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Cosmology with gravitational waves: Inferring the Hubble constant with standard sirens(*)

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Summary. — The following proceeding aims to provide an overview of the discussion presented at the 20th edition of "Incontri di Fisica delle Alte Energie" (IFAE) held in Florence from 3 to 5 April 2024. We will delve into some of the principal cosmological inference methods currently available using gravitational waves detected by the LIGO, Virgo, and KAGRA interferometers. Our focus will be the Hubble constant estimation with standard sirens, thus called gravitational wave detections, which have the property to provide directly the source's luminosity distance. The use of sirens for cosmological studies had been proposed long before the first gravitational wave detection by B. Schutz and has been further developed within the Bayesian framework by several authors. In particular, the following proceeding will provide a brief description of the bright siren method, the spectral siren method, and how to measure the Hubble constant with the support of the galaxy catalogue information. These new statistical approaches show potential with the first published data, offering only a glimpse of the precision and accuracy achievable in the coming years with third-generation interferometers and increasingly accurate galaxy redshift measurements.

1. – Introduction

Predicted by Einstein's theory of general relativity (GR) in 1916 by linearising the field equations in the weak-field approximation, gravitational waves (GW) are ripples in space-time, that propagate outwards from their source at the speed of light [1]. The GW amplitudes are extremely small. Detectable GWs require heavy sources capable of generating strain amplitudes. Considering the sensitivity range of current interferometers, the actual strain is on the order of $|h_{GW}| \sim 10^{-21}$. The most violent and energetic phenomena in the Universe produce GWs: the coalescence of binary compact (CBC) objects such as binary black holes (BBH), binary neutron stars (BNS) [7-9] or binary supermassive black holes SMBH [20], the core-collapse of massive stars at the end of their lives [10, 11], and even from the primordial stages of the Cosmos [12-14].

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GWs remained directly unobserved for almost a century until their first observation in 2015 by the two interferometers of the Laser Interferometer Gravitational-Wave Observatory (LIGO) [2], which detected a signal labelled GW150914 on 14 September, originating from a BBH system. The binary involved two black-hole of about 36^{+5}_{-4} and 29^{+4}_{-4} $M_{\odot}$, respectively [3]. After coalescing, a more massive black hole of 62^{+4}_{-4} $M_{\odot}$ remained and copious amounts of energy were converted into gravitational radiation. These values are reported with a 90% confidence interval. This first concrete evidence for the existence of BBH systems in the Universe has been followed up as far by 90 detections published in the latest Gravitational Waves Transient Catalogs GWTC-3 by the LIGO, Virgo and KAGRA (LVK) collaboration. Almost of these events are BBHs [6].

This unpredicted abundance of CBC coalescences opened a new way to chase the origins of the Universe, testing the nowadays cosmological Λ-Cold Dark Matter (ΛCDM) model and GR under extreme conditions of strong fields and relativistic speeds [15-17]. But the large number of GW events also allows for the conduct of population and astrophysical studies of CBC objects [44, 45], which are essential in cosmological research too. About a hundred years after Edwin Powell Hubble's first measurement of the Universe's expansion rate, the Hubble constant H_0, various investigative techniques still lead to different results. Thanks to GW events, now we have a method that is a new independent way of constraining cosmological parameters, offering the opportunity to solve the Hubble tension, a discrepancy of about $\sim 4\sigma$ between late and early H_0 measurements. At high redshift, H_0 can be extrapolated by studying the properties of the cosmic microwave background (CMB) or using the baryonic acoustic oscillations (BAO). The latest value from Planck Collaboration is $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} [18]. While for the local Universe, the SHOES team's results indicate a $H_0 = 73.0 \pm 1.4$ km s^{-1} Mpc^{-1} [22], using Type Ia Supernovae and Cepheids at $z \leq 1$.

Hence, standard sirens are a key to its resolution because it is a model-independent method for measuring the value of the Hubble constant, exploring new physics and understanding the shortcomings of the current cosmological model of ΛCDM.

