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Toward a better characterization of the nuclear EOS using central collisions around Fermi energy

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Summary. — This study presents a novel approach to characterize the nuclear equation of state (EOS) using central heavy-ion collisions in the Fermi energy domain. We analyze experimental data from Nickel-Nickel collisions at 52–90 MeV/nucleon, recorded with the INDRA 4π array at GANIL, employing Artificial Intelligence (AI) and Machine Learning (ML) techniques. Our methodology introduces a neural-network–based reconstruction of the impact parameter, trained on HIPSE and ELIE simulations, achieving sub-femtometer accuracy. This enables precise selection of central collision events for in-depth analysis. We then implement a Bayesian inference framework to estimate in-medium nucleon-nucleon cross-sections, utilizing Kolmogorov-Smirnov probabilities calculated over an extensive set of global observables. Preliminary results demonstrate consistency with previous phenomenological studies, particularly for reactions below 100 MeV/nucleon. The Bayesian approach provides both mean cross-section values and associated uncertainties, offering a more comprehensive characterization of nuclear medium effects. We discuss ongoing efforts to extend this methodology to estimate average and maximum densities reached in collisions, as well as plans to investigate the isospin dependence of the EOS using recent INDRA data. These advancements aim to provide improved constraints on the nuclear EOS across a range of densities and isospin asymmetries, contributing to our understanding of nuclear matter properties in both terrestrial experiments and astrophysical contexts.

1. – Introduction

Nuclear equations of state (EOS) play a fundamental role in physics, describing the thermodynamic behavior of complex systems [1]. In the field of nuclear physics, these equations are particularly important for understanding the behavior of nuclear matter under extreme conditions. The nuclear matter EOS establishes the relationship between the energy per nucleon, density, temperature and proton-neutron asymmetry, thus providing crucial information about the properties of nuclear matter under various conditions.

The study of the EOS is essential not only for the fundamental understanding of nuclear physics but also for its implications in various fields of astrophysics. It plays a crucial role in describing phenomena such as the structure of neutron star, supernova mechanisms, and the dynamics of neutron star collisions. However, despite its importance, the precise determination of EOS parameters remains a major challenge for the scientific community.

To characterize the EOS, a Taylor expansion around the saturation density ρ_0 is generally used for the isoscalar and isovector terms of the EOS:

$$
\begin{aligned}
E(\delta,\rho) &= E_{iso}(\rho) + E_{vec}(\rho)\delta^2 + \mathcal{O}(\delta^3),\\
E_{iso}(\rho) &= E_{sat} + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \mathcal{O}(\rho^3),\\
E_{vec}(\rho) &= E_{sym} + L_{sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \mathcal{O}(\rho^3).
\end{aligned}
$$

This expansion reveals several key parameters, including the symmetry energy E_{sym}, its slope L_{sym}, and its curvature K_{sym}, as well as isoscalar parameters such as the incompressibility modulus K_0. These parameters, although crucial, are only known with limited precision, with uncertainties ranging from $\pm 10\%$ to $\pm 100\%$ for some of them [1].

The main objective of this work is to provide new constraints on these EOS parameters using innovative approaches based on artificial intelligence (AI) and machine learning (ML). We focus on the analysis of central heavy-ion collisions around Fermi energy, using experimental data recorded with the INDRA detector at GANIL. An example of this study is shown in fig. 1.

It is a Gaussian emulation (non-parametric regression) of the symmetry energy on the experimental data gathered from the literature represented here by the blue circles. The symmetry energy $S(\rho)$ is the energy cost per nucleon for changing a symmetric $(N = Z)$ nuclear system at a given baryonic density ρ to a pure neutron matter $(N = A)$ system. This study shows that as one explores a wider range of densities, the better

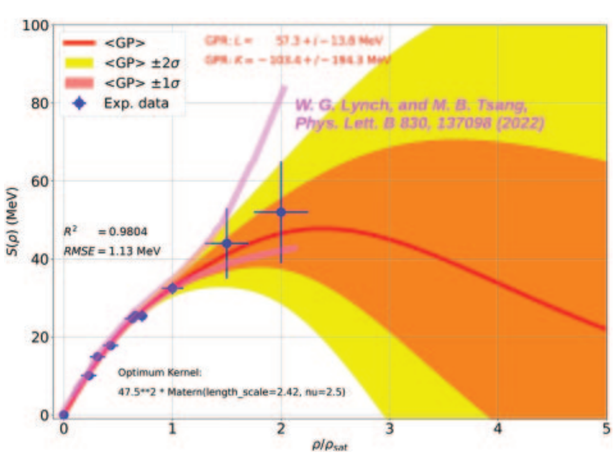

Fig. 1. – Gaussian emulation of the symmetry energy $S(\rho)$ with the density ρ made by Lopez and presented during NuSym2023.

