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A crystallographic method to investigate the presence of cluster states in ^{12}C ground state

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Summary. — This work explores the investigation of medium-energy angular distribution literature data for p + ^{12}C elastic scattering. The goal is to highlight the possible existence of α-cluster structures in the ground state of the carbon nucleus; this matter is approached via Coupled-Channel (CC) calculations, which describe the ^{12}C nucleus through a classic spheroidal-like structure, along with a further contribution, which explicitly takes into account the possible existence of a triangular α-cluster structure for the nucleus, treated similarly to the X-ray diffraction by molecules. The results point towards a rather small cluster component in the ground state of the carbon nucleus: an upper limit of $\simeq 1\%$ is shown with a 99.75% confidence level and an inter-cluster distance of $\simeq 3.9$ fm, compared, in the end, with various theoretical predictions reported in the literature.

1. – Introduction

The structure of light nuclei has been one of the attention centers of the nuclear physics field in the last decades, especially when it comes to α-clustering phenomena. These are very likely to occur in the structure of the ground or excited states of self-conjugated nuclei, such as ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg [1-7], with clear and well-known examples given by the ground state of ^{8}Be and the Hoyle state in ^{12}C [8-13]. The study of these configurations may have paramount importance, since those could lead to important changes in the description of rooted nuclear structure and nuclear astrophysics concepts [14-16].

In light nuclei, clustered and deformed structures may arise from complications of the behaviour of the nuclear force, which lead to deviations from the classical spherical structure, either because of collective motions (which cause *axial deformations*) or of rearrangements in discrete sub-units, such as α-particle–like clusters; the insurgence of

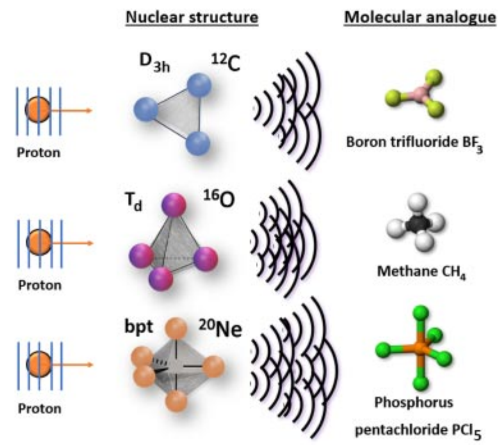

Fig. 1. – Schematic representations of the diffraction phenomena which would be given by an impinging proton beam of suitable wavelength, when the target nucleus (^{12}C, ^{16}O or ^{20}Ne) is depicted as the molecular analogue shown on the right-hand side of the picture.

such structures may exhibit some effects in the interpretation of nuclear fusion phenomena [17, 18]. Some kinds of deformed or atypical structures are already known, such as in nuclei that exhibit structures with a nucleonic core surrounded by a neutron halo or skin (observed for some isotopes of He, Li, Be and C), nuclei with α-cluster structures, such as some states of the aforementioned self-conjugated nuclei [19, 20], or *linear-chain* molecular-like structures, in which the α-like nuclear particles are bond together by excess neutrons. The latter is, of course, the case of neutron-rich light isotopes such as ^{10}Be, $^{13-14}$C or ^{16}C [21-24].

The main idea behind this work would be to exploit the fact that, similarly to atoms in molecules, α-like clusters inside the nuclei could exhibit geometrical and symmetrical structures, as described by the Group Theory for crystals and molecules [25, 26]. Upon this assumption, the ^{12}C nucleus would assume a rather triangular structure, the ^{16}O a tetrahedral one and the ^{20}Ne nucleus would show the α-particles rearranged in a triangular bipyramid, as shown in fig. 1.

