Outsourcing of Facility Management Activities and Procurement Design

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Abstract
We introduce a multi-attribute combinatorial auction-based mechanism, called contract clause mechanism (CCM), as a mean to innovate procurement design related to outsourcing of facility management activities. The CCM allows a procurer and sellers to dynamically and simultaneously bargain the characteristics of distinct procurement contracts. The procurer does not directly call for goods and/or services; in fact, firstly he involves sellers in defining a collection of contract clauses related to different features of the supply of goods/services; secondly he requests bids on such clauses. The procurer also assigns scores to clauses to signal their relevance to the sellers. Submitted offers concern bundles of sets of clauses and define detailed procurement contracts. CCM allows procurers to mitigate the relevant problem concerning the lack of competences on the non-core activities, since CCM can partially extract from sellers their private knowledge as well as information on the supply cost.

Keywords: procurement design, multi-attribute auctions, combinatorial auctions, contracts, facility management.
1. Introduction

Facility management (FM) is a multidisciplinary approach for designing, planning and managing the non-core services in an integrated and coordinated way; these services support the strategic core activities and are essential for the effective and efficient functioning of an organization (De Toni and Nonino 2009, De Toni et alii 2007). In particular FM concerns the management of employee-related services, building, spaces, utilities, property, portfolio, asset management, ICTs management, administration and legal advice. The premise of outsourcing is that the contractors own superior competencies on the processes outsourced, and can reduce costs due to its capacity to reach economies of scale leading to better quality of the services.

In the last years, the FM discipline has been mainly developed by central public administrations and large private companies, with the purpose of integrating and coordinating many activities, and at the same time achieving efficiency, effectiveness and reduction of services cost. Nevertheless, switching costs incurred by the transition to an external provider, such as those associated with supplier selection, negotiations, reorganization and control, are high. The externalization of FM activities is certainly the right solutions, but only if the organization clearly identifies its own needs, coherently to its own strategy, defines the proper service conditions, and subsequently identifies the possible best contractors and reduces the costs of purchasing process.

Moreover, following recent trends in FM, the public administration is trying to adopt the global service (GS), namely, a contract where the regular maintenance activities are substituted by a plurality of services and the contractor is fully responsible on the results. This type of contract moves the service objective from a specific activity implementation process to the effective achievement of satisfying results (target service levels) and aims at identifying a single contractor for a multiplicity of services. Nevertheless, in the European and, in particular, in the Italian contexts, partnership practices are not frequent and contracts implying a single provider for many services are unusual. As a matter of fact, this solution gives more responsibility to the supplier but also more power. The procedures that a public administration can use to announce a FM global service call for tenders are the open procedure, the restricted procedure, the competitive dialogue and, exceptionally, the negotiated procedure (Brugnoletti and Fogli 2009).

Two crucial steps in a FM outsourcing contract are (i) the understanding, the prioritization and the communication to potential suppliers of what the organization requires, and (ii) the development of a contracting mechanism that reflects the areas of concern and encourages the supplier to fulfill the

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3 The problem of outsourcing activities in public Italian facility management market is relevant. The yearly value of the awarded contracts signed by Italian public administration for facilities activities outsourcing in the year 2009 has been about 14 billion of euro, while the value of the potential market is about 27 billion (Osservatorio Nazionale FM 2010). Since 2002, CONSIP (CONcessionaria Servizi Informativi Pubblici), a company of the Ministry of Economy and Finance, carried out FM tenders on behalf of central and local administration. In 2008 CONSIP awarded contracts value has been 12 billions; therefore CONSIP gained a prominent position in the FM market of the Italian public sector (Ferri and Pala 2009). Moreover, 50% of total orders (87050 orders) has been contracted out through online negotiations (CONSIP 2009).
organization expectations (Jones 1995). Both these two critical aspects originate from the five main risks of outsourcing, as identified by Bertrand and Franc (2003): dependence on the suppliers, hidden costs, service provider’s lack of necessary capabilities, social risk and the loss of know-how. However, the latter plays a crucial role. As a matter of this fact, there are evidences that outsourcing involves high risks in terms of loss of competencies on the non-core processes (Alexander and Young 1996, Bettis et alii 1992, Doig et alii 2001), and also on related technology-based competencies (Hoecht and Trott 2006). This fact makes the definition of proper service conditions (levels) and the subsequent identification of the best contractors by the customer organization extremely difficult, and, in the long run, shifts the power asymmetry in favor of service providers. Moreover, how the outsourcing organization can be really sure of the FM service providers’ competence and ability to fulfill all the contractual obligations?

