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Neutrino masses, oscillations and neutrinoless double-beta decay

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Summary. — The theory of neutrino-neutrino and neutrino-antineutrino oscillations is revisited. The neutrino production and detection processes are part of a single Feynman diagram with initial and final states represented by plane waves. The revised S-matrix approach guarantees the energy-momentum conservation in neutrino oscillations. The formalism is manifested in the processes involving $\nu_\mu - \nu_e$ and $\nu_\mu - \overline{\nu}_e$ oscillations. The obtained results address the question about the justification of disentanglement of the three involved processes - production, propagation, and absorption of neutrinos. Further, a connection between the neutrinoless double-beta decay ($0\nu\beta\beta$-decay) and neutrino-antineutrino oscillations is discussed. It is pointed out that if the effective Majorana mass governing the $0\nu\beta\beta$-decay is strongly suppressed, a more favorable process of proving the Majorana nature of neutrinos is that which incorporates neutrino-antineutrino (or vice versa) oscillations at certain distances. The question remains: what should be the neutrino source in such a case?

1. – Introduction

The discovery of neutrino oscillations has opened new opportunities to use neutrinos to understand the Universe and the laws that govern it. This achievement established that neutrinos possess tiny masses and that leptonic flavors are not symmetries of Nature. The subject of primary interest remains mass hierarchy, the absolute scale of neutrino masses, and possible additional sterile neutrinos. The new generation of neutrino oscillation experiments, *e.g.*, JUNO, DUNE, T2K, etc., address these issues.

One of the leading unanswered questions about neutrinos is whether they are Majorana (neutrinos are their own antiparticles) or Dirac (neutrinos have their antiparticle) particles. Ettore Majorana formulated the theory of fully neutral neutrinos 86 years ago [1]. Soon after its appearance, Giulio Racah proposed the chain of reactions with real neutrinos [2],

$$(A,Z) \to (A,Z+1) + e^- + \nu, \qquad \nu + (A',Z') \to (A',Z'+1) + e^-, \tag{1}$$

for experimental verification of the hypothesis of Majorana neutrinos. In 1939, Wolfgang Furry considered for the first time neutrinoless double beta decay ($0\nu\beta\beta$-decay) [3],

$$(A,Z) \to (A,Z+2) + e^- + e^-, \tag{2}$$

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a Racah chain of reactions with virtual neutrinos ($(A, Z+1) \equiv (A', Z')$). In this process two neutrons from the initial nucleus transform, by exchanging a virtual Majorana neutrino, into two protons of the final state nucleus with emission of two electrons.

The $0\nu\beta\beta$-decay has not yet been observed. The search for the $0\nu\beta\beta$-decay represents one of the main challenges of neutrino physics. The ultimate goal of the search for $0\nu\beta\beta$-decay is to determine the Majorana neutrino mass [4, 5]

$$m_{\beta\beta} = \left| \sum_{k=1}^{3} U_{ek}^2 m_k \right|. \tag{3}$$

Here, U_{ek} and m_k ($k = 1, 2, 3$) are elements of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix and masses of neutrinos, respectively.

In this contribution, a new Quantum Field Theory (QFT) formalism of neutrino oscillations, inspired by the theory of $0\nu\beta\beta$-decay, is advocated. It works for both (anti)neutrino-(anti)neutrino and neutrino-antineutrino oscillations (and vice versa). The effective Majorana neutrino mass associated with neutrino-antineutrino oscillations is derived and discussed.

2. – Neutrino oscillations as a single Feynman diagram

Neutrino oscillations are intrinsically a finite-time and finite-distance phenomenon, which can be decomposed in a production of neutrinos in a source **S**, a propagation of neutrinos to the detector with distance L and the interaction of neutrinos with the medium of the detector **D**. By considering the charged current weak interaction vertices we have

$$S \to S' + \ell_\alpha^+ + \nu_\alpha, \quad \nu_\alpha \to \nu_\beta, \quad \nu_\beta + D \to D' + \ell_\beta^-. \tag{4}$$

The flavor indices α, β stand for e, μ, τ. Here, $S(D)$ and $S'(D')$ represent initial and final hadrons or nuclei in the vertex $\mathbf{S}(\mathbf{D})$, respectively. In the case of a meson decay, the S' hadron is missing.

