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The short-time approximation and its potential application to neutrino scattering

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Summary. — The short-time approximation (STA) has been developed to study nuclear responses to external electroweak probes and, to date, has been validated against electron scattering data; however, this technique is broadly applicable and work is underway to extend it to study neutrino scattering from nuclei. These efforts are relevant to planned and on-going long-baseline neutrino oscillation experiments. In this contribution, we detail the theory behind the STA, discuss its use for electron scattering, and provide outlook for its use as a tool to study neutrino-nucleus scattering.

1. – Introduction

One of the clearest indications of physics beyond the Standard Model (SM) was the discovery of neutrino oscillations [1, 2]. This implied that the neutrinos must have a non-zero mass that is unaccounted for in the SM. In the future, a primary endeavor of fundamental physics is to precisely determine the mixing of the neutrino mass eigenstates between different lepton generations. One critical set of experiments in this area are long-baseline neutrino oscillation experiments [3,4]. While it is possible to control the length of the baseline in this kind of experiment, it is not possible to produce neutrinos in a monochromatic beam. Rather, these experiments produce a flux of neutrinos and the incident energy must be inferred from the interactions with nuclei, the active material in the target. Currently, these experiments rely on event generators using somewhat simplistic models of the underlying nuclear dynamics to reconstruct the event kinematics [5]; however, it is critical to obtain accurate nuclear responses in a microscopic framework that fully treats the correlations of nucleons in the system and important many-body contributions in electroweak current operators.

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In this contribution, we will discuss the theory of the short-time approximation (STA) of nuclear responses to external probes [6]. This approach is similar in spirit to the more familiar plane wave impulse approximation; however, critically, it retains important quantum interference effects and two-body physics by factorizing the A-body nucleus into two struck nucleons and an $A-2$ spectator system. This approach has been successfully applied to describe electron scattering in light nuclei using quantum Monte Carlo (QMC) wave functions [6, 7], but its generality provides the flexibility to compute responses to other external probes and to study heavier systems using different many-body methods.

2. – Theory

The standard definition of the response of a nucleus in some initial state $|i\rangle$ with energy E_i to an external probe $\mathcal{O}(\mathbf{q},\omega)$ with four momentum transfer $q^\mu = (\omega, \mathbf{q})$ may be written as

$$R_{\alpha\beta}(q,\omega) = \overline{\sum_{M_i}} \sum_f \langle i|\mathcal{O}^\dagger_\alpha(\mathbf{q},\omega)|f\rangle\langle f|\mathcal{O}_\beta(\mathbf{q},\omega)|i\rangle\delta(E_f - E_i - \omega) \ , \tag{1}$$

where the sum is over all intermediate final states $|f\rangle$ having energy E_f and averaged over all possible spin projections M_i for the initial state. The subscripts α and β denote the Cartesian components of the operator inducing the response. Using properties of the delta function, eq. (1) may be recast in terms of a response function in real time t,

$$R_{\alpha\beta}(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \mathrm{e}^{i(\omega+E_i)t} \overline{\sum_{M_i}} \langle i|\mathcal{O}^\dagger_\alpha(\mathbf{q},\omega)\, \mathrm{e}^{-iHt}\mathcal{O}_\beta(\mathbf{q},\omega)|i\rangle \ , \tag{2}$$

where H is the Hamiltonian describing the nuclear system. If one Wick rotates the response to imaginary time $\tau = it$,

$$R_{\alpha\beta}(\mathbf{q},\omega) = \int \frac{d\tau}{2\pi} e^{\omega\tau} \langle i|\mathcal{O}^\dagger_\alpha(\mathbf{q},\omega)e^{-(H-E_i)\tau}\mathcal{O}_\beta(\mathbf{q},\omega)|i\rangle \tag{3}$$

the response is now defined as a Laplace transform. In fact, the Laplace transform (or "Euclidean Response") of $R(\mathbf{q},\omega)$ can be evaluated with standard QMC techniques [8]. Then, $R(\mathbf{q},\omega)$ is obtained by finding the most likely inversion of Euclidean response using statistical techniques. This essentially exact approach has been applied to study the electron-induced [9] and neutrino-induced charge-changing weak current responses [10] of ^{12}C and has obtained remarkable agreement with data. For a more complete review of Euclidean response theory and its applications to lepton-nucleus scattering, interested readers should consult ref. [11].

Despite the success of the Euclidean response approach, it is computationally intensive and limited to inclusive responses summed over all final states. Further, one cannot fully treat the relativistic kinematics of knocked out nucleons at sufficiently large q. The meson production region will be of particular importance to DUNE [12] and thus necessitates the capability to efficiently analyze exclusive final states in relativistic kinematics.