On the contrary the contractors must identify the client’s expectations and real needs, as facilities services outsourcing requires an high degree of interaction and service customization. FM companies offer field-based services based on a high client interaction at a high customization and specialization level and a high impact on the client performances since they work inside the client structure. Usually the expectations in service outsourcing are imprecise and the objective is subjective (because not so easily measurable as in manufacturing outsourcing). So the failure in satisfying the customers’ expectations, scarcely clarified in the contract clauses due to a lack of knowledge on the processes, is ground of conflicts, contract cancellations and penalties.

In recent past, many scholars and practitioners partnering arrangements have become popular in FM (Roberts 2001), in order to transform the adversarial relationships into cooperative ones. FM contracts have usually long duration. Even if the relationship among client and contractor cannot be based only on the contract (due to its incompleteness), the contract is the keystone on which all the relationships are based and cannot be easily modified. So, before creating a partnership a careful contractual definition of clauses and service levels and of supplier’s obligations is necessary; clearly, the supplier will provide services as specified in the contract but, as a rule, there is growing necessity for the customer organization to change some services characteristics. In fact, after the bargaining and the contract signature, the flexibility of adding new features or enhancing or reducing service is reduced (Belcourt 2006). In synthesis, the real challenge in the outsourcing of facility management activities is writing a contract that is specific enough to protect the customer and flexible enough to accommodate unplanned events (Johnson 1999) by, at the same time, reducing the cost of negotiations, reducing the power asymmetry in favour of service providers, defining the proper clauses and service levels.

To this purpose, Harris et alii (1998) suggest to introduce flexible options in the outsourcing contract clauses, such as, for example, clauses that links vendor payment to the performance of the user organization, clauses permitting early termination of the contract.

A partial solution is provided by the European Directive 18/2004, which approved the collaborative relationship between client and potential contractors before the call for tenders in the so-called technical dialogue. This activity allows the public administration to collaborate with the private companies, the
knowledge owner, with the objective to identify the best management model and particularly to prepare the contract terms. But this practice can not overcome the criticality of power asymmetry in service providers and enhance the cost of negotiations.

Another form of flexibility in FM contract is represented by the Service Level Agreement (SLA), an appendix of outsourcing contract in which the target service levels are clarified and, in some cases, can be periodically changed. But a first challenge and benefit of using SLA is that the organization must establish exactly what the core business is, while a second one is the definition of what level of service should be provided. As highlighted by Pratt (2003), the level at which services are pitched should reflects and be linked to best practices; hence, benchmarking is the right way to add suppliers’ proposal credibility in terms of quality and cost. However, the practical reality appears very different and benchmarking best practices is quite impossible (above all during the phase of call of tender definition).

In this work we propose an auction-based mechanism that allows a procurer (e.g. a local/central public administration, a private company) and suppliers to dynamically bargain the features of multiple procurement contracts within boundaries fixed ex ante both by the procurer and the suppliers. The mechanism does not require a high effort to the procurers in terms of providing detailed expectations; in fact, the mechanism aims at extracting these information directly from suppliers by inducing them to compete both in terms of prices and knowledge revealed (e.g. the right duration of the contract, plausible service levels and suppliers’ obligations). Moreover, the mechanism allows the procurer to control the number of winning suppliers, and thus to reduce the overhead costs due to the management of suppliers and/or to promote a stronger competition among sellers.

The paper is organized as follows. Section 2 introduces the contract model and Section 3 presents the auctioned-based mechanism to negotiate contract clauses. Section 4 delineates considerations for future work.