The theory of neutrino oscillations remains to be an open problem. The standard quantum mechanical (QM) approach describes the propagation of neutrino with plane wave [6] resulting in questions about the equality of the energies or momenta of the different mass eigenstates, the proper choice of the reference frame, and the entanglement of neutrinos and accompanying particles. Nevertheless, this approach is successfully exploited for global analysis of neutrino oscillation data.

There is a belief that the QFT framework for neutrino oscillations might provide a more realistic interface between theory and experiment once it is properly formulated. Although the QFT wave-packet approach [7-9] is a significant improvement, it suffers from ill-defined flavor states and difficulties with the determination of the wave packet size. This approach has not been used for a global analysis of neutrino oscillation data, even though it was proposed more than two decades ago, in particular due to the additional parameters involved.

In parallel with the wave packet approach, the QFT approach, in which the production and detection processes are assigned to be part of a single Feynman diagram, has been developed in refs. [10, 11]. However, the question remains whether neutrino oscillations can be consistently described in the standard S-matrix formalism of QFT, and if not, how should this formalism be adopted to make the description possible?

Recently, the QFT formalism of neutrino oscillations originating from a single Feynman diagram has been revisited [12]. Two consecutive weak charged current processes in the source (**S**) and detector (**D**) integrated in a single second-order Feynman diagram with effective vertices separated by a macroscopic distance L were considered [12]:

$$S + D \to \ell^+_\alpha + \ell^-_\beta + S' + D'. \tag{5}$$

The developed S-matrix formalism is fully Lorentz invariant at the level of amplitudes, resulting in momentum and energy conservation in each vertex and the whole process. The fermionic neutrino propagator describes the propagation of neutrinos, unlike in the QM formalism, in which the evolution of the plane wave is considered. For the differential rate of this process, the L-dependent master formula was derived:

$$\begin{aligned} d\Gamma^{\alpha\beta}(L) = &\sum_{km} U_{\alpha k} U^*_{\beta k} U_{\alpha m} U^*_{\beta m} \; \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \times \mathcal{F}^{\alpha\beta}_{km} \\ &\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m) \\ &\frac{(2\pi)^7}{4E_S E_D} \; \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)\times \\ &\frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha \; (2\pi)^3} \; \frac{d\mathbf{p}_\beta}{2E_\beta \; (2\pi)^3} \; \frac{d\mathbf{p}'_S}{2E'_S \; (2\pi)^3} \; \frac{d\mathbf{p}'_D}{2E'_D \; (2\pi)^3}, \end{aligned} \tag{6}$$

where

$$\mathcal{F}^{\alpha\beta}_{km} = 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T^{\alpha\beta}_k \left(T^{\alpha\beta}_m\right)^* + T^{\alpha\beta}_m \left(T^{\alpha\beta}_k\right)^* \right) \tag{7}$$

with

$$T^{\alpha\beta}_k = J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D) \; \overline{u}(P_\beta; \lambda_\beta)\gamma_\nu(1-\gamma_5)\not{Q}_k \gamma_\mu v(P_\alpha; \lambda_\alpha). \tag{8}$$

Here, $J^\mu_S(P'_S, P_S)$ and $J^\nu_D(P'_D, P_D)$ are the hadronic currents associated with the weak interaction at source and in the detector, respectively. The notation for the 4-momenta of hadrons and leptons is $P_{S,D} \equiv (E_{S,D}, \mathbf{p}_{S,D})$, $P'_{S,D} \equiv (E'_{S,D}, \mathbf{p}'_{S,D})$ and $P_{\alpha,\beta} \equiv (E_{\alpha,\beta}, \mathbf{p}_{\alpha,\beta})$. $Q_k \equiv (E_\nu, \mathbf{p}_k)$,= is the 4-momenta of neutrino with mass m_k which energy fullfils the relation $E_\nu = E_S - E'_S - E_\alpha = E_\beta + E'_D - E_D$. The factor $1/(\hat{J}_S \hat{J}_D)$ ($\hat{J} = 2J + 1$) is due to averaging over spin projections of the initial hadrons.

