New mathematical connections concerning string theory

Michele Nardelli

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Dipartimento di Matematica ed Applicazioni "R.Caccioppoli"
Università degli Studi di Napoli "Federico II"- Polo delle Scienze e delle Tecnologie
Monte S.Angelo, via Cintia (Fuorigrotta), 80126 Napoli, Italy

Riassunto.

Scopo del presente lavoro è quello di descrivere le relazioni trovate tra il modello di Palumbo sull'origine e l'evoluzione dell'Universo e la teoria di stringa. Il modello di Palumbo è sintetizzato dalla relazione $F = \int_0^\infty F_i dF_i$, dove F rappresenta l'energia iniziale del Big Bang, ossia, l'esplosione del buco nero dal quale si originò l'universo. Dal Big Bang, si sprigionarono tutte le onde immaginabili di F . Al pari delle radiazioni elettromagnetiche, le quali consistono di una successione continua di insiemi di onde, anche le radiazioni F sono costituite da insiemi parziali di onde, definite F_i. Dopo aver descritto le azioni di stringa bosonica e quelle di superstringa, si evidenziano le connessioni trovate tra queste ed il modello di Palumbo. Vengono evidenziate, inoltre, le connessioni trovate tra le azioni delle brane di Dirichlet, precisamente le D3 e D9-brane ed il modello di Palumbo. Anche per alcune azioni di stringa inerenti il modello cosmologico di pre Big-Bang, vengono evidenziate delle connessioni con il modello di Palumbo. Infine, vengono descritte le relazioni trovate tra alcune soluzioni solitoniche in teoria di campo di stringa ed alcune equazioni correlate alla funzione zeta di Riemann. Viene evidenziato, quindi, come anche per queste ultime è possibile la connessione con il modello di Palumbo.

1.Bosonic String.

A one-dimensional object will sweep out a two dimensional world-sheet, which can be described in terms of two parameters, $X^\mu(\tau,\sigma)$. The simplest invariant action, the Nambu-Goto action, is proportional to the area of the world-sheet. We define the induced metric h_{ab} where indices $a,b,...$ run over values (τ,σ): $h_{ab} = \partial_a X^\mu \partial_b X_\mu$. (a) Then the Nambu-Goto action is $S_{NG} = \int_M d\tau d\sigma L_{NG,}$ (b)

$L_{NG} = -\frac{1}{2\pi\alpha'}(-\det h_{ab})^{1/2}$, (c) where M denotes the world-sheet. The constant α', which has units of spacetime-length-squared, is Regge slope. The tension T of the string is related to the Regge slope by $T = \frac{1}{2\pi\alpha'}$. We can simplify the Nambu-Goto action by introducing an independent world-sheet metric $\gamma_{ab}(\tau,\sigma)$. We will take γ_{ab} to have Lorentzian signature $(-,+)$. The action is

$S_p[X,\gamma] = -\frac{1}{4\pi\alpha'}\int_M d\tau d\sigma(-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu$, (d) where $\gamma = \det\gamma_{ab}$. This is the Brink-DiVecchia-Howe-Deser-Zumino action, or Polyakov action for short. It was found in the course of deriving a generalization with local world-sheet supersymmetry. To see the equivalence to S_{NG}, use the equation of motion obtained by varying the metric,

$\delta_\gamma S_p[X,\gamma] = -\frac{1}{4\pi\alpha'}\int_M d\tau d\sigma(-\gamma)^{1/2}\delta\gamma^{ab}\left(h_{ab} - \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd}\right)$, (e) where h_{ab} is again the induced metric (a). We have used the general relation for the variation of a determinant, $\delta_\gamma = \gamma\gamma^{ab}\delta\gamma_{ab} = -\gamma\gamma_{ab}\delta\gamma^{ab}$.

