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Microscopic calculation of the ^{6}He β-decay spectrum for new physics searches

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Summary. — In this contribution, we review a recent calculation of the ^{6}He β-decay spectrum in a microscopic framework that treats nucleons as interacting degrees of freedom. The calculation presented in this contribution includes one- and two-body electroweak currents describing the interaction of single nucleons and correlated pairs of nucleons with an external field, along with dynamics emerging from two- and three-nucleon correlations. With precision measurements of this quantity on-going, the comparably precise theoretical calculation reviewed in this contribution will make it possible to constrain or observe new physics in the near future should the experimental uncertainty goals be achieved.

1. – Introduction

Despite the success of the Standard Model (SM), it is clear that physics beyond the SM (BSM) must exist. The non-zero neutrino masses implied by neutrino oscillations [1, 2], the origin of the observed asymmetry between matter and anti-matter, and the nature of dark matter are, as of yet, unexplained. One of the main thrusts for nuclear physics in the future is the precision study of SM allowed processes– such as β-decay– to probe the origin of BSM physics [3, 4]. In order to observe signatures of new physics in these experiments, one needs theoretical predictions with uncertainties comparable to those of the measurements. In this contribution, we will discuss a recent theoretical calculation of this nature. In particular, we review a recent prediction of the ^{6}He β-decay spectrum [5]– being measured with permille precision goals in on-going experiments [6-8]– using quantum Monte Carlo methods. Several models of the Norfolk local chiral potential [9-11] with consistent one- and two-body electroweak currents [12-14] allowed for an estimate of the uncertainty on the spectrum. The estimated level of theoretical uncertainty indicates that on-going experiments will constrain or observe new physics by comparing to the analysis in ref. [5]

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2. – Quantum Monte Carlo methods

In this study, quantum Monte Carlo (QMC) methods were used to perform the many-body calculations. The procedure to obtain the many-body wave function involves two main steps: First, one begins with a variational Monte Carlo (VMC) calculation. For nuclear physics calculations, one adopts the ansatz [15]

$$|\Psi_T\rangle = \hat{F}|\Phi(JMTT_z)\rangle \tag{1}$$

where Φ is a fully antisymmeric state encoding the long-range structure and quantum numbers of the system. The symmetric operator $\hat{F}$ generates correlations that reflect the impact of the nuclear interaction at short-range. Within $\hat{F}$ are variational parameters which one optimizes to obtain the best variational estimate Ψ_V by minimizing the energy expectation value for a given nuclear Hamiltonian H.

While the VMC approach obtains rather good wave functions, it provides only an upper bound on the ground state energy of the system. One can project out the true ground state by propagating the wave function in imaginary time τ to generate [16],

$$|\Psi(\tau)\rangle = \exp[-(H - E_0)\tau]\Psi_V\,, \tag{2}$$

where E_0 controls the normalization of the wave function. For an appropriately chosen E_0, taking the limit $\tau \to \infty$ will be proportional to the exact ground state of the system. In practice, one propagates the expectation value of the energy and observables by taking several small steps in τ and expectation values are then extracted by averaging once convergence is achieved.

To perform the β-decay calculation and estimate the error on the spectrum, the Norfolk two- and three-body (NV2+3) local interactions were used [9-11] . These potentials were derived in a chiral effective field theory (χEFT) approach retaining nucleons, pions, and Δ isobars as degrees of freedom. Different fitting procedures correspond to the eight NV2+3 model classes that are differentiated by the energy range used to fit the two-nucleon potential, the coordinate space regulators used to remove singularities, and the data used to fit the three-body force. An *ad hoc* uncertainty estimation may be performed using different model classes [17], though this is to be regarded as a model dependence analysis rather than a rigorous uncertainty quantification such as those performed with Bayesian statistical methods.

Associated with the NV2+3 are electroweak charge and current operators derived in the same χEFT approach [12-14]. These operators describe single nucleons and correlated pairs of nucleons interacting with an external electroweak field that transfers momentum $\mathbf{q}$ to the nucleus; *i.e.*, one may express the operators $\mathcal{O}(\mathbf{q})$ schematically as

$$\mathcal{O}(\mathbf{q}) = \sum_{i=1}^{A} \mathcal{O}_i(\mathbf{q}) + \sum_{\{ij\}} \mathcal{O}_{ij}(\mathbf{q})\,, \tag{3}$$

where $\{ij\}$ indicates the set of all pairs.