The L-dependent master formula in eq. (6) depends on the underlying process and is not reducible to the conventional approach resorting to the concept of neutrino oscillation probability, which assumes a factorization of the methods of production of neutrinos, their propagation in space and the final absorption (or scattering) at the detector. To demonstrate it, an illustrative process

$$\pi^+ + n \to \mu^+ + e^- + p \tag{9}$$

is considered [12]. With some kinematical assumptions ($E_e \simeq E_\nu$ as $E_{n,p} \simeq m_{n,p} = m_N$), the production rate can be written as

$$\Gamma^{\pi^+ n}_{QFT} = \frac{1}{2\pi^2}\; G^4_\beta \; \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \; \frac{m^2_\mu}{m_\pi}\; E^2_\nu \; \frac{P^{\rm QFT}_{\mu e}(E_\nu, L)}{4\pi L^2}\; \left(g^2_V + 3 g^2_A\right)\; p_e E_e, \tag{10}$$

where

$$\mathcal{P}^{\rm QFT}_{\mu e}(E_\nu, L) = \frac{1}{2}\sum_{km} U_{ek} U^*_{\mu k}\; U^*_{em} U_{\mu m}\; e^{i(p_m - p_k)L}\left(1 + \frac{p_k p_m}{E^2_\nu}\right). \tag{11}$$

Here, $G_\beta = G_F \cos\theta_C$, where $\cos\theta_C$ is the Cabbibo angle. E_ν (p_ν) and E_e (p_e) are the energies (momenta) of neutrino and electron, respectively. m_π (m_μ) being the mass of pion (muon). The nucleon's vector and axial-vector coupling constants are denoted by g_V and g_A, respectively. f_π is the pion decay constant.

The production rate in eqs. (10) and (11) is not reducible to that of the conventional approach, resorting to the concept of neutrino oscillation probability. It addresses the question about the justification of disentanglement of the three involved processes - production, propagation, and absorption of neutrinos. The two weak processes at the source and the detector are weakly coupled due to the trace of lepton currents in eq. (7). The subject of this coupling should be an issue for any particular process involving neutrino oscillations in context with related kinematics by considering both energy and angular correlations of emitted leptons.

The master formula in eq. (6) is general and can be exploited with appropriate modification(number of particles produced at the source and detector) for any second-order process with on-shell intermediate neutrinos irrespective of whether charge or neutral currents of neutrinos are considered. It is worth mentioning that the presented formalism, which describes oscillation of neutrinos in vacuum, can be extended to consider the medium effects by propagation of neutrinos. To do it the medium effect on the fermion propagator have to be taken into account [13, 14].

3. – Neutrinoless double-beta decay

Commonly, it is assumed that the conventional light neutrino exchange mechanism generated by left-handed V-A weak currents is the dominant mechanism of the $0\nu\beta\beta$-decay. The inverse half-life of the $0\nu\beta\beta$-decay takes the form [4, 5]

$$\left[T^{0\nu}_{1/2}\right]^{-1} = \left(\frac{m_{\beta\beta}}{m_e}\right)^2 G^{0\nu} (g^{\rm eff}_A)^4 \left|M^{0\nu}_\nu(g^{\rm eff}_{\rm A})\right|^2. \tag{12}$$

Here, $G^{0\nu}$, $g^{\rm eff}_A$ and $M^{0\nu}$ stand for the known phase-space factor, the effective axial-vector coupling constant and the nuclear matrix element (NME) of the process, respectively. m_e is the mass of an electron.

The global fit of neutrino oscillations allows us to evaluate $m_{\beta\beta}$ as a function of the lightest neutrino mass by considering the values Majorana CP violating phases to be arbitrary. In the case of the inverted hierarchy of neutrino masses, $m_{\beta\beta}$ is in the range of tens of meV or larger. If the normal hierarchy is considered, a preferable value of $m_{\beta\beta}$ is of few meV but can be strongly suppressed for the lightest neutrino mass in the range

2 meV $\leq m_{\beta\beta} \leq$ 6 meV [17]. This scenario might prevent observation of the $0\nu\beta\beta$-decay in the foreseeable future. Thus, the potential of other total lepton number violating processes has to be revisited.