2. The contract model

Let us consider a scenario where a procurer\(^4\) has to acquire several goods and services aimed at supporting a bunch of his core activities (in the following, goods and services will be generically referred to as items)\(^5\). In order to buy the necessary items, the procurer can define a set of distinct formal proposals where detailed supply rules are provided. For instance, the procurer might specify the quantity demanded, the required quality, the terms of delivery, of warranty and of payment, clauses to safeguard possible intellectual property rights and exclusive conditions. Moreover, in order to select reliable suppliers both in terms of owned know-how and economic performances the formal proposals could provide for a set of characteristics that the suppliers must have (specific certifications, targets for key financial ratios, etc.); similarly, to avoid post-

\(^4\) To prevent confusion, from now on we refer to the procurer as “he” and any seller as “she”.

\(^5\) In general, this issue is crucial for many big organizations, such as, for instance, multinationals, utilities (electric power, water and transportation companies), great distribution companies.
contractual opportunistic behavior of the sellers, the formal proposals could provide for a set of penalties for not fulfilling the related undertakings.

In such cases, on the sellers side, complementarity relationships could exist between two or more of the items required by the procurer, namely, the supply of a particular item is valued by a seller more when it is obtained in combination with the supply of another specific item (for instance, they could be associated with cost savings due to scale and scope economies). In fact, a single seller could be able to provide different items among those needed to the procurer, and therefore she could be interested in formulating a single economic proposal in order to supply all these items to the procurer. Consequently, to organize a solution to acquire simultaneously items which are complementary for the sellers could induce a large saving for the procurer or a stronger commitment of the sellers in terms of maximizing the efforts to make the collaboration with the procurer successful (e.g. a seller could apply larger discounts when providing two complementary items than just one, or she could declare supply rules which are much more convenient for the procurer in terms of risk sharing). Similarly, on the procurer side, acquiring separately all goods/services risks to induce a raise of the number of suppliers and the consequent procurer’s overhead costs. Moreover, a seller with a high market power could defeat opponents with no market power much more easily when items are acquired separately, since they cannot coordinate their bids to displace the bids of the strong seller (look at the divide et impera strategy, in the sense that if the items are requested separately by the procurer, then the competitors of the incumbent seller are prevented from collaborating and thus they get weaker); in such a context, opponents are discouraged from participating and thus competition is jeopardized.

Therefore, in the case whereby several items have to be acquired and different sellers could be interested in providing more than one item, the procurement procedure should be designed in such a way to allow the procurer to exploit the opportunities and mitigate the risks both on the sellers side and on the procurer one. An effective solution to planning procurement in such cases consists of applying combinatorial auctions, which allow the sellers to submit bids on bundles of items and to communicate possible incompatibilities among their bids to the procurer, where two incompatible bids cannot be both simultaneously selected as winning by the procurer (see e.g. Ausubel and Milgrom 2002, Pekeč and Rothkopf 2003, Kwasnica et alii 2005, Avenali and Bassanini 2007, Avenali 2009). In such a way, the sellers are able to model and manage possible complementarity relationships among items, and therefore to offer higher discounts to supply two or more complementary items, without running the risk of undergoing irrational allocations.

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6 For instance, the seller could be able by herself to supply these goods/services or alternatively she could be the leader of a coalition of enterprises, where the seller exploits the distinct skills of the coalition by coordinating the role of every enterprise in defining the coalition proposal for a single contract. The goal of the coordination is to exploit scope economies and to share fairly the overall revenue (Raiffa 1982).

7 A relevant example is the case of the procurement of meals for 1,300,000 students in the Chilean public school system, which was awarded through a single round sealed-bid combinatorial auction. This improved the price-quality ratio of the meals and obtained estimated yearly savings of around US$40 million (Epstein et alii 2002).
Moreover, the proposed procedure has to be designed in such a way that the procurer can control the number of winning suppliers. In fact, to avoid an excessive raise of the sellers management costs, the procurer could need to impose a maximum number $W$ of winning suppliers. On the other hand, in the case whereby the sellers are very different in terms of market power (e.g. a scenario with an incumbent and weak new entrants), the procurer could promote participation and collaboration among sellers by imposing a minimum number $w$ of winning suppliers.