The main aim of this work is the study of the α-cluster structure of the ^{12}C nucleus in its ground state; one of the main problems is to disentangle between the co-existence of the spheroidal and cluster structures. If these two can co-exist, it is still unclear which one of them could be the main contributor to the structure of the ground state. Other questions, then, revolve around the possibility to have clearer cluster configurations in excited states or in heavier nuclei. More recently, in the framework of the study of the ^{12}C structure, the Group Theory principles have been applied to clusterized structures such as ^{8}Be and ^{12}C which, respectively, should exhibit symmetries belonging to the Z_2 and D_3h groups. This would mean that the molecular structure inside the ^{12}C nucleus exhibits symmetry elements like a main three-fold axis (C_3), three secondary two-fold axes perpendicular to the main one and a horizontal symmetry plane.

These considerations are made in the framework of the Algebraic Cluster Model (ACM) [27], a now-trending theoretical model, which exploits the Group Theory to depict the mentioned triangular structure for the Hoyle and for the ground states of ^{12}C [28]. In this way, it succeeds in the forecast of the succession of spectroscopic levels by the exploitation of the structure's symmetry properties. The Hoyle state has in fact a structure with a larger radius which, upon the assumptions of the ACM, is seen as a breathing mode of the ground state [28].

Recently, other theoretical works [29] reported previsions based on Monte Carlo Shell Models (MCSM) and Machine Learning approaches for the structures of different states in the spectroscopy of ^{8}Be and ^{12}C; these predictions show the presence of a clustered triangular structure with a relatively minor role in the ground state of ^{12}C. The results shown from [29] are, however, unaligned with previous Antisymmetrized Molecular Dynamic (AMD) calculations reported by ref. [30] in 2007; in this case, in fact, the prevision is way more unbalanced towards the cluster component, which is almost half of the spectroscopic amplitude both in the Hoyle and in the ground states of ^{12}C.

Understanding the occurrence of cluster phenomena in the ground states of light nuclei is fundamental to highlight and investigate the properties of the following excited states, which may be influenced by clustering in the parent ground states; *e.g.*, the α breaking in the ground state of the nucleus may lead to changes in the 3α cluster structures in the other excited 0^+ states. To better understand this matter, it is important to know the real distribution of the nuclear charge density in the ^{12}C ground state, *i.e.*, its structure.

2. – Description of the model

The main hypotheses and assumptions on which the model (aiming to describe the elastic scattering of protons by the ^{12}C nucleus) is based, are the following: 1) the proton beam energies taken into account were in the range from $\simeq$30 to $\simeq$80 MeV, so that compound nucleus effect could be neglected; 2) the ^{12}C, in its ground state, is supposed to oscillate between a main spheroidal shape and a more exotic minor contribution in α-cluster configuration; 3) the energies inside the domain considered in this study would produce matter wave of protons with a wavelength between 3.5 and 5.7 fm, of the same order as the distance that two α-like particles (here called r_{ij}) would have in the carbon cluster configuration, thus producing coherence effects upon scattering of the protons on the α-particles; 4) the α-clusters are supposed to be so weakly bound inside the nucleus that they can be considered as *free* and independent scattering centers; 5) upon the considerations based on the ACM, the cluster configuration for the ground state of ^{12}C is supposed to have a triangular shape; 6) the obtained angular distributions are supposed to arise from the superposition of effects due to the scattering of proton from the spheroidal target (*spheroidal contribution)* and to the effects due to the α-cluster triangular configuration (*coherent contribution)*, neglecting interference effects between the two as a first approximation.

The spheroidal contribution has been treated and computed through an optical model, based on Coupled-Channel calculations, considering the initial parameters and prescriptions reported in [31], coupling together the states belonging to the ground state band of ^{12}C, while neglecting other couplings to the β- and the octupole bands, which would have minor effects on the elastic scattering angular distributions. In this framework, the optical model potential (U) used to describe the scattering from the spheroidal structure is composed by a real part (V), which takes into account for the elastic scattering, and an imaginary part (W), devoted to account for the absorbed intensity of the matter wave, as prescribed by the systematics by Becchetti and Greenless [32]. Considering the light nature of the ^{12}C target nucleus, and upon confirmation by a χ^2 reduction analysis, the data were reproduced by a surface imaginary potential alone, as reported in [31].