2. – Cosmology with standard sirens

Many years before the first GW detection, in 1986, Bernard Schutz [34] described a pioneering method for estimating the Hubble constant, using GWs and the galaxy catalogue. This method was later refined and expanded using Bayesian analysis techniques by several authors [21, 23, 42]. A key concept of his theory was that the GW luminosity distance d_L can be measured directly using a network of interferometers, and it is inversely proportional to the strain h_{GW}; in summary

$$|h_{GW}| \propto \frac{M_z^{5/3}}{d_L} \tag{1}$$

where M_z is the detector chirp mass that we can measure

$$M_z = (1+z)M_s = (1+z)\frac{(m_{1,s}m_{2,s})^{3/5}}{(m_{1,s}+m_{2,s})^{1/5}} \tag{2}$$

that is linked with the source-frame mass M_s, called chirp mass, related with the initial masses involved $m_{1,s}$ and $m_{2,s}$ in the source frame masses at redshift z, taking into account cosmological effects on GWs parameters. Therefore, with the GW amplitude signal is possible to assess the source distance without relying on traditional astronomical methods like the "cosmic distance ladder" and thus CBCs earn the name of "standard sirens" analogous to the role of Type Ia supernovae or Cepheid Variable stars, which are referred to as "standard candles" by their known absolute magnitude, Mag, which allow astronomers to extrapolate their distance, d_L [21] with the relation

$$\mathrm{mag} - \mathrm{Mag} = 5\log(d_L/10\,\mathrm{pc}) - 5 \tag{3}$$

where mag is the apparent magnitude of the source. Cosmology speaking, the distance is well connected with the rate of expansion of the Universe, described by the Hubble constant

$$d_L = c(1+z)\int_0^z \frac{dz'}{H(z)} \rightarrow d_L = \frac{cz}{H_0} \quad for \;\; z \leq 1 \ . \tag{4}$$

The independence of distance measurements from gravitational waves makes them a crucial tool for tackling fundamental questions such as the opened Hubble tension.

Furthermore, a network of interferometers enables astronomers to pinpoint the GW sky localisation to a remarkably small patch of sky [34]. This capability facilitates the identification of potential host galaxies within the narrowed-down area where the merger of compact objects occurred.

However, to fully characterise the merging objects and their host environment, we require additional information beyond what can be directly measured from the GW signal alone. This is where the redshift of the source comes into play. Redshift information is intricately linked to the masses of the two merging objects in the rest frame of the source, a crucial parameter that cannot be determined solely from the GW signal. This inherent limitation is referred to as "mass redshift degeneracy". The masses we infer from the detectors have been "redshifted" due to the expansion of the Universe. In the field of GW cosmology, overcoming this limitation and estimating cosmological parameters are central goals. This proceeding will overview the main methods to break the mass-redshift degeneracy and infer the Hubble constant from GW detections: the bright siren method, the spectral siren method, and the galaxy catalogue method are discussed in the following sections.

2˙1. *The Bright Siren Method.* – The strong signal detected both the LIGO and Virgo interferometers, GW170817 was originating from a BNS merger of total mass between 2.73 and 3.29 $\mathrm{M}_\odot$ [24] and marked a pivotal turning point in the era of multi-messenger astronomy and cosmology. This event was followed in less than $\sim 2\,\mathrm{s}$ by the γ- ray burst GRB179817A, detected by Fermi and INTEGRAL [27], and by a cascade of electromagnetic (EM) counterparts in the X-ray [26], ultraviolet, optical [25, 28, 29], infrared and radio [30] wavelengths. This bright siren underlined the remarkable potential of collaboration between diverse astronomical facilities. Beyond unveiling BNS as the progenitor of short γ-ray burst and providing insights into the emitting region and the Kilonova/Macronova explosions, GW170817 offered a chance to test the gravity. It allowed researchers to compare the speed of light and gravitational radiation, validating GR, and also to infer for the first time H_0 with a GW event [31]. The direct GW sky localisation, coupled with observations from EM facilities, pinpointed the host galaxy NGC4993. Hence, determining the redshift of the galaxy is crucial to breaking the mass-redshift degeneracy. The first measurement of the Hubble constant with a GW as the trigger was $H_0 = 70^{+12}_{-8}$ km s^{-1} Mpc^{-1} [31], in agreement with previous measurements of H_0 [18, 19] but derived entirely independently. However, due to the large uncertainty, it was not possible to definitively resolve the Hubble tension with just one event and with a method in its early stages of development. Systematic biases introduced by selection effects in electromagnetic transients must be carefully considered [32], as well as the degeneracy between the luminosity distance of the sources and their observing angle [33]. The primary limitation of this method is that GW170817 remains unique to this day. Waiting for more bright siren detections, researchers are actively pursuing efforts to enhance the synergy between GW interferometers and ground- and space-based telescopes.