is the constraint on K_{sym} and L_{sym}. From this study, the values found by using the Gaussian processes: $L_{sym} = 57.3 \pm 13.8\,\mathrm{MeV}$ and $K_{sym} = -103 \pm 194\,\mathrm{MeV}$.

This study will serve us as a guide to improve the constraints. Thus, our approach aims to combine several advanced techniques:

1) The use of neural networks to reconstruct the impact parameter of collisions, allowing precise selection of central events.

2) The exploitation of machine learning methods to analyze a wide set of global variables and extract the likelihood (error) between a simulation and the experiment.

3) The application of Bayesian analysis to estimate nucleon-nucleon cross-sections in the nuclear medium, a crucial parameter for modeling nuclear reactions.

By combining these techniques with the high-quality experimental data provided by INDRA [2], we aim to bring new constraints on the EOS, particularly on its density and isospin asymmetry dependence. This work is part of a broader effort to improve the understanding of nuclear matter, from atomic nuclei to compact astrophysical objects.

In the following sections, we will detail our AI and ML-based methodologies. We will then discuss our preliminary results and their implications for nuclear physics and astrophysics, before concluding on the perspectives opened by this innovative approach.

2. – Machine learning approach for impact parameter reconstruction

2˙1. *Neural network architecture*. – We used a neural network to reconstruct the impact parameter of Ni-Ni collisions between 32 and 50 MeV/nucleon. The architecture consists of an input layer (features like kinetic energy, particle multiplicities, and charge distributions), hidden layers with ReLU activation for nonlinearity, and a single neuron output layer to predict the continuous impact parameter value. Training preparation follows the network setup.

2˙2. *Training with* HIPSE *and* ELIE *simulations*. – The neural network was trained using HIPSE (Heavy Ion Phase Space Exploration) [3] and ELIE simulations [4].

For each system at a given energy, a different neural network is required. The simulations provide a large set of collision events covering various impact parameters. The training process involves:

- *Data preparation*: Normalizing simulation data and splitting it into training, validation, and test sets.

- *Training*: Using backpropagation and the Adam optimizer to minimize the mean squared error (MSE).

- *Hyperparameter tuning*: Optimizing learning rate, batch size, and hidden layers via grid search.

2˙3. *Results and accuracy assessment*. – The neural network's performance was evaluated on the test set using two metrics:

- *Mean Absolute Error (MAE)*: Measures the average prediction error [5].

- *R-squared (R^2) score*: Indicates how well predictions match actual values, with higher scores reflecting better performance [6].

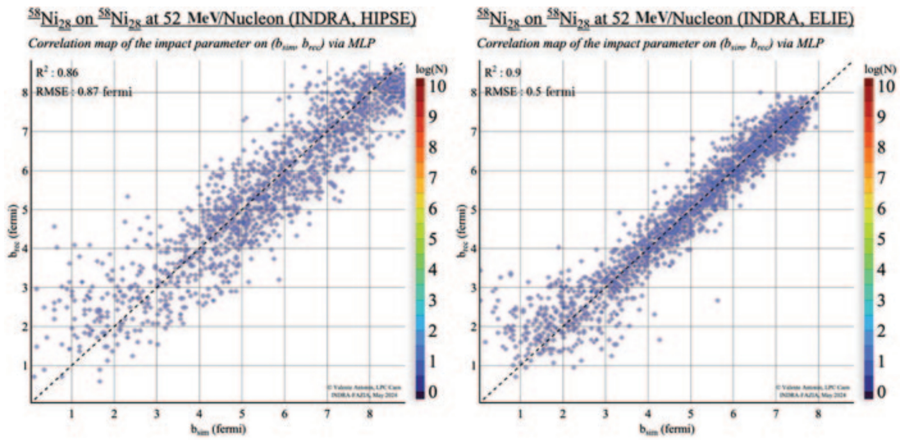

Fig. 2. – Correlation plot of the impact parameter of the simulation *vs.* the reconstructed and simulation impact parameter for HIPSE (left) and ELIE (right).

