the isovector strengths. Panels (a) and (b) refer to the monopole, panels (c) and (d) refer to the dipole, panels (e) and (f) refer to the quadrupole, and panels (g) and (h) refer to the octupole excitations.

values of the Hamiltonian parameters, compatible with those of the other zirconium isotopes, were adopted.

As one can see from the fig. 1, panel (a), for the isoscalar monopole excitation ($L = 0$), that all nuclei show a high peak of strength between 5 and 15 MeV of excitation energy and a smaller peak at higher energies, above 30 MeV. The strength functions seem quite similar for all other nuclei except ^{90}Zr: Its strength function has a significantly lower major peak, both in energy and intensity, than the other isotopes.

For the isoscalar dipole excitation ($L = 1$), fig. 1 panel (c), all of the strength is located in roughly the same positions for all of the zirconium isotopes. It has two dominating peaks, at energies between 5-10 MeV and slightly over 20 MeV.

The isoscalar quadrupole ($L = 2$) strength functions for all of the zirconium isotopes almost coincide. The strength functions can be seen in fig. 1, panel (e). Contrary to the other isoscalar strength functions, the quadrupole excitations show a notable strength only as one peak, which is located between 10 to 15 MeV in energy.

The isoscalar octupole ($L = 3$) resonances, fig. 1, panel (g), show the largest strength in the lower end of the energy spectrum, slightly over 5 MeV. There is also some strength spread over a wider distribution, about 15-25 MeV, but it has less strength than the highest peak.

The isovector strength distributions are shown in fig. 1, panels (b), (d), (f), and (h). The strength distributions, including the giant states, seem quite similar for all the Zr isotopes. This is due to the fact that for all of them similar isovector parameters of the Hamiltonian were used. It is notable that just for the octupole ($L = 3$) there is a wide spread of the transition strength, for the other multipoles the strength is concentrated in one or few closely lying peaks.

The strength functions can be compared with the experimentally measured ones. In the recent, not yet published, study, we have calculated also isoscalar and isovector strength functions for a set of molybdenum isotopes, ^{92}Mo, ^{94}Mo, ^{96}Mo, ^{98}Mo, and ^{100}Mo, in addition to the zirconium isotopes ^{90}Zr, ^{92}Zr, and ^{94}Zr. The theoretically obtained results have been compared to the experimental results, and the preliminary results show mostly quite good correspondence between theory and experiment.

The future experimental data on the multipole strength functions, with or without the spin operator, would help chart how well the QRPA type of nuclear models work at higher excitation energies, like in the multipole resonance region. The spin-multipole strength functions are especially important in studies of double-β decay since they include also the Gamow-Teller giant resonance which plays a role in the double-β decay. Therefore, experimental data on this kind of excitations would give new information about the wave functions of higher-lying states used to describe those rare processes.

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