3. – β-decay theory

The electron energy spectrum can be obtained starting from the standard expression of the differential β-decay rate [18]

$$d\Gamma = \frac{2\pi}{2J_i+1}\sum_{s_e,s_\nu}\sum_{M_i,M_f}|\langle f|H_W|i\rangle|^2\delta(E_i-E_f-E_e-E_\nu)\frac{d^3\mathbf{k}_e}{(2\pi)^3}\frac{d^3\mathbf{k}_\nu}{(2\pi)^3} \tag{4}$$

where J_i is the angular momentum of the initial nucleus, m_e is the electron rest mass, $s_{e(\nu)}$ is the electron (neutrino) helicity, $M_{i(f)}$ is the projection of the initial (final) nuclear angular momentum on the spin-quantization axis, $k_{e(\nu)}$ is the electron (neutrino) three-momenta, $E_{i(f)}$ is the energy of the initial (final) nuclear configuration, and $E_{e(\nu)}$ is the outgoing electron (neutrino) energy. It is typical to decompose the sempileptonic matrix element of the interaction mediating the decay, H_W, into reduced multipoles with well-defined angular momentum L. Then, using the well-understood trace properties of the lepton tensor, the expression in eq. (4) becomes [18],

$$\begin{aligned} d\Gamma =& 2\pi\delta(E_i-E_f-E_e-E_\nu)G_F^2V_{ud}^2\frac{4\pi}{2J_i+1} \\ &\Big[(1+\mathbf{v}_e\cdot\mathbf{v}_\nu)\sum_{L\geq 0}|C_L(q)|^2+(1-\mathbf{v}_e\cdot\mathbf{v}_\nu+2\mathbf{v}_e\cdot\hat{\mathbf{q}}\,\mathbf{v}_\nu\cdot\hat{\mathbf{q}})\sum_{L\geq 0}|L_L(q)|^2 \\ &-2\hat{\mathbf{q}}\cdot(\mathbf{v}_e+\mathbf{v}_\nu)\sum_{l\geq 0}Re\,[C_L(q)L_L^\star(q)] \\ &+(1-\mathbf{v}_e\cdot\hat{\mathbf{q}}\,\mathbf{v}_\nu\cdot\hat{\mathbf{q}})\sum_{L\geq 1}\left[|M_L(q)|^2+|E_L(q)|^2\right] \\ &-2\hat{\mathbf{q}}\cdot(\mathbf{v}_e-\mathbf{v}_\nu)\sum_{L\geq 1}Re\,[M_L(q)E_L^\star(q)]\Big]\frac{d^3\mathbf{k}_e}{(2\pi)^3}\frac{d^3\mathbf{k}_\nu}{(2\pi)^3}\,, \end{aligned} \tag{5}$$

where $\mathbf{v}_e = \mathbf{k}_e/\sqrt{k_e^2+m_e^2}$, $\mathbf{v}_\nu = \mathbf{k}_\nu/E_\nu$, and $\hat{\mathbf{q}} = \mathbf{q}/q$. The reduced multipoles $C_L(q)$, $L_L(q)$, $E_L(q)$, and $M_L(q)$ can be extracted from matrix elements of the operators represented in Equation (3) with standard techniques [19,20]. In the case of ^{6}He β-decay, the change in the nuclear angular momentum is $\Delta J = 1$ while the parity is unchanged. This restricts the multipole contributions to four; namely, $C_1(q;A)$, $L_1(q;A)$, $E_1(q;A)$, and $M_1(q;V)$ where A (V) indicates a contribution arising from an operator associated with an axial (vector) external field.

In the limit of vanishing momentum transfer $q\to 0$, due to β-decay selection rules, only the reduced multipole $L_1(0;A) = E_1(0;A)/\sqrt{2}$ survives for the ground state decay of ^{6}He. Thus, integrating eq. (4) in E_ν and the relative angle between lepton momenta, we obtain the standard expression of the rate differential in E_e

$$\frac{d\Gamma_0}{dE_e} = |L(0;A)|^2\frac{2G_F^2V_{ud}^2}{3\pi^2}\omega_0(\omega_0-E_e)^2E_e^2\sqrt{1-\frac{m_e}{E_e}}R(Z,E_e)\,, \tag{6}$$

where G_F is the Fermi coupling constant, m_e is the electron mass, and $R(Z,E_e)$ represents radiative corrections which depend on E_e and the nuclear charge Z. Interested readers can find the details of the corrections composing $R(Z,E_e)$ in ref. [21].

Accounting for BSM physics, the differential β-decay rate of ^{6}He in eq. (6) is distorted [22],

$$\frac{d\Gamma}{dE_e} = \frac{d\Gamma_0}{dE_e}\left[1 + b\frac{m_e}{E_e}\right], \tag{7}$$

where b is the so-called "Fierz interference term"; however, it is also possible to generate a non-zero value of b by accounting for recoil corrections coming from the small momentum transferred to the nucleus in the decay process. The size of this SM correction is on the order of the size of the correction that currently allowed BSM physics could generate [23]. Thus, it is crucial to go beyond the $q \to 0$ approximation in order to disentangle BSM physics from nuclear effects. This was achieved in ref. [5] by computing the reduced multipoles at several small values of q and fitting the coefficients of a Taylor expansion. In this way, one may include recoil corrections while incorporating important two-body corrections up to the desired precision for experimental analyses.