Then $\delta_\gamma S_p = 0$ implies $h_{ab} = \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd}$. Dividing this equation by the square root of minus its determinant gives $h_{ab}(-h)^{-1/2} = \gamma_{ab}(-\gamma)^{-1/2}$, so that γ_{ab} is proportional to the induced metric. This in turn can be used to eliminate γ_{ab} from the action,

$S_p[X,\gamma] \to -\frac{1}{2\pi\alpha'}\int d\tau d\sigma(-h)^{1/2} = S_{NG}[X]$. (f)

The condition that the world-sheet theory be Weyl-invariant is:

$\beta^G_{\mu\nu} = \beta^B_{\mu\nu} = \beta^\Phi = 0$ (1.1) , where:

$$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha'\nabla_\mu\nabla_\nu - \frac{\alpha'}{4}H_{\mu\lambda\omega}H_\nu^{\lambda\omega} + O(\alpha'^2),$$

$$\beta^B_{\mu\nu} = -\frac{\alpha'}{2}\nabla^\omega H_{\omega\mu\nu} + \alpha'\nabla^\omega\Phi H_{\omega\mu\nu} + O(\alpha'^2),$$

$$\beta^\Phi = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\Phi + \alpha'\nabla_\omega\Phi\nabla^\omega\Phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + O(\alpha'^2).$$

The equation $\beta^G_{\mu\nu} = 0$ resembles Einstein's equation with source terms from the antisymmetric tensor field and the dilaton. (Note that the dilaton is the massless scalar with gravitational-strength couplings, found in all perturbative string theories). The equation $\beta^B_{\mu\nu} = 0$ is the antisymmetric tensor generalization of Maxwell's equation, determining the divergence of the field strength.

The field equations (1.1) can be derived from the spacetime action:

$$S = \frac{1}{2\kappa_0^2}\int d^D x\,(\text{-G})^{1/2}e^{-2\Phi}[-\frac{2(D-26)}{3\alpha'} + R - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi + O(\alpha')] \quad (1.2)$$

The normalization constant κ_0 is not determined by the field equations and has no physical significance since it can be changed by a redefinition of Φ. One can verify that:

$$\delta S = -\frac{1}{2\kappa_0^2\alpha'}\int d^D x(-G)^{1/2}e^{-2\Phi}[\delta G_{\mu\nu}\beta^{G\mu\nu} + \delta B_{\mu\nu}\beta^{B\mu\nu} + (2\delta\Phi - \frac{1}{2}G^{\mu\nu}\delta G_{\mu\nu})(\beta^{G\omega}_\omega - 4\beta^\Phi)] \quad (1.3)$$

This is the effective action governing the low energy spacetime fields. It is often useful to make a field redefinition of the form:

$\tilde{G}_{\mu\nu}(x) = \exp[2\omega(x)]G_{\mu\nu}(x)$, (1.4), which is a spacetime Weyl transformation. The Ricci scalar constructed from $\tilde{G}_{\mu\nu}$ is:

$\tilde{R} = \exp(-2\omega)[R - 2(D-1)\nabla^2\omega - (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega]$ (1.5)

For the special case $D = 2$, this is the Weyl transformation $g'^{1/2}R' = g^{1/2}(R - 2\nabla^2\omega)$.

Let $\omega = 2(\Phi_0 - \Phi)/(D-2)$ and define $\tilde{\Phi} = \Phi - \Phi_0$ (1.6), which has vanishing expectation value. The action becomes:

$S = \frac{1}{2\kappa^2}\int d^D X(-\tilde{G})^{1/2}[-\frac{2(D-26)}{3\alpha'}e^{4\tilde{\Phi}/(D-2)} + \tilde{R} - \frac{1}{12}e^{-8\tilde{\Phi}/(D-2)}H_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} - \frac{4}{D-2}\partial_\mu$
$\tilde{\Phi}\partial^\mu\tilde{\Phi} + O(\alpha')]$, (1.7). In terms of $\tilde{G}^{\mu\nu}$, the gravitational Lagrangian density takes the standard Hilbert form $(-\tilde{G})^{1/2}\tilde{R}/2\kappa^2$. The constant $\kappa = \kappa_0 e^{\Phi_0}$ is the observed gravitational coupling constant, which in

nature has the value $\kappa = (8\pi G_N)^{1/2} = \dfrac{(8\pi)^{1/2}}{M_p} = (2{,}43x10^{18}GeV)^{-1}$. Commonly, $G_{\mu\nu}$ is called the "sigma model metric" or "string metric", and $\widetilde{G}_{\mu\nu}$ the "Einstein metric". The (1.7) becomes :