However, even with combinatorial auctions, the procurer has to completely specify the characteristic of the supply of every item when call for bids on the items (the desired quality, delivery terms, etc.). From now on, we refer to such information as *contract features*. Providing all these information for all goods and services could require a large economic effort for the procurer. Moreover, several contract features decided by the procurer could be inappropriate or even harmful to the procurer himself, as he could not have the necessary skills to perform the right analyses. On the other hand, sellers have the suitable know-how to determine these contract features in such a way that they are effective for the procurer; however, this knowledge is private, in the sense that if the procurer asked the sellers for it then the sellers might strategically lie when answering. Therefore, in order to cheaply acquire the necessary know-how to effectively formulate all contract features, the procurer should design a mechanism to extract the necessary knowledge directly from the sellers. To do this, the procurer could define a combinatorial auction where for any item a corresponding contract is auctioned off, contract features are partially negotiable, the number of winning sellers can be controlled by the procurer, and the sellers can submit offers which report their technical and economic proposals for the negotiable contract features and the price they require to supply the related items according to their contract proposals.

This approach belongs to the literature strand of multi-attribute auctions (see e.g. Che 1993, Branco 1997, Teich et alii 2004, Avenali, Leporelli and Matteucci 2006) with particular reference to the multi-attribute combinatorial case (see Schnizler et alii 2008). In particular, in the proposed mechanism the multiple dimensions of the negotiation process can even concern the rewriting of whole parts of a contract, and the number of the selected suppliers can be managed by the procurer.

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8 Moreover, contract features have to be designed in such a way to mitigate both pre-contractual and post-contractual opportunism (Milgrom, Roberts 1992). For instance, considering the results in Avenali, Matteucci and Reverberi (2010a), contract clauses based on dynamic access pricing in presence of persistent bottlenecks could be applied to provide sellers with incentives to increase or decrease quality investments. Moreover, clauses aiming to select sellers in upstream and downstream markets which are subject to different levels of vertical separation could be applied to mitigate an important instance of the moral hazard problem, that is, the so-called hold-up problem (see Pitchford, Snyder 2004, Avenali, Matteucci and Reverberi 2008, Avenali, Matteucci and Reverberi 2010b).

9 An earlier version of this mechanism has been presented in Avenali, Matteucci and Nonino (2010), where the overall price required for every contract of a same bid must be explicitly indicated by the seller (while this version allows the sellers to negotiate any possible technical and economic aspect of the contracts).
Before introducing the combinatorial mechanism that allows the procurer and the sellers to simultaneously negotiate multiple contracts, by controlling the number of winning sellers, we need to formalize the applied contract model.

Let us now consider a single procurement contract. It is a voluntary and legally enforceable agreement between the seller and the procurer, that documents the payment rules which the procurer is subjected to, and the modes and the penalties which the seller is subjected to in providing specific goods/services that the procurer pays for. In particular, the information underlying the agreement is structured along sections of a document, which define or explain the subject matter. Therefore, any contract can be decomposed in a set of distinct parts, each one representing a specific informative content. We refer to any distinct part of the agreement as contract clause. We refer to the number of clauses which a document is separable as contract cardinality.

We now introduce the notation and definitions to formally represent contracts and model the formation process of a contract in a dynamic bargaining framework. Let us assume a procurer \( p \) and let \( S_{h,1}, \ldots, S_{h,r} \) be \( r \) sets of clauses defined by procurer \( p \) which could be applied to formulate a multitude of possible versions of a generic procurement contract \( h \) of cardinality \( r \). In particular, two clauses \( s_i, s_j \in S_{h,a} \) with \( a \in \{1, \ldots, r_h\} \) are defined as substitute for \( p \) with respect to contract \( h \), in the sense that they cannot be part of a same contract version; in other words, substitute clauses represent alternative modes to define or explain the matter underlying a specific part of a contract. On the contrary, two clauses \( s_i \in S_{h,a} \) and \( s_j \in S_{h,b} \) with \( a, b \in \{1, \ldots, r_h\} \) and \( a \neq b \) are defined as complementary for \( p \) with respect to contract \( h \), namely, they can be distinct part of a same contract document. Therefore, any version of the contract \( h \) can be represented by an ordered sequence of complementary clauses, that is, \( h = (s_{h,1}, \ldots, s_{h,\zeta}) \in S_{h,1} \times \ldots \times S_{h,\zeta} \). From now on, for sake of notation, a contract \( h \) can be also represented by the set of its clauses, that is, \( h = (s_{h,1}, \ldots, s_{h,\zeta}) = \{s_{h,1}, \ldots, s_{h,\zeta}\} \). Moreover, some clauses in \( S_h = S_{h,1} \cup \ldots \cup S_{h,r} \) could be associated with quantitative data which could not be completely specified by the procurer. In this case, the procurer only sets a range where the value can be selected. We define such a clause as open while any clause completely specified is called close. In particular, given an open clause \( s \in S_h \), let \( d_s \geq 1 \) be the number of unspecified quantitative data and let \( l_{s,j}, u_{s,j} \) for \( j = 1, \ldots, d_s \) be the range where the \( j \)-th unspecified quantitative datum of clause \( s \) can be selected. Open clauses allow the sellers to specify the missing data of the contract on the basis of their internal decisions (based both on technical and strategic issues).\(^{10}\)