2˙2. *The Spectral Siren Method.* – The spectral siren method infers cosmological parameters using GW data named dark sirens that have no EM counterparts. Currently, about ninety CBC events have been identified as dark sirens. This number is expected to increase during the future runs of the LVK network. This approach relies on a key assumption about properties of the source population, such as mass gaps, peaks or multi-peaks in the mass spectrum of CBC [35]. Therefore, this method offers a way to break the mass-redshift degeneracy by studying the population properties with the CBC merger rate detector

$$\frac{dN}{\mathrm{d}d_L \mathrm{d}\Omega \mathrm{d}m_1 \mathrm{d}m_2 \mathrm{d}\chi \mathrm{d}t_d} \propto R_0 \Psi(z;\Lambda) \times p_{\mathrm{pop}}(m_{1,S}, m_{2,S}|\Lambda)\, p_{\mathrm{pop}}(\chi|\Lambda) \frac{\mathrm{d}V_c}{\mathrm{d}z}\frac{1}{1+z}. \tag{5}$$

We are assuming a flat ΛCDM Universe with $\Lambda = \{\Lambda_m, \Lambda_c\}$ where Λ_c is the cosmological parameter which includes the Hubble constant H_0 and also the present-day fraction of matter density $\Omega_{m,0}$ and a set of hyper-parameters Λ_m for the distribution of sources, in the co-moving volume $\frac{dV_c}{dz}$ [37]. The R_0 (Gpc^{-3} yr^{-1}) describes the merger rate per comoving volume per year, with $\Psi(z;\Lambda)$ we are parametrising the rate evolution in the Hubble flow. $p_{\mathrm{pop}}(m_{1,S}, m_{2,S}|\Lambda)$ is the source-frame mass distribution, while $p_{\mathrm{pop}}(\chi|\Lambda)$ is a prior distribution for the spin parameters. Therefore, using the hierarchical Bayesian inference, the probabilities for source-frame masses, $m_{1,S}$ and $m_{2,S}$, and spin parameters χ, are synthesised with

$$p_{\mathrm{pop}}(\theta|\Lambda_m, \Lambda_c) \;\propto\; p(m_{1,S}, m_{2,S}|\Lambda_m) \times \frac{\mathrm{d}V_c}{\mathrm{d}z}(\Lambda_c)(1+z)^{\gamma-1} \tag{6}$$

where θ is a set of parameters of binary in the source frame S, $\theta = (z, m_{1,S}, m_{2,S}, \chi)$, while the power-law index γ characterises the merger rate evolution with redshift. Up to now, there are 8 models for the mass spectrum and two for the spin distribution, derived from two main formation channels [44, 45]. The LVK sources reveal at least five distinct "features" specific to each detectable mass range for BHs and NSs [35]. Information about the source's redshift is obtained indirectly by analysing the redshifted detector-frame masses M_z and by statistically modelling both the distribution of source-frame mass M_S with the cosmological parameters. Researchers begin by establishing the mass distribution of the CBCs in the source frame. This distribution encompasses both the masses of the individual compact objects and the overall mass of the merging system. In this way, the spectral siren method eliminates the need for coincident EM observations. Accurately characterising the mass distribution of CBCs in the Universe remains a challenge, as a wrong model of the CBC rate merger could introduce bias in the results. For a more detailed review, please refer to the paper [41].