4. – Neutrino to antineutrino oscillations

The oscillations of neutrinos to antineutrinos (or vice versa) are commonly presented as a sequence of three processes, namely the production of neutrinos at source, oscillation of neutrinos to antineutrinos in flight, and detection of antineutrinos at the detector [15,16]:

$$S \to S' + \ell^+_\alpha + \nu_\alpha, \quad \nu_\alpha \to \overline{\nu}_\beta, \quad \overline{\nu}_\beta + D \to D' + \ell^+_\beta\,. \tag{13}$$

However, it can also be described as a second-order lepton number violating process,

$$S + D \to \ell^+_\alpha + \ell^+_\beta + S' + D', \tag{14}$$

which involves a distance L propagation of on-shell Majorana neutrinos.

The QFT approach based on a single Feynman diagram of ref. [12] can also be applied to the process in (14). The *Master formula* for differential decay rate corresponds to that in eq. (6) with replacements $U_{\alpha k} \to U^*_{\alpha k}$, $U^*_{\beta m} \to U_{\beta m}$ and

$$T^{\alpha\beta}_k = J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D)\ \overline{v^C}(P_\beta;\lambda_\beta)\gamma_\nu(1+\gamma_5) m_k \gamma_\mu v(P_\alpha;\lambda_\alpha). \tag{15}$$

We note that $T^{\alpha\beta}_k$ in eq. (15) is proportional to the neutrino mass m_k unlike $T^{\alpha\beta}_k$ in eq. (8), which is proportional to the neutrino momentum p_k.

For the lepton number violating process,

$$\pi^+ + p \to \mu^+ + e^+ + n, \tag{16}$$

with μ^+ and e^+ emitted in the source **S** and detector **D**, respectively, the production rate takes the form [12]

$$\Gamma^{\pi^+ p}_{QFT} = \frac{1}{4\pi^2}\ G^4_\beta\ \left(\frac{f_\pi}{\sqrt{2}}\right)^2\ \frac{m^2_\mu}{m_\pi}\ E^2_\nu\ \frac{P^{\mathrm{QFT}}_{\mu\overline{e}}(E_\nu, L)}{4\pi L^2}\ \left(g^2_V + 3g^2_A\right)\ p_e E_e \tag{17}$$

with

$$P^{\mathrm{QFT}}_{\mu\overline{e}}(E_\nu, L) = \left|\sum_{k=1}^{3} U^*_{\mu k} U^*_{ek} \frac{m_k}{E_\nu} e^{-i m^2_k L/(2E_\nu)}\right|^2. \tag{18}$$

We note that the production rate of lepton number violating processes with on-shell relativistic Majorana neutrinos is strongly suppressed due to the factor m_k/E_ν entering in eq. (18). We note that for L=0 we have [17]

$$\mathcal{P}^{\mathrm{QFT}}_{e\overline{e}}(E_\nu, L=0) = \frac{m^2_{\beta\beta}}{E^2_\nu}. \tag{19}$$

If there is a strong suppression of $m_{\beta\beta}$, the observation of processes with neutrino-antineutrino (or vice versa) oscillations might be preferable as it follows from a detailed analysis of $P^{\rm QFT}_{e\bar{e}}(E_\nu, L)$ in [17].

5. – Conclusions

In summary, a new QFT formalism of the neutrino oscillations for processes with and without total lepton number violation is presented. This approach incorporates the neutrino emission and detection in a single second-order Feynman diagram. It is demonstrated that these two processes are entangled in the presented formalism unlike in the QM concept of oscillation probability, which implies complete independence of the neutrino production and detection processes. The entanglement manifests itself as the term $p_k p_m/E^2$ in eq. (11), which differs from unity.

The presented formalism also leads to the expected result for Racah's process involving the oscillation of neutrino into antineutrino (and vice versa). A connection between the effective masses of Majorana neutrinos governing the $0\nu\beta\beta$-decay and neutrino-antineutrino oscillations is established. It is maintained that it is essential to investigate the potential of the neutrino-antineutrino oscillation processes to determine the Majorana nature of neutrinos.

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