To circumvent these limitations, one may turn to approximation schemes, such as the recently developed Short-time approximation (STA) [6]. While, thus far, the only applications of the STA are in combination with QMC many-body wave functions, it could in principle be used with any many-body method. The STA is a factorization scheme that retains correlations and many-body currents involving at most two active particles while the remaining nucleons are treated as a spectator system. To see how the notion of short times connects to this approximation, one should look at the Taylor series expansion of the real time propagator for nucleon kinetic energies T_i and a pairwise potential v_{ij},

$$e^{-iHt} \approx 1 - i\left(\sum_i T_i + \sum_{ij} v_{ij}\right) t - \frac{1}{2}\left(\sum_i T_i + \sum_{ij} v_{ij}\right)\left(\sum_{i'} T_{i'} + \sum_{i'j'} v_{i'j'}\right) t^2 + \dots , \tag{4}$$

and note that, if one correlates at most two particles, the first terms dropped in the expansion involve corrections from correlating a third nucleon k and possibly a fourth nucleon l at the same time. Hence, we are neglecting terms of the form $T_iT_kt^2$, $T_iv_{ik}t^2$, $v_{ij}v_{kl}t^2$, and $v_{ij}v_{ik}t^2$. Noting that the average kinetic energy per particle $\langle T_N\rangle$ typically dominates the binding per pair, then the expansion of the propagator will be valid provided that $\langle T_N\rangle t \ll 1$. Thus, it is clear that this is a short-time (and thus high-energy) approximation. Given that quasi-elastic physics peaks at an energy transfer $\omega_{qe} \approx q^2/(2m)$ associated with the timescale $t_{qe} \sim \omega_{qe}^{-1}$, this also constrains the values of q where the STA is a valid description of such processes. Note also that no mention of the specific v_{ij} was made; therefore, this approximation may clearly be generalized to describe any many-body response function at energies large enough that the terms dropped in eq. (4) are negligible.

Making the approximation of short times, one may restrict the current-current correlator to terms involving at most two active nucleons,

$$\begin{aligned}
\mathcal{O}^\dagger e^{-iHt}\mathcal{O} &= \left(\sum_i \mathcal{O}_i^\dagger + \sum_{i<j} \mathcal{O}_{ij}^\dagger\right) e^{-iHt}\left(\sum_{i'} \mathcal{O}_{i'} + \sum_{i'<j'} \mathcal{O}_{i'j'}\right)\\
&= \sum_i \mathcal{O}_i^\dagger e^{-iHt}\mathcal{O}_i + \sum_{i\neq j} \mathcal{O}_i^\dagger e^{-iHt}\mathcal{O}_j\\
&\quad + \sum_{i\neq j}\left(\mathcal{O}_i^\dagger e^{-iHt}\mathcal{O}_{ij} + \mathcal{O}_{ij}^\dagger e^{-iHt}\mathcal{O}_i + \mathcal{O}_{ij}^\dagger e^{-iHt}\mathcal{O}_{ij}\right) ,
\end{aligned}$$

This result, combined with the factorization into intermediate states retaining two active nucleons, allows one to represent the STA responses in terms of a density $D(e, E_{\rm CM})$ in the relative energy e and the center of mass energy $E_{\rm CM}$,

$$R^{\rm STA}(q,\omega) = \int_0^\infty de \int_0^\infty dE_{\rm CM}\delta(\omega + E_i - e - E_{\rm CM})D(e, E_{\rm CM}) . \tag{5}$$

3. – Status and outlook

The STA was originally developed for electron scattering, and its main advantage is highlighted in fig. 1 which shows the transverse response densities of ^{4}He cut at $E_{\rm CM} = q^2/(4m)$. Studying $D(e, E_{\rm CM})$ in this way illuminates features of the response

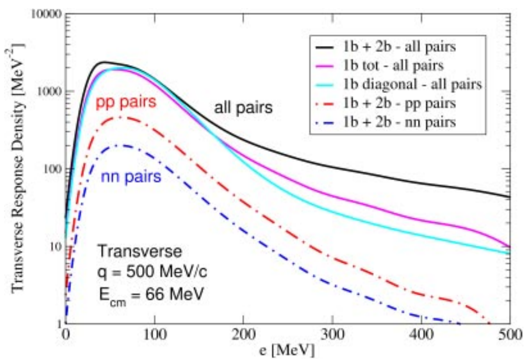

Fig. 1. – (Color online) The STA electron-induced transverse response density of ^{4}He as a function of e cut at $E_{CM} = q^2/(4m)$. The total one-body (pink line) and one- and two-body (black line) responses are compared with individual one-body diagonal (cyan line), *pp* one- and two-body (dashed red line), and *nn* one- and two-body (dashed blue line) contributions. Figure reproduced from ref. [6].

that deepen the understanding two-body effects. In particular, this figure shows the substantial contribution that two-body currents make to the neutron-proton (np) pair response density and that np pairs dominate the two-body current contribution, which enhances the response at large e. The comparative study in ref. [7] demonstrates that the STA reproduces the Euclidean response where the approximation is valid; however, the STA currently does not implement relativistic kinematics and presently fails at large values of q. Another factorization scheme– the spectral function (SF) formalism [13]– incorporates these effects and has more success at large q. In the future, it will be important to include relativistic effects in the STA [14] so that a large q description may be obtained while retaining important interference contributions.

The STA, to this point, has only been applied to study electron scattering; however, it is also possible to study weak responses using the currents in ref. [15]. Work is currently underway to reproduce neutral and charged current responses of ^{2}H obtained in the aforementioned work with exact hypserpherical harmonics wave functions [16]. In the future, benchmarks should also be performed for the ^{12}C neutrino-induced responses obtained with the Euclidean [10] and the SF [17] approaches. Including relativistic kinematics in the STA will open the door for studies of meson production, which should be benchmarked against the description of this process using the SF approach combined with the extended factorization scheme [18].

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