On the other hand, the angular distributions for the coherent contribution, *i.e.*, the diffraction intensity $I_{clust}(\theta)$ have been computed through the Wierl crystallographic formula,

$$I_{clust}(\theta) = \sum_i |f_i|^2 + 2 \cdot \sum_{i,j}^{i \neq j} f_i f_j \cdot \frac{\sin(\mu r_{ij})}{\mu r_{ij}}, \tag{1}$$

which relates the diffraction intensity to the scattering amplitude of a single α-cluster, $|f_i|^2$ and to the diffraction effects given by two neighboring clusters, assuming an equilateral triangle α-cluster structure [25,26], with μ resuming the wave parameters [33]. To calculate the $|f_i|^2$ factor, data for the p $+^4$He elastic scattering have been downloaded (from the EXFOR database, refs. [26, 34-40]) and fitted with the same optical model prescription used for the ^{12}C nucleus, in the same angular and energy ranges, to obtain the f_i value as a function of E_p and θ.

Upon the approximation of non-interfering terms, the contributions coming from the *spheroidal* (optical model) and the *coherent* (cluster model) scatterings may be added, each one weighted for its Spectroscopic Factor, A or B,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{exp}}(\theta) = A \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{sph}}(\theta) + B \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{clust}}(\theta); \tag{2}$$

since the presence of other phenomena is also neglected, it must hold true that $A+B=1$. The fitted parameters then were: 1) the *Imaginary Surface Depth*, 2), the radii (R) and diffusenesses (a) for both the real and imaginary parts of the optical potential; 3) A and B, and 4) the *inter-cluster distance*, r_{ij}. The parameters were thus allowed to vary no more than 30% of their original values, reported in [31], inside a new fitting procedure which couples diffractive and Coupled-Channel contributions, applied to p $+^{12}$C elastic scattering data. More details on the model and the fitting procedure are reported in [41].

3. – Results

After the estimation of the $|f_i|$ factor from the cubic spline interpolation of p $+^4$He elastic scattering data, p $+^{12}$C elastic scattering literature data, collected in the EXFOR database, were fitted in the bombarding energy range from $\simeq$30 to $\simeq$80 MeV and from 0° to 170°, successfully reproducing previous fits performed in the 30–40 MeV region; these calculation were then extrapolated to the full energy range of the presently available data. The fit appears to be in optimal accord with the experimental data up to 40°; at larger angles, especially in the backward hemisphere, sizeable deviations start to arise. To improve the description of the broad range dataset, the fit has been upgraded and calculated through a dedicated fit function based on the CHUCK code, also including the reaction cross section data for the p $+^{12}$C collision, previously collected and reported in [42]. Results obtained from the new Coupled-Channel fit are shown in fig. 2, together with the literature data and the results obtained using the original parameters from [31], in an improved fit. The figure also shows the coherent term $B\sigma_{clust}$ alone; the fit shows a clear improvement with respect to the description given by the previous parameters, with a rather small effect coming from the inclusion of the cluster contribution, and a light effect on the reduction of the χ^2, lowered by $\simeq$ 10%.

A similar light effect can be seen in the normalized residuals distribution (defined as the normalized difference between the theoretical and the experimental value), which