$$S = \frac{1}{2\sqrt{8\pi G_N}}\int d^D X(-\widetilde{G})^{1/2}[-\frac{2(D-26)}{3\alpha'}e^{4\widetilde{\Phi}/(D-2)} + \widetilde{R} - \frac{1}{12}e^{-8\widetilde{\Phi}/(D-2)}H_{\mu\nu\lambda}\widetilde{H}^{\mu\nu\lambda} - \frac{4}{D-2}\partial_\mu\widetilde{\Phi}\widetilde{\partial}^\mu\widetilde{\Phi} + O(\alpha')]$$

(1.8)

The amplitudes of string, that corresponding at the classical terms in the effective action, would be obtained in field theory from the spacetime action $S + S_T$, where S is the action (1.2) for the massless fields, and where

$S_T = -\dfrac{1}{2}\int d^{26}x(-G)^{1/2}e^{-2\Phi}(G^{\mu\nu}\partial_\mu T\partial_\nu T - \dfrac{4}{\alpha'}T^2\ )$ (1.9), is the action for the closed string tachyon T.

(Note that, in string theory, the tachyon is a particle with a negative mass-squared, signifying an instability of the vacuum). We take the more general case of $D = d+1$ spacetime dimensions with x^d periodic. Since we are in field theory we leave D arbitrary. Parameterize the metric as

$ds^2 = G^D_{MN}dx^M dx^N = G_{\mu\nu}dx^\mu dx^\nu + G_{dd}(dx^d + A_\mu dx^\mu)^2$ (1.10).

We designate the full D-dimensional metric by G^D_{MN}; the Ricci scalar for the metric (1.10) is:

$R = R_d - 2e^{-\sigma}\nabla^2 e^\sigma - \dfrac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}$ (1.11), where R is constructed from G_{MN} and R_d from $G_{\mu\nu}$. The graviton-dilaton action (1.2) becomes:

$$S_1 = \frac{1}{2\kappa_0^2}\int d^D x(-G)^{1/2}e^{-2\Phi}(R + 4\nabla_\mu\Phi\nabla^\mu\Phi) =$$

$$= \frac{\pi R}{\kappa_0^2}\int d^d x(-G_d)^{1/2}e^{-2\Phi+\sigma}(R_d - 4\partial_\mu\Phi\partial^\mu\sigma + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}) \quad \text{hence:}$$

$S_1 = \dfrac{\pi R}{\kappa_0^2}\int d^d x(-G_d)^{1/2}e^{-2\Phi_d}(R_d - \partial_\mu\sigma\partial^\mu\sigma + 4\partial_\mu\Phi_d\partial^\mu\Phi_d - \dfrac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu})$ (1.12) giving kinetic terms for all the massless fields. Here G_d denotes the determinant of $G_{\mu\nu}$, and in the last line we have introduced the effective d-dimensional dilaton, $\Phi_d = \Phi - \sigma/2$.

The antisymmetric tensor also gives rise to a gauge symmetry by a generalization of the Kaluza-Klein mechanism. Separating B_{MN} into $B_{\mu\nu}$ and $A'_\mu = B_{d\mu}$, the gauge parameter ζ_M separates into a d-dimensional antisymmetric tensor transformation ζ_μ and an ordinary gauge invariance ζ_d (Note that the action

$S_\sigma = \dfrac{1}{4\pi\alpha'}\int_M d^2\sigma\sqrt{g}[(g^{ab}G_{\mu\nu}(X) + i\varepsilon^{ab}B_{\mu\nu}(X))\partial_a X^\mu\partial_b X^\nu + \alpha' R\Phi(X)]$, where the field $B_{\mu\nu}(X)$ is the antisymmetric tensor, and the dilaton involves both Φ and the diagonal part of $G_{\mu\nu}$, is invariant under $\delta B_{\mu\nu}(X) = \partial_\mu\zeta_\nu(X) - \partial_\nu\zeta_\mu(X)$ which adds a total derivative to the Lagrangian density. This is a generalization of the electromagnetic gauge transformation to a potential with two antisymmetric indices. The gauge parameter ζ_M, above mentioned, is defined in this relation). The gauge field is $B_{d\mu}$ and the field strength $H_{d\mu\nu}$. The antisymmetric tensor action becomes:

$$S_2 = -\frac{1}{24\kappa_0^2}\int d^D x(-G_D)^{1/2}e^{-2\Phi}H_{MNL}H^{MNL} = -\frac{\pi R}{12\kappa_0^2}\int d^d x(-G_d)^{1/2}e^{-2\Phi_d}(\widetilde{H}_{\mu\nu\lambda}\widetilde{H}^{\mu\nu\lambda} + 3e^{-2\sigma}H_{d\mu\nu}H_d^{\mu\nu})$$

(1.13). Here, we have defined $\widetilde{H}_{\mu\nu\lambda} = (\partial_\mu B_{\nu\lambda} - A_\mu H_{d\nu\lambda})$ + cyclic permutations. The term proportional to the vector potential arises from the inverse metric G^{MN}. It is known as a "Chern-Simons term", this signifying the antisymmetrized combination of one gauge potential and any number of field strengths.

The relevant terms from the spacetime action (1.7) concerning an D-brane are:
$S = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\tilde{G}}(\tilde{R} - \frac{1}{6}\nabla_\mu\tilde{\Phi}\tilde{\nabla}^\mu\tilde{\Phi})$ (1.14). Recall that the tilde denotes the Einstein metric.
The tilded dilaton has been shifted so that its expectation value is zero. In terms of the same variables, the relevant terms from the D-brane action
$S_p = -T_p\int d^{p+1}\xi e^{-\Phi}\left[-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})\right]^{1/2}$, (with T_p is a constant and
$G_{ab}(\xi) = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b}G_{\mu\nu}(X(\xi))$, $B_{ab}(\xi) = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b}B_{\mu\nu}(X(\xi))$ are the induced metric and antisymmetric tensor on the brane) are:
$S_p = -\tau_p\int d^{p+1}\xi \exp\left(\frac{p-11}{12}\tilde{\Phi}\right)\left(-\det\tilde{G}_{ab}\right)^{1/2}$. We have defined $\tau_p = T_p e^{-\Phi_0}$; this is the physical tension of the Dp-brane when the background value of the dilaton is Φ_0. To obtain the propagator we expand the spacetime action to second order in $h_{\mu\nu} = G_{\mu\nu} - \eta_{\mu\nu}$ and to second order in $\tilde{\Phi}$. Also , we need to choose a gauge for the gravitational calculation. The simplest gauge for perturbative calculation is
$F_\nu \equiv \partial^{\hat{\mu}}h_{\mu\nu} - \frac{1}{2}\partial_\nu h_\mu^{\hat{\mu}} = 0$, where a hat indicates that an index has been raised with the flat space metric $\eta^{\mu\nu}$.
(Note that an D-brane is a dynamical object on which strings can end. The term is a contraction of "Dirichlet brane". The coordinates of the attached strings satisfy Dirichlet boundary conditions in the directions normal to the brane and Neumann conditions in the directions tangent to the brane. The mass or tension of a D-brane is intermediate between that of an ordinary quantum or a fundamental string and that of "soliton". The soliton is a state whose classical limit is a smooth, localized and topologically nontrivial classical field configuration; this includes particle states, which are localized in all directions, as well as extended objects. The low energy fluctations of D-branes are described by supersymmetric gauge theory, which is non-Abelian for coincident branes). Expanding the action to second order and adding a gauge-fixing term $-F_\nu F^{\hat{\nu}}/4\kappa^2$, the spacetime action becomes:
$S = -\frac{1}{8\kappa^2}\int d^{26}X(\partial_\mu h_{\nu\lambda}\partial^{\hat{\mu}}h^{\hat{\nu}\hat{\lambda}} - \frac{1}{2}\partial_\mu h_\nu^{\hat{\nu}}\partial^{\hat{\mu}}h_\lambda^{\hat{\lambda}} + \frac{2}{3}\partial_\mu\tilde{\Phi}\partial^{\hat{\mu}}\tilde{\Phi})$ (1.15).