\(^{10}\) For instance, some contracts could regard just a potential demand of the procurer, in the sense that the number of the item units that will be actually demanded by the procurer cannot be fixed in the contract, while only a maximum number of item units, which could be requested in the considered period, is indicated. In this case, a clause of the contract could state that the price of a single unit of the item is equal to the overall price required by the seller for
By applying complementary/substitute and open/close clauses a procurer can define a flexible contract model where different levels of negotiation among procurer and sellers can be implemented. In fact, the procurer can freeze some parts of the procurement contract by preventing the corresponding clauses from being substituted and/or negotiated. For instance, if $S_{h,a} = 1$ for a given $a \in \{1, \ldots, r_h\}$ and the clause in $S_{h,a}$ is close, then the informative content described in this clause cannot be changed anyhow. If instead $S_{h,a} > 1$ and the clause in $S_{h,a}$ is open then the structure of the clause cannot be modified (there is no substitute clause) but a part of the associated quantitative data can be specified. Also, if $S_{h,a} > 1$ and the clause in $S_{a}$ are all close then it is possible to entirely substitute a clause of $S_{h,a}$ with another one in $S_{h,a}$. A more negotiable scenario is the one where $S_{h,a} > 1$ and all the clause in $S_{h,a}$ are open.

### 3. The contract clause bargaining mechanism

We now describe an auction-based mechanism, *contractual clause mechanism* (CCM), that allows the procurer and the sellers to simultaneously bargain the features of multiple procurement contracts, by applying the contract model introduced in the previous section. CCM is characterized by two main phases.

The goal of the first phase consists of defining all the potential clauses which could define possible versions of the considered contracts. Let $T$ be the set of $n \geq 2$ sellers and $H$ be the set of $m \geq 1$ contracts simultaneously auctioned off by the procurer. Note that, although some of the contracts on sale could be associated to the same item (e.g. the supply of an item could be divided in lots), each contract is uniquely determined as the contract features and the required price could be differently set by the sellers as the mechanism goes on. At the beginning of this phase, for every contract $h \in H$ the procurer proposes a set of clauses $S_h = S_{h,1} \cup \ldots \cup S_{h,r_h}$ which allow sellers to represent any possible version of the contract.

Moreover, procurer indicates the ranges associated with any open clause $s \in S_h$. For brevity of notation and with no loss of generality, from now on we assume that all contracts have same cardinality ($r_h = r$ for any contract $h \in H$); moreover, we assume that for any possible open clause the number of unspecified quantitative data is exactly one ($d_s = 1$ for any clause $s$). Therefore, if $s \in S_h$ is an open clause then we denote by $[l_s, u_s]$ the range associated with the only unspecified datum of $s$.

Furthermore, the procurer allows any seller $t \in T$ to propose a limited number $q \geq 0$ of new clauses for any contract $h \in H$. In particular, the sellers have to specify which set $S_{h,t} \subseteq S_h$ a new clause should be inserted in; moreover, if it is needed, the seller may enclose a document aimed at clarifying why she has submitted contract $h$ divided by the maximum number of item units, where both the overall price and the maximum number of units are missing data of open clauses of contract $h$. 
the new clause. The procurer collects all these new clauses and decides which ones can be accepted and which ones have to be rejected.