We also look at the correlation between the reconstructed impact parameter on the simulation dataset and the native one in fig. 2. We can see that the correlation is linear which means that the model is reproducing well the impact parameter of the simulation. Futhermore the RMSE estimating the error of the model is less than a 1 fm.

The results highlight the effectiveness of the machine learning approach in accurately reconstructing impact parameters, offering a valuable tool for further analysis of nuclear collisions and the characterization of the nuclear EOS.

2˙4. *Centrality classes*. – After validation, we examine the impact parameter distribution in the experimental dataset. Centrality classes, defined by the impact parameter, categorize collision events by the degree of nuclear overlap, helping in the study of nuclear matter dynamics.

We analyze charge *vs.* parallel velocity distributions to gain insights into reaction mechanisms and nuclear matter properties. As shown in fig. 3, central events exhibit emission near the center of mass, while increasing impact parameters reveal distinct quasi-projectile and quasi-target emissions. Peripheral events show only quasi-projectile, characteristic of elastic scattering events.

With this representation we can see that the reconstruction is reproducing behaviors expected in a centrality analysis. So, we are going to use this selection in the rest of the work.

3. – Bayesian inference of in-medium nucleon-nucleon cross-sections

We now turn to the Bayesian inference of in-medium nucleon-nucleon (NN) cross-sections. For the simulation, we sample the NN cross-section with a uniform probability distribution between 0 and 100 mb. This interval is divided into 20 bins, and for each bin, we calculate the Kolmogorov-Smirnov (KS) statistics.

To estimate the cross-section, we combine the probability distributions from all observables. For each observable, we obtain a probability distribution for the cross-section value. The goal is to calculate the expectation value of the cross-section across all observables.

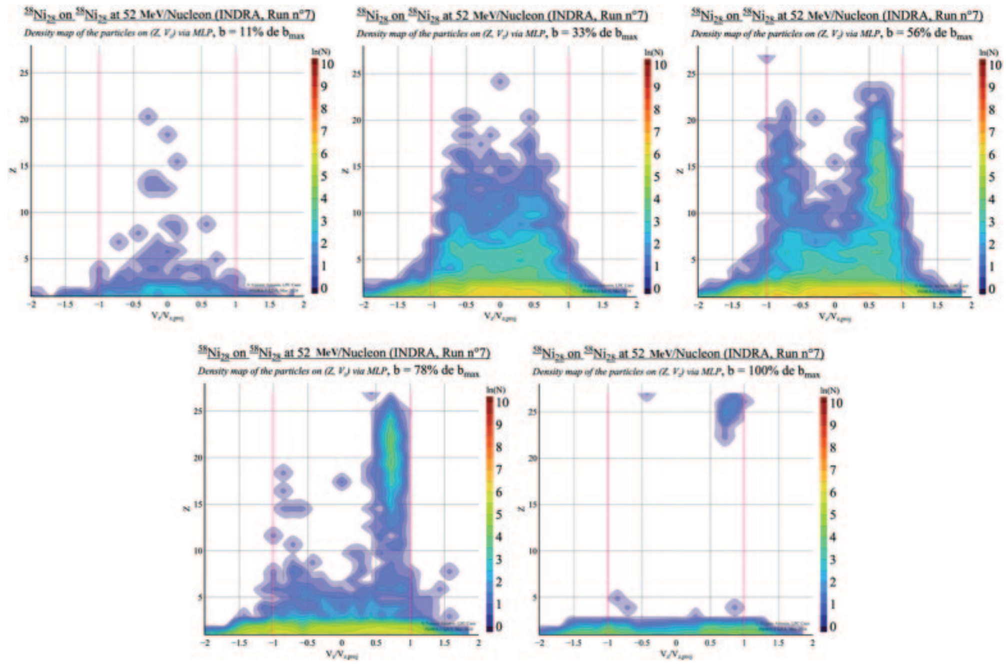

Fig. 3. – Plots of atomic numbers *vs.* parallel velocity in the center of mass for different slices of reconstructed impact parameter on experimental dataset, going from central events (top left) to peripheral ones (bottom right).

The average in-medium nucleon-nucleon cross-section is computed by taking the expectation value over all Kolmogorov-Smirnov probability distributions. For each observable, the expectation value of the cross-section is given by integrating its probability distribution, weighted by the cross-section itself.