2. The oscillation modes of string corresponding to the graviton.

In the paper "Dual Models for Nonhadrons" of J. Scherk and John H. Schwarz of California Institute of Technology, published in April 1974, (2) the possibility of describing particles other than adrons (leptons, photons, gauge bosons, gravitons, etc.) by a dual model is explored. The Virasoro-Shapiro model is studied, interpreting the massless spin-two state of the model as a graviton.
Both the 26-dimensional Veneziano model (VM) and the 10-dimensional meson-fermion model (MFM) have a massless "photon", the nonplanar Virasoro-Shapiro model (VSM) has a massless "graviton", and the MFM has a massless "lepton". The VM and the MFM have both been studied and found to yield the Yang-Mills theory of massless self-interacting vector mesons needed to describe electromagnetism and weak interactions.

2.1. Virasoro-Shapiro Model.

The most general action for massless gravitons and scalars with general coordinate invariance, involving no more than two derivatives, is:
$$S = \int d^4x\sqrt{g}\left\{f_1(\phi) - \frac{1}{16\pi G}Rf_2(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi f_3(\phi)\right\}, \quad (2.1)$$
where the f_1 are functions, analytic at the origin, subject to the normalization constraints $f_2(0) = f_3(0) = 1$. In this equation, g is the determinant of the metric tensor and R is the scalar curvature.

The first term in the equation (2.1) contains no derivatives, while the second and third each contain two derivatives. The form of the action in the equation (2.1) can be simplified by performing a Weyl transformation $g_{\mu\nu} = g'_{\mu\nu} f_2(\phi)^{-1}$ (2.2).

Under this transformation $\sqrt{g}$ and R become: $\sqrt{g} = \sqrt{g'} f_2(\phi)^{-2}$ (2.3) and

$$R = R' f_2(\phi) - 3 f_2(\phi) D'_\alpha \left(\frac{\partial^\alpha f_2(\phi)}{f_2(\phi)} \right) + \frac{2}{2 f_2(\phi)} \partial_\alpha f_2(\phi) \partial^\alpha f_2(\phi), \text{ (2.4)}$$

where D_α represents a covariant derivative.

Then, the action becomes:

$$S = \int d^4x \sqrt{g} \left\{ \frac{f_1(\phi)}{[f_2(\phi)]^2} - \frac{1}{16\pi G} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \frac{f_3(\phi)}{f_2(\phi)} + \frac{3}{16\pi G} \left(\frac{f'_2(\phi)}{f_2(\phi)} \right)^2 \right\},$$ where setting

$k_1(\phi) = \frac{f_1(\phi)}{[f_2(\phi)]^2}$ and $k_2(\phi) = \frac{f_3(\phi)}{f_2(\phi)} + \frac{3}{16\pi G} \left(\frac{f'_2(\phi)}{f_2(\phi)} \right)^2$, we have:

$$S = \int d^4x \sqrt{g} \left\{ k_1(\phi) - \frac{1}{16\pi G} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi k_2(\phi) \right\} \text{ (2.5).}$$

The action can be simplified further by a transformation of ϕ itself. Setting $\phi = \phi[\psi(x)]$, where the function $\phi(\psi)$ satisfies the differential equation $(\phi')^2 k_2(\phi) = 1$, gives an action of the form in the equation (2.5) with $k_2(\phi)$ replaced by one and $k_1(\phi)$ by a new function $d(\phi)$. Therefore the first order terms in α' are completely determined and it only remains to evaluate the zeroth order terms. The absence of constant and linear terms implies that in tree approximation the multigraviton amplitudes must be precisely those of general relativity, as given by the Hilbert-Einstein action (the second term in the equation (2.5)). The vanishing of $d(\phi)$ means that there is no cosmological term in the multigraviton interactions, whereas the vanishing of $d'(0)$, implies that multigraviton amplitudes cannot contain scalar poles. To first order in the zero-slope expansion the action become:

$$S = -\int d^4x \sqrt{g} \left\{ \frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} \text{ (2.6).}$$

There is a TSS (1 tensor and two scalars) interaction in which the graviton couples to the energy momentum tensor of the ϕ field. Furthermore, all pure scalar self-interactions terms vanish identically. The graviton+gauge boson interactions have to be both Yang-Mills invariant and generally covariant. Therefore the unique action for these fields to first order in α' is:

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{g} R - \frac{1}{8} \int d^4x \sqrt{g} g^{\mu\rho} g^{\mu\sigma} Tr[G_{\mu\nu} G_{\rho\sigma}]$$ (2.7). (Note that "Tr" denote the trace of matrix square).