With the current detectors, we could achieve an accuracy on H_0 better than 10% in the local Universe while with the *3rd* generation detectors such as Einstein Telescope (ET) it will be lower than 1% with only one month of duty cycle [35].

2˙3. *The Galaxy Catalogue Method.* – Galaxy catalogues are utilised to statistically extrapolate the redshift of the potential host galaxy in a credible volume from the GW localisation [42,43]. Consequently, the smaller the event region, the better the constraints on cosmological parameters. In particular, the inference of H_0 in the Bayesian framework with a collection of GW events $\{x\}$ follows:

$$p(H_0|x) \propto p(H_0)p(N_{\rm det}|H_0)\prod_i^{N_{\rm det}} \mathcal{L}_i(\{x\}|H_0) \tag{7}$$

where $p(H_0)$ represents a uniform prior for the Hubble constant, the likelihood $\mathcal{L}_i$ takes into account, for each event, the detection probability and the probability distribution of the CBCs as a function of redshift within galaxies, $p_{\rm pop}$, based on observational data, and previously described in the sect. **2**˙2. However, assuming that the density of mergers is lower than the density of galaxies, approximately $10^{-6} - 10^{-5}$ yr^{-1} per galaxy [46], we can approximate $p_{\rm pop}(z) \simeq p_{cat}(z)$, where p_{cat} is the probability of finding a galaxy at redshift z, based on galaxy catalogue information. In the population method the redshift prior is considered to be uniform in the comoving volume, whereas when using galaxy information dependent on sky-position, the resulting prior becomes a highly discrete function of redshift. More details on the methodology can found in [38, 42, 43].

The limitation of this method lies in (i) the increasing incompleteness of galaxy catalogues at higher redshifts, due to the intrinsic flux-limited nature of EM instruments and telescopes . Indeed, many possible GW hosts fall below a magnitude threshold and are thus undetected, leading to an incomplete catalogue of observed objects. Moreover, (ii) the results depend on the EM band used in the analysis and on the weighting of the galaxies. This affects the statistical analysis of the host galaxies associated with GW events. Additionally, (iii) one needs to fix the GW population, such as the mass distribution and their merger rate through the Universe, and this could introduce biases.

Several works are now focusing on the joint inference of cosmological and population parameters [36,39,40]. For example, by combining 42 BBH events from the GWTC-3 catalogue with different compact object mass distribution models, the posterior distribution of H_0 shows a 17% improvement in precision compared to the GWTC-2 analysis [47].

3. – Outlook

The Hubble tension seems easier to resolve with future events from the O4 and O5 runs, not to mention the data that third generation interferometers, such as the ET, could collect. For instance, ET is expected to obtain approximately 10-100 bright sirens in a single year, along with an enormous number of dark sirens, about $10^{5\pm1}$, working with two different L-shaped interferometers of about 15km at a relative orientation of 45° or with a 15km triangle configuration [50]. This vast number of detections will enable us to constrain H_0 to within less than 1% at a 90% confidence level using a network of 3G detectors like ET [48].

Therefore, cosmology with GW events will become increasingly precise, but the support of data from the EM spectrum remains essential. Adding more galaxies to our catalogues will be a driving force, alongside improvements in statistical methods for studying the CBC population [50, 51]. Instruments such as Euclid ([1]), Theseus ([2]), and Athena ([3]) will provide us with spectroscopic redshift data, crucial ingredient to a more precise estimate of H_0 [49]. Naturally, this introduces significant new challenges: refining robust analytical methods and analysis pipelines, investigating possible sources of systematic uncertainties, managing the large volume of data and events, and handling and sharing the increasing number of astronomical data.

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