Next, we compute the uncertainty, or standard deviation, of the in-medium cross-section. The standard deviation is calculated for each observables probability distribution, and the overall uncertainty is obtained by averaging these standard deviations.

This Bayesian approach enables us to determine both the mean in-medium cross-section and the associated uncertainty, providing a comprehensive statistical analysis of the cross-section based on the experimental data distributions. We then plot for each system this analysis on the figure showing the evolution of NN-cross-sections with incident energy taken from [7] on fig. 4.

The solid curves are from [7]. The first curve (blue) represents the free nucleon-nucleon cross-section and the second one (green) represents the first curve corrected with Pauli blocking effect [7,8]. The third curve (red) is the nucleon-nucleon cross-section with in-medium effect proposed by [9]. On it, we put the results from the Bayesian analysis (black stars). The Ni-Ni results are shown here. Those results are in good agreement with in-medium effect on the cross-section for the upper range in energy (> 50 MeV/nucleon). For the lower range the uncertainties are quite large and we cannot conclude if the cross-section is governed by only Pauli blocking or with in-medium effects. We suspect that the model used here (ELIE) is not well suited for this range of energy below 50 MeV/nucleon and that the use of HIPSE can be valuable.

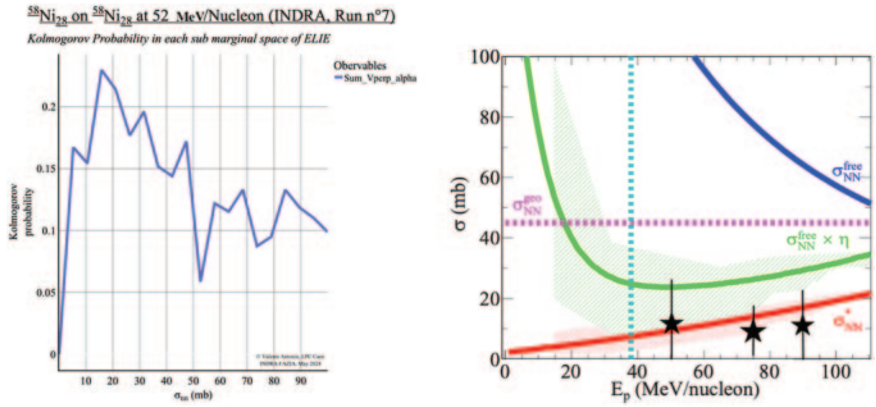

Fig. 4. – On the left: marginal probability of the Bayesian analysis for the sum of the perpendicular velocity of the alphas in the events. On the right: nucleon-nucleon cross-section with the excitation energy of the projectile. The black stars indicate the results obtained by this work.

4. – Discussion

Our results demonstrate that the neural network is capable of reconstructing the impact parameter with sub-femtometer accuracy, a significant improvement over traditional methods. This high precision allows for the selection of central collision events with greater confidence, which is essential for subsequent analysis of nuclear reaction dynamics and the extraction of EOS parameters.

The neural network was tested on a separate validation dataset and achieved a mean absolute error less than 1 fm across a wide range of impact parameters. This performance suggests that the network has successfully learned to generalize from the training data, making it a powerful tool for impact parameter reconstruction in heavy-ion collisions.

5. – Conclusion and future work

This study presents a novel approach for better characterizing the nuclear EOS using central heavy-ion collisions in the Fermi energy domain. By employing advanced machine learning techniques, particularly neural networks trained on simulation data, we achieved sub-femtometer accuracy in reconstructing impact parameters. This allows for precise event selection, improving the analysis of nuclear collisions. The integration of Bayesian inference methods, along with the Kolmogorov-Smirnov statistical comparison, enabled us to estimate in-medium nucleon-nucleon cross-sections with meaningful uncertainties.

Our results show good agreement with previous studies, especially for reactions below 100 MeV/nucleon, where the in-medium effects on nucleon-nucleon cross-sections are expected to be more pronounced. The Bayesian analysis provides robust estimations of cross-sections, offering a clearer understanding of nuclear medium modifications at different densities.

By applying all these improvements, we aim to provide tighter constraints on the nuclear EOS, enhancing the understanding of nuclear matter under extreme conditions. These advancements will have broad implications not only for nuclear physics but also for astrophysics, where the EOS plays a critical role in modeling neutron stars, supernovae, and neutron star mergers.

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