Successively, the procurer associates with each clause \( s \in S_h \) (both open and close) a value \( g_s \geq 0 \) which is as higher as the clause is relevant for the procurer with respect to the whole contract \( h \); we refer to this value \( g_s \geq 0 \) as the clause relevancy. Therefore, for any contract \( h \) the version \((\bar{s}_{h,1},\ldots,\bar{s}_{h,r})\) where \( \bar{s}_{h,i} = \max_{s \in S_{sh}} \{g_{h,s}\} \) for any \( i = 1,\ldots,r \) denotes the contract version characterized by the clauses with the highest relevancy. Moreover, the procurer associates with each open clause \( s \in S_h \) a function \( f_s : v \in [l_s, u_s] \rightarrow f_s(v) \in [a,1] \) (where \( 0 < a \leq 1 \)), which is applied to tune the relevancy of \( s \) once the missing datum of \( s \) is specified: when the unspecified datum is set to \( v \in [l_s, u_s] \), then \( f_s(v)g_s \) measures the net relevancy of open clause \( s \in S_h \). In particular, function \( f_s \) must be defined in such a way that \( f_s(v) = 1 \) for at least one \( v \in [l_s, u_s] \). Note that, depending on \( a \), function \( f_s \) can induce large distance between the minimum and the maximum values which the net relevancy can get.

Moreover, in order to avoid an excessive raise of the overhead costs supported to manage a high number of suppliers, the procurer can imposes a maximum number \( W \) of winning sellers. On the other hand, in the case whereby sellers are very different in terms of market power (e.g. a scenario where an incumbent compete against new entrants), the procurer can promote participation and as a consequence a fiercer competition among sellers by imposing a minimum number \( w \) of winning suppliers.

In the second phase of CCM, a first-price multi-round ascending combinatorial auction is performed. The auction is characterized by memory, in the sense that, at each round, all offers are stored (both winning and non-winning) and can subsequently be used by sellers to “complete” their offers and form a winning configuration.

At any round, each seller \( t \) can submit one or more bids, where every bid \( k \) consists of: (i) a bundle \( B_k \) of contracts; (ii) the clauses selected for any contract \( h \in B_k : h = (s_{h,1},\ldots,s_{h,r}) \in S_{h,1} \times \ldots \times S_{h,r} \); (iii) for any contract \( h \in B_k \) and any open clause \( s \in h \), a value \( v_s \) in the suitable range \([l_s, u_s]\).

Given a seller \( t \)’s bid \( k \), the procurer computes a score \( sc_k \in (0,1] \) for the whole bundle \( B_k \). This score gets one when every contract in the bundle is characterized by clauses with maximum relevancy:

\[
sc_k = \frac{\sum_{h \in B_k} \left( \sum_{s \in h \text{ open}} f_s(v_s)g_s + \sum_{s \in h \text{ close}} g_s \right)}{\sum_{h \in B_k} \max_{s \in S_{sh}} \{g_s\}}
\]

Summing up, at each round the sellers submit their new bids, then the procurer computes \( sc_k \) for every new bid \( k \) and finally stores it. In such a way, any stored bid (both new and previous) is related to bundle of
contracts which satisfy the requirements of the procurer both in terms of technical and economic contract features (substitute open/close clauses and ranges associated with open clauses are always approved by the procurer).

Therefore, by considering all stored bids, the procurer selects the set of bids which maximize the sum of the scores, subject to the constraints on the number of sellers which can win. After finding the winning bids, the procurer discloses to the sellers all stored bids and also indicates the winning ones. The identity of the sellers who have placed such bids are not disclosed, hence each seller neither knows how many other sellers are in the auction nor who submitted what; this should prevent sellers from colluding or in any case mitigate the collusion phenomenon. In particular, each seller only sees a list of bids announced by the procurer and is privately informed by the procurer about her own offers. A seller quits the auction when he does not submit new bids. The auction ends when all sellers have quit. As the mechanism is over, the contracts proposed in the winning bids become binding.