2.2. Interaction of the Scalar Field with Matter.

Before to write the action, is useful to recall that the scalar field couples in all orders to the trace of the stress tensor of matter, as predicted by the Brans-Dicke theory. The parameter ω in the Brans-Dicke theory, which fixes the relative strength of scalar and tensor couplings with matter, is determined by the dual models. The action, for the Brans-Dicke theory takes the form:

$$S = \int d^4x \sqrt{g} \left[-\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \exp(-c\phi) - \frac{1}{2} m^2 \psi^2 \exp(-2c\phi) \right], \text{ (2.8)}$$

where $c = \sqrt{\frac{16G}{3+2\omega}}$. ψ is a scalar field that has been introduced to represent matter. One can easily show that this action gives the equation of motion:

$\Box\phi = \sqrt{\frac{4\pi G}{3+2\omega}} T_{\mu\nu}$ (2.9), where $T_{\mu\nu}$ is the canonical matter stress tensor obtained by varying $g_{\mu\nu}$. In this theory the exchange of a graviton and a scalar between two ψ particles is given by:

$$\frac{1}{q^2}\left[8\pi G\left(T_{\mu\nu}T'_{\mu\nu} - \frac{1}{2}T_{\mu\mu}T'_{\nu\nu}\right) + \frac{4\pi G}{3+2\omega}T_{\mu\mu}T'_{\nu\nu}\right]. \quad (2.10)$$

One sees that in the dual model, the last two terms of the equation (2.10) cancel exactly, which would correspond to $\omega = -1$ in the Brans-Dicke model. Then, if $c = \sqrt{\frac{16\pi G}{3+2\omega}}$, for $\omega = -1$ we have $c = \sqrt{16\pi G}$ and (2.8) become:

$$S = \int d^4x\sqrt{g}\left[-\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi\exp\left(-\sqrt{16\pi G}\phi\right) - \frac{1}{2}m^2\psi^2\exp\left(2\sqrt{16\pi G}\phi\right)\right]$$

(2.11).

In conclusion, the action representing the VM to first order in α' contains photon, graviton, and scalar fields. It is described by the equation:

$$S = \int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G} - \frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] \quad (2.12)$$, where

$f(\phi) = 1 + k\phi + \ldots$, and $k = \frac{5}{\sqrt{6}}\sqrt{8\pi G}$. If the scalar couplings are given by the Brans-Dicke theory, then $f(\phi)$ should be $\exp(k\phi)$.

3. Superstrings.

The superstring action is obtained introducing a supersymmetry on the world-sheet that connect the spacetime coordinates $X^\mu(\tau,\sigma)$, that are bosonic fields on the world-sheet, at a fermionic partner $\Psi^\mu_\alpha(\tau,\sigma)$. The index μ denote that the fermionic coordinate changes as a vector, whose components are spinors on the world-sheet. The theory obtained is defined "superstring theory", and the corresponding action is:

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau\sqrt{-\gamma}\left[\gamma^{ab}\partial_a X^\mu\partial_b X^\nu - i\overline{\psi}^\mu\Gamma^a\nabla_a\psi^\nu - i\overline{\chi}_a\Gamma^b\Gamma^a\psi^\mu\left(\partial_b X^\nu - \frac{i}{4}\overline{\chi}_b\psi^\nu\right)\right]\eta_{\mu\nu}, \quad (3.1)$$

where χ_a is a Majorana gravitino and, as $\sqrt{-\gamma}\gamma^{ab}$, is a Lagrange multiplier without dynamic. The action can be simplified selecting the equivalent of bosonic conformal gauge, defined superconformal gauge, $\gamma_{ab} = \