4. – Results

Following the approach of ref. [17], an uncertainty estimation was performed on the matrix elements used to fit the coefficients in the Taylor expansions of the $L = 1$ multipoles. Inserting the Taylor polynomials with coefficients fit to the matrix element averaged across the different NV2+3 models into eq. (5), the spectrum was predicted and the uncertainty estimated to arise from the interaction was propagated to this measurable quantity. Figure 1 displays the distortion of the spectrum retaining recoil order

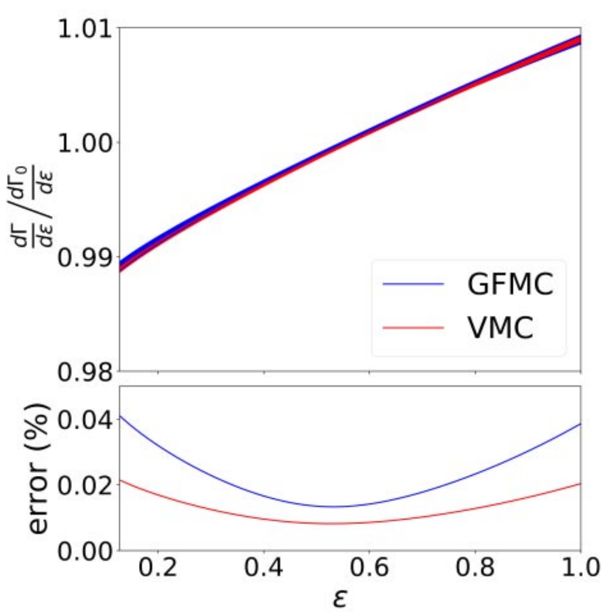

Fig. 1. – (Color online) The top panel displays deviations of the recoil-corrected ^{6}He β-decay spectrum from the standard $q \to 0$ approximation in eq. (6) obtained using VMC (red) and GFMC (blue). Numerical values of the model uncertainties represented by the width of the band in the top panel are presented in the panel below. Figure reproduced from ref. [17].

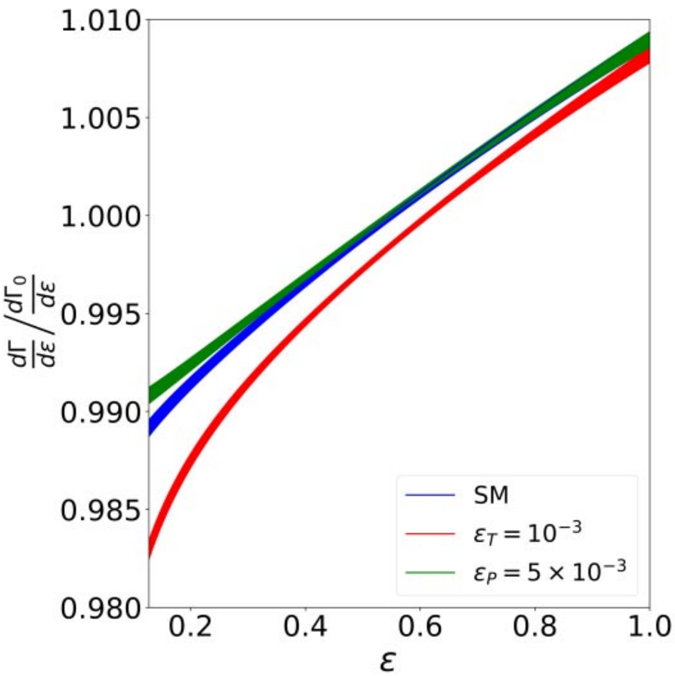

Fig. 2. – (Color online) The deviation of ^{6}He β-decay spectrum from the standard $q \to 0$ approximation in eq. (6) (blue) compared with the deviations from including tensor (red) and pseudoscalar (green) contribution to the charge changing weak current. Model uncertainties are represented by the width of the band. Figure reproduced from ref. [17].

corrections compared with the typical $q \to 0$ approximation in eq. (6). In this figure, the dimensionless quantity ε represents E_e scaled by the endpoint energy of the decay. The theoretical error estimated in this approach is below the permille level required to probe new physics with on-going experiments. The SM correction $b = -1.47(3) \times 10^{-3}$ obtained with QMC not only agrees with a previous evaluation, but also significantly reduces the uncertainty by including explicit two-body physics. To further confirm that this result is accurate, one may look at the half life coming from the integrated spectrum. The GFMC half-life of 808 ± 24 ms is in excellent agreement with the recently measured value of $807.25 \pm 0.16 \pm 0.11$ ms [24]

It was also possible to analyze the effects of BSM physics on the spectrum using an effective field theory approach [25,26]. The leading order effect of psuedoscalar and tensor currents could be included by leveraging similarities with the multipole decomposition for the SM contributions. These similarities made it possible to express these contributions in terms of $L(0; A)$. Higher order deviations are below the level of precision needed to compare with experiment. Using values of the tensor and pseudoscalar couplings currently corresponding to 4 and 8 TeV new physics, respectively, it was possible to predict the distortion generated on top of the recoil corrections. As shown in fig. 2, these corrections would lead to an observable difference from the SM prediction should new physics appear at this level. While current constraints from pion decays [27] make the observation of a pseudoscalar contribution unlikely within the current experimental sensitivity goals, a tensor coupling of this size is allowed by current constraints [28-30]. Thus, it will be possible to either observe or constrain new physics with the on-going measurements of the ^{6}He β-decay spectrum when comparing with the SM result obtained using the NV2+3 interaction and quantum Monte Carlo many-body methods.

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