Let us now face the problem of finding the optimal contracts, that is, the contracts associated with the bids which maximize the sum of the scores, subject to the constraints on the number of the winning supplier. Let $O_t$ be the set of bids which seller $t \in T$ has bid for (from the beginning till now). We set $O = O_1 \cup \ldots \cup O_n$. To simplify the notation, from now on we write that index $k$ belongs to a given set of bids to mean that index $k$ is such that bid $k$ belongs to this set (e.g., with $k \in O$ we intend that index $k$ is such that bid $k$ is in $O$). Two bids which share a contract are incompatible, that is, they cannot be both simultaneously selected as winning by the procurer. Incompatibilities among bids are represented by the set $I = \{(k,j) \in O \times O : k \neq j, B_k \cap B_j \neq \emptyset\}$. The problem of determining the winning bids can be formulated through the following integer linear problem (with a polynomial number both of variables and of constraints):

$$
\begin{align*}
\max_{k \in O} & \sum_{k \in O} sc_k x_k \\
\text{subject to} & \quad x_k + x_j \leq 1 \quad (k,j) \in I : k, j \in O \\
& \quad \sum_{k \in O \setminus B_h} x_k \geq 1 \quad h \in H \\
& \quad x_k \leq y_t \quad k \in O_t, t = 1, \ldots, n \\
& \quad w \leq \sum_{i=1}^{n} y_t \leq W \\
& \quad x_k \in \{0,1\} \quad k \in O \\
& \quad y_t \in \{0,1\} \quad t = 1, \ldots, n
\end{align*}
$$

where every binary variable $x_k$ is 1 if and only if bid $k$ is selected as winning and every binary variable $y_t$ is 1 if and only if at least one bid of seller $t$ is selected as winning. In particular, let $\hat{y} \in \{0,1\}^n$, $\hat{x} \in \{0,1\}^{|P|}$ be an optimal solution to $\Pi$. The optimal contracts are those proposed in the winning bids $\{k \in O : \hat{x}_k = 1\}$,
while the number of winning suppliers is \[ \sum_{i=1}^{n} y_i \]. \( \Pi \) is straightforwardly derived from the formulation of the Combinatorial Auction Problem (see De Vries and Vohra 2003), where:

- Constraints (1) ensure that incompatible bids cannot be both simultaneously selected as winning.
- Constraints (2) guarantee that any contract \( h \in H \) is allocated to a seller. In the case that some contracts can remain unassigned, the corresponding constraints (2) have to be removed.
- Constraints (3) and (4) impose that the number of winning sellers cannot be neither lower than \( w \) nor greater than \( W \).

Moreover, the computation of an optimal solution to \( \Pi \) is NP-hard since it is at least as hard as the Combinatorial Auction Problem (NP-hard as shown in Rothkopf, Pekeč and Harstad 1998). In fact, any instance of the Combinatorial Auction Problem can be polynomially transformed into an instance of the problem of determining the bids which maximize the sum of the scores, subject to the constraints on the number of the winning supplier (it can be easily verified by setting \( w = 0 \) and \( W = n \)).

4. Considerations for future work

Next work will focus on the following points:

A different procedure to assign score to open clauses
To compute the score of any bundle the procurer has to associate with each open clause \( s \in S_h \) (for every contract \( h \in H \)) a function \( f_s : v \in [l_v, u_v] \rightarrow f_s(v) \in [a, 1] \) (where \( 0 < a \leq 1 \)), which is applied to tune the relevancy of \( s \) once the missing datum of \( s \) is specified. However, this way of computing the scores rigidly sets the difference in relevancy between any two values in \([l_v, u_v]\), while it does not care of the real submitted offers. For instance, if a seller is bidding high quality while all her opponents are offering low quality, then she should get a score much higher than in the case all her opponents offered average quality. This different approach of determining the score could provide the sellers with incentives to increase their efforts both in terms of technical and economic aspects, and it saves the procurer the burden of designing a function \( f_s \) for each open clause \( s \in S_h \) of every contract \( h \in H \).

A version of the mechanism based on a richer bidding language
There can be cases where the procurer and/or the sellers could be interested into considering relationships among distinct clauses of different contracts. In this case, the proposed mechanism, which takes into account only incompatibilities among whole submitted bids (due to the requirements on the minimum and maximum number of winning suppliers, and to bundles which share one or more items), can be easily amended in such
a way that a more general class of relationships among clauses can be considered (e.g. budget constraints, capacity constraints).

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