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# Forward-backward correlations with the Σ quantity in the wounded-constituent framework at LHC energies

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**Summary.** — This paper discusses the forward-backward (FB) correlations studied with the $\Sigma$ quantity, which, defined as a strongly intensive quantity, is expected to be independent of system volume and volume fluctuations. The study compares the $\Sigma$ observable calculated in the wounded nucleon model (WNM) and wounded quark model (WQM) with the most commonly used heavy-ion physics Monte Carlo generators and finds that the wounded-constituent approach performs better in describing experimental data. However, in this framework, the quantity $\Sigma$ is no longer strongly intensive. The findings also show that $\Sigma$ can be used to determine the fragmentation function of a wounded constituent with a novel method that is also applicable in symmetric nucleus-nucleus collisions.

## 1. – Forward-backward correlations with Σ quantity

This paper is based on ref. [1], where more details can be found. Ultrarelativistic heavy-ion collisions offer a unique opportunity to examine a new state of matter called quark-gluon plasma (QGP) that emerges on the brief moment before the quarks and gluons recombine again into hadrons, detected later in the experimental apparatus. The fleeting nature of QGP makes it challenging for physicists to identify reliable observables that can provide insights into this early phase of matter under extreme conditions. Over the years, researchers have developed a powerful toolkit to study these early phases of heavy-ion collisions, which involves analyzing correlations and fluctuations.

For decades, *forward-backward multiplicity correlations* have been a key technique used in high-energy physics to study particle correlations. Recently, there has been significant interest in studying particle multiplicity correlations using so-called *strongly intensive quantities*. Strongly intensive quantities were first introduced to heavy-ion physics in ref. [2] as a solution to the problem of the misleading impact of system volume (centrality) fluctuations. These fluctuations spuriously affect the observed values of physical variables, such as the multiplicity correlation coefficient, as noted in refs. [3, 4].

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Sets of strongly intensive quantities were constructed within the framework of the independent source model (ISM) in such a way as to provide direct information about characteristics of sources producing particles and to not depend on *system volume and its fluctuations*. The ISM is a superposition framework within which particles are produced independently from statistically identical sources. Derived in terms of ISM, a strongly intensive quantity containing the correlation term is called the $\Sigma$. In the context of forward-backward multiplicity studies, the $\Sigma$ is defined by the formula given in eq. (1) as a combination of FB covariance $\mathrm{Cov}(N_B, N_F)$, forward (backward) scaled variance $\omega_{F(B)} \equiv \frac{\mathrm{Var}(N_{F(B)})}{\langle N_{F(B)} \rangle}$, and mean of multiplicity distribution $\langle N_{F(B)} \rangle$.

$$\Sigma = \frac{\omega_B \langle N_F \rangle + \omega_F \langle N_B \rangle - 2\mathrm{Cov}(N_B, N_F)}{\langle N_F \rangle + \langle N_B \rangle}. \tag{1}$$

The ALICE Collaboration recently conducted the first-ever measurement of forward-backward correlations with the $\Sigma$ variable [3-6]. The outcome of this study suggests that $\Sigma$ behaves like a strongly intensive quantity, showing no dependence on the centrality bin width within different centrality classes or the centrality estimator. This means that $\Sigma$ is not affected by volume fluctuations or by the method used to select centrality, setting it apart from the forward-backward correlations coefficient, which is known to be strongly influenced by these factors.

Further analysis by ALICE explored $\Sigma$'s behaviour across various colliding systems and centrality ranges, revealing an increase in $\Sigma$ values with both the energy and percentage of centrality, meaning they grow as the collisions move from central to peripheral events in the experimental data. Notably, the increase in $\Sigma$ with collision energy was consistent across all types of studied nucleus-nucleus collisions. However, the trend of $\Sigma$ with centrality was found to be opposite in larger collision systems, such as Pb–Pb and Xe–Xe, when compared to pp interactions in ref. [4].

Theoretical models that are widely used to describe heavy-ion collisions, such as HIJING [7], EPOS [8], and AMPT [9], are facing difficulties in accurately predicting $\Sigma$ observable in Pb–Pb and Xe–Xe collisions. While AMPT and EPOS fail to describe experimental data quantitatively, HIJING fails to describe it qualitatively. The observed discrepancies between the model's predictions and the actual data indicate a significant gap in our understanding of the underlying processes in heavy-ion collisions refs. [3-5].

This paper discusses the FB correlations with $\Sigma$ in the framework of the wounded-constituent model. The research examines the properties of the $\Sigma$ quantity in the framework of the wounded nucleon model and wounded quark model refs. [10-14]. This analysis was primarily inspired by the studies outlined in refs. [15-17].

## 2. – Forward-backward correlations in wounded constituent framework

The first iteration of the wounded constituent framework, called *the wounded nucleon model*, was postulated over 45 years ago in ref. [10]. According to this model, a nucleus-nucleus collision is a combination of individual nucleon-nucleon interactions. A 'wounded nucleon' refers to a nucleon that has undergone at least one inelastic collision. The model assumes that each wounded nucleon is equally capable of emitting particles, regardless of the number of collisions it has sustained. The model also adopts a universal fragmentation function, denoted as $F(\eta)$, which governs the particle emission profile for each wounded nucleon. It is important to note that the application of fragmentation function $F(\eta)$ is not

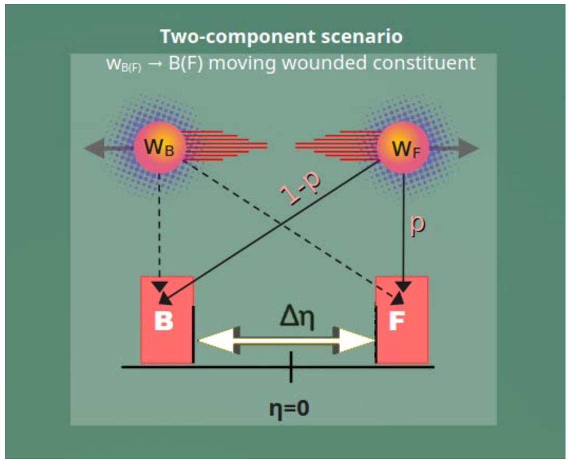


Fig. 1. – The figure illustrates the forward - moving wounded source producing particles into producing particles of forward F=$(\Delta\eta/2,\ \Delta\eta/2+\delta\eta)$ and backward B=$(\Delta\eta/2-\delta\eta,\ -\Delta\eta/2)$ pseudorapidity intervals located symmetrically around $\eta=0$, with probability $p$ and $1-p$, respectively. In this analysis, the widths of the F and B pseudorapidity intervals are assumed to be $\delta\eta = 0.2$.

limited to the nucleon's hemisphere and can project particles into both the forward and backward pseudorapidity regions. Hence, a single particle pseudorapidity distribution $N(\eta) \equiv \frac{dN(\eta)}{d\eta}$ of produced particles can be written as:

$$N(\eta) = \langle w_F \rangle F(\eta) + \langle w_B \rangle F(-\eta), \tag{2}$$

In eq. (2), the symbol $\langle w_{F(B)} \rangle$ stands for the average number of forward (backward)-moving, wounded nucleons (see ref. [11]).

*The wounded quark model* naturally builds upon the wounded nucleon framework by proposing that quark-quark interactions rather than nucleon collisions determine particle production. The wounded quark model shares assumptions of the wounded nucleon model. However, the specific characteristics, such as the fragmentation function or the number of forward (backward)-moving wounded sources, now refer to the wounded quarks instead of the wounded nucleons.

The author has recently determined *the formula for the forward-backward* $\Sigma$ *quantity* in an arbitrary nucleus-nucleus collision within the framework of the wounded nucleon and wounded quark models. The expression takes into account the distinction between two types of sources: forward-moving and backward-moving wounded constituents, as illustrated in fig. 1. This two-component scenario assumes that the wounded constituent in a forward (backward) moving nuclei emits a particle into the forward (backward) $\eta$ bin with probability $p$, and into the backward (forward) $\eta$ bin with probability $1-p$. The values of probability $p$ depend on the position of pseudorapidity F and B intervals, and the fragmentation function $F(\eta)$ of the wounded constituent as follows:

$$p = \frac{\int_{F(B)} F(\eta)\, d\eta}{\int_B F(\eta)\, d\eta + \int_F F(\eta)\, d\eta}. \tag{3}$$

In the wounded constituent framework the expression for $\Sigma$ for a symmetric nucleus-nucleus collision, where $\langle w_F \rangle = \langle w_B \rangle$, takes the following form:

$$\Sigma = 1 + (2p-1)^2 \frac{\overline{n}}{2} \left[ \frac{\langle (w_B - w_F)^2 \rangle}{2\langle w_F \rangle} + \frac{2}{k} \right] \tag{4}$$

Symbols $\overline{n}$ (average multiplicity) and $k$ (the deviation from a Poisson distribution) are the parameters of the negative binomial distribution (NBD) according to which the wounded constituent is producing particles to the sum of F+B $\eta$ intervals. For the energies $\sqrt{s_{\mathrm{NN}}} = 2.76$, 5.02 and 5.44 TeV these two parameters were interpolated based on the experimental pp data given in refs. [18, 19].

It is immediately evident from eq. (4) that in the wounded constituent model:

- the characteristics of the forward-backward $\Sigma$ quantity are sensitive to the value of the probability $p$;
- when the probability of a wounded source producing particles in forward and backward pseudorapidity intervals is equal to $p = 0.5$, the value of $\Sigma$ becomes unity;
- for $p = 0.5$ the two types of particle-producing sources (forward- and backward-moving) merge into one type of wounded constituent and the WNM and WQM meet the assumptions of the ISM. In this particular case $\Sigma$ is a strongly intensive quantity, as defined in ref. [2];
- for $p \neq 0.5$, the quantity $\Sigma$ is larger than unity ($\Sigma > 1$) and shows an intrinsic dependence on the number of wounded nucleons (quarks); in this scenario, the properties of the strongly intensive quantities of $\Sigma$ collapse.

Despite eq. (4) showing that $\Sigma$ is not a strongly intensive variable in a wounded-constituent framework, it is interesting to comment here that it appears to be independent of the width of the centrality interval and on the method of centrality selection. In fact, the behaviour predicted by WNM and WQM of $\Sigma$ for centrality classes of 10% width in symmetric nucleus-nucleus collisions is similar to what has been observed in ALICE experimental data, ref. [3]. It is worth mentioning that the explanation for this behaviour comes from the fact that eq. (4) can be rewritten in terms of partial covariance with the effect of the control random variable (the fluctuation of the total number of wounded nucleons $w = w_F + w_B$ ) reduced; this is discussed in detail in ref. [1].

## 3. – Results and discussion

Figure 2 shows the results on forward-backward correlations, represented by the $\Sigma$ quantity, in WNM and WQM for Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV across various centrality classes, ranging from $0-10\%$ to $70-80\%$. The $\Sigma$ values in the wounded constitution framework, obtained from the analytical expression eq. (4), were juxtaposed with the experimental results for Pb–Pb collisions obtained by the ALICE detector at the LHC. Results for ALICE have been redrawn from ref. [6]. They are categorized based on different separation gaps between forward and backward intervals $\Delta\eta$, while theoretical predictions were determined for several values of probability $p$. The values of parameter $p$ were selected in such a way as to provide the best fit between experimental results and models.

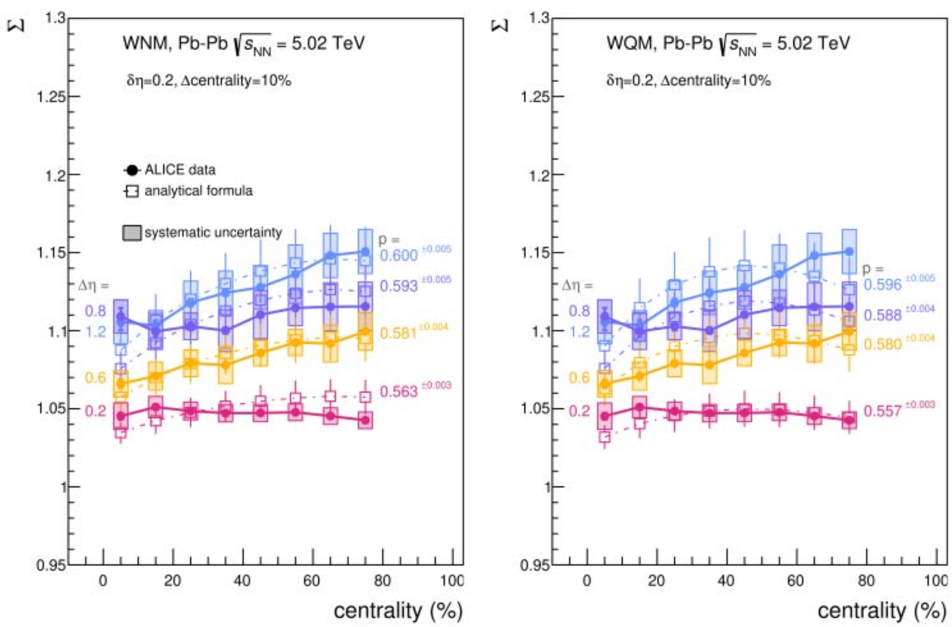


Fig. 2. – The centrality dependence of the Σ quantity for Pb–Pb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV in WNM and WQM. Results are plotted for selected fixed values of probability $p$. Theoretical predictions for each panel are compared with experimental results from ALICE, taken from ref. [6], for various values of the pseudorapidity gap $\Delta\eta$. The uncertainty bars marked for the models represent the total uncertainty.

Referring to the behaviour of data points shown in fig. 2, the following conclusions can be drawn:

1) As the probability value $p$ approaches 0.5, the value of Σ tends to unity in WNM and WQM;

2) For $p \neq 0.5$, a trend with centrality can be observed, namely an increase in value of Σ with the transition from central to peripheral collisions. This trend with centrality is more pronounced for larger values of $p$ and shows a monotonic behaviour in WNM.

3) The WNM and WQM accurately represent the trend of Σ with centrality observed in the experimental data in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. This performance is notably superior to other widely used heavy-ion collision models, such as HIJING, AMPT, or EPOS (see ref. [4, 6]). This observation is also true for Pb–Pb at $\sqrt{s_{NN}} = 2.76$ TeV and Xe–Xe at $\sqrt{s_{NN}} = 5.44$ TeV. Interested readers are encouraged to consult ref. [1] for a detailed discussion.

4) The results of both the WNM and WQM suggest (see eq. (4)) that the increase in Σ observed with the transition from central to peripheral collisions is the result of an intrinsic dependence of Σ on the size of the system (the number of wounded nucleons), rather than a difference in the characteristics of the source producing the particles, which remain unchanged with the centrality of the collision in both WNM and WQM.

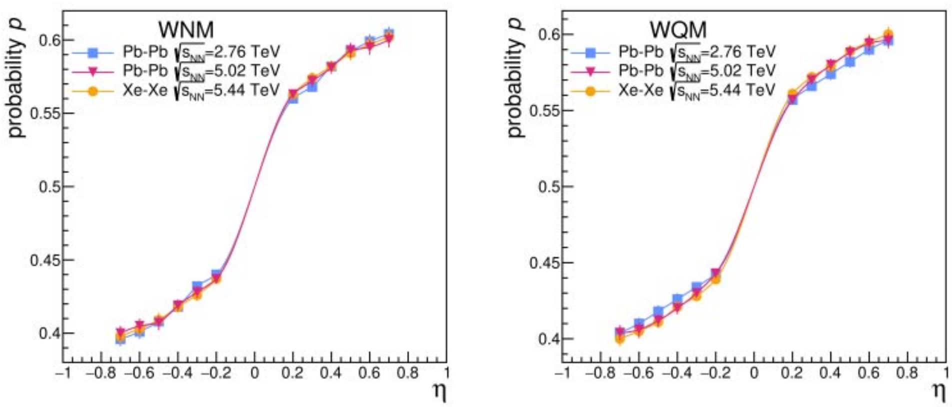


Fig. 3. – Probability $p$ as a function of pseudorapidity obtained in WNM and WQM for Pb–Pb and Xe–Xe collisions.

5) There is an evident dependence of $\Sigma$ on the probability value $p$ as predicted by eq. (4). In the comparison of WNM and WQM with experimental data, it was established consequently that the value of $p$ changes with pseudorapidity.

Figure 3 shows *the values of probability* $p$ extracted from the experimental data on $\Sigma$ quantity for three different collision systems, namely Pb–Pb at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV and Xe–Xe at $\sqrt{s_{NN}}= 5.44$ TeV. The probability $p$ is drawn as a function of pseudorapidity for WNM and WQM. The study found a consistent pattern in the probability values ($p$) across different collision energies and types, revealing a universal behaviour in pseudorapidity. Perhaps the most interesting finding of this study is that estimating the probability $p$ as a function of pseudorapidity from $\Sigma$ measurements makes it possible to determine *the fragmentation function* of a wounded nucleon or wounded quark in a new way.

So far, the fragmentation function was deduced from the measurement of the single particle pseudorapidity distribution, $N(\eta)$ based on the relation eq. (5).

$$F(\eta) = \frac{1}{2}\left(\frac{N(\eta)+N(-\eta)}{\langle w_F\rangle + \langle w_B\rangle} + \frac{N(\eta)-N(-\eta)}{\langle w_F\rangle - \langle w_B\rangle}\right). \quad (5)$$

The latter formula is derived directly from eq. (2), and it is only applicable for asymmetric nucleus-nucleus collisions, where $\langle w_F\rangle \neq \langle w_B\rangle$. For symmetric collisions ($\langle w_F\rangle = \langle w_B\rangle$), the denominator of the second term of the formula is zero, and the expression on $F(\eta)$ becomes undefined.

*A new approach* to determining the fragmentation function is based on the understanding that the $\Sigma$ observable, within the framework of the wounded constituent model, directly reflects the probability value $p$. This probability is closely linked to the fragmentation function of individual constituents (nucleons or quarks), as defined in eq. (3). By combining eqs. (2) and (3), we can establish the following expression on $F(\eta)$ for a

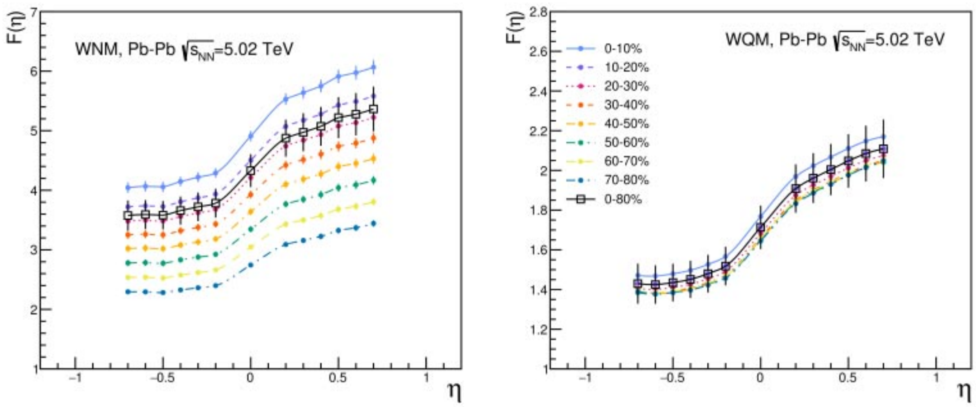


Fig. 4. – The wounded nucleon and quark fragmentation functions extracted from symmetric Pb–Pb collisions at $\sqrt{s_{\rm NN}}$=5.02 TeV as a function of $\eta$ for different centrality classes.

sufficiently narrow $\eta$ interval:

$$F(\eta) \approx \frac{p}{\langle w_F \rangle + \langle w_B \rangle} \left( N(-\eta) + N(\eta) \right). \tag{6}$$

The advantage of the above new method is that it can be used to determine the fragmentation function in both asymmetric and symmetric collisions.

Figure 5 shows the first-ever attempt to derive the fragmentation function for a wounded nucleon and wounded quark in a symmetric Pb–Pb collision at $\sqrt{s_{NN}} = 5.02$ TeV based on the relation given by eq. (6). Results were obtained as a function of $\eta$ for different centrality classes. The figure depicts the difference in the behaviour of fragmentation function observed for WNM and WQM. In the case of wounded nucleons, the shape of the fragmentation function clearly varies with centrality. On the other hand, the fragmentation function for wounded quarks displays a consistent shape and values across all centrality classes. These observations correspond with previous results from asymmetric d+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, where a more uniform behaviour of the wounded quark fragmentation function was noted.

## 4. – Summary

This study investigates forward-backward correlations using the $\Sigma$ quantity in heavy-ion collisions at LHC energies within the wounded nucleon model and wounded quark model. It finds that $\Sigma$, dependent on the number of wounded constituents, does not remain a strongly intensive quantity in these models. Despite this, in symmetric collisions, $\Sigma$ shows resilience to volume fluctuations and centrality selection methods due to its link to partial covariance, aligning with observations by the ALICE Collaboration. Furthermore, the study shows that wounded constituent models describe the ALICE data on $\Sigma$ quantity significantly better than models like HIJING, AMPT, or EPOS. Notably, it reveals $\Sigma$'s sensitivity to the particle emission probability $p$ by a wounded source, which results in a new method of determination of the fragmentation function not only in asymmetric but also in symmetric nucleus-nucleus collisions.

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