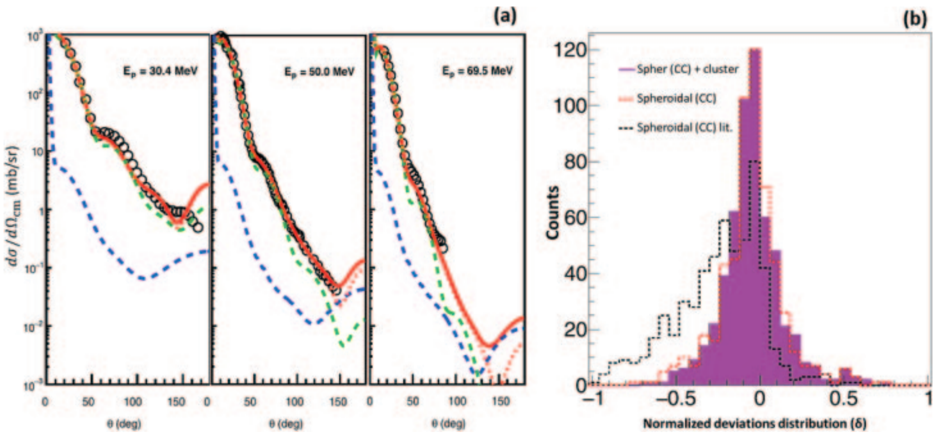

Fig. 2. – (a) p $+^{12}$C elastic scattering angular distributions at three sample energies, in the lower, medium and higher domains. Black dots: experimental data. Red dashed lines: predictions of Coupled-Channel calculations, with optimized parameters obtained from the fit with the spheroidal. Green dashed line: Coupled-Channel calculations with the parameters of ref. [31]. Red solid line: global fit including the spheroidal and coherent terms contributions. Blue dashed line: coherent contribution $B\sigma_{clust}(\theta)$ obtained from the global fit. Data from [34, 35, 38]. (b) Normalized deviations (δ) distribution between experimental data points and theoretical calculations at the various bombarding energies. Black dashed line: CC calculations with literature [31] parameters. Red dashed line: CC calculations with parameters fitted to data (see ref. [41] for details). Purple shaded histogram: comprehensive calculations taking into account both the spheroidal (CC) and coherent (diffraction) contributions with parameters fitted to data.

shows a slightly lower Full Width at Half Maximum (FWHM) when including the contribution from the cluster model. From this analysis, it is possible to extract, within a 99.75% confidence level, an upper limit of 1.0% for the B, the spectroscopic factor for the cluster component, and an inter-cluster distance value of $r_{ij} = 3.9 \pm 0.5$ fm. These values are of the same order of magnitude as the previous and most recent estimates of [29], pointing towards a minor influence of the cluster component wave function in the ground state. Interesting questions that may arise from these results are the eventual existence of a clear correlation between r_{ij} and B, and the nature of the α-clustering phenomena, in ^{12}C and in similar nuclei, as a *threshold* process.

In the future, these analysis could be implemented upon availability of new data with higher angular resolution, and, of course, by including interference effects between the wave scattering due to the spheroidal and the cluster component. Moreover, similar models may be applied to other self-conjugate nuclei, which may exhibit analogue geometrical arrangements of α-particles, like tetrahedral or kite-like structures; another important upgrade of the model would be the possibility to include neutron-scattering data, which would not be hindered by the Coulomb repulsion.

4. – Conclusions

In this proceeding, a new analysis technique which aims to probe the occurrence of cluster structures in the ground states of ^{12}C and other neighboring self-conjugate nuclei through elastic scattering data has been described. The main idea is that, in the presence of α-clustering dispositions, diffractional effects may play an important role if the wavelength of the incident wave is of the same order of the characteristic distance between different α-clusters inside the nucleus, leading to a different pattern with respect to that obtained upon the scattering on a spheroidal structure.

The implemented model was quite simple, and based on Coupled-Channel calculations, which describe the scattering on a spheroidal structure (as the ^{12}C nucleus is seen in a classical framework) through an optical model, combined with the Wierl crystallographic formula to take into account the diffraction scattering given by the molecular-like arrangement. This model was exploited to analyze p $+^{12}$C elastic scattering differential cross section data in a very broad bombarding energy range, approximately from 30 to 80 MeV; after a thorough analysis, it was possible to estimate a value of $r_{ij} = 3.9 \pm 0.5$ for the inter-cluster distance and an *upper limit* for the occurrence of clustering phenomena in the ground state of ^{12}C of $\approx 1.0\%$, within a 99.75% confidence level. These values are found to be in agreement with the most recent theoretical calculations, highlighting a rather minor role of α-clustering phenomena in the definition of the structure of the ground state of the ^{12}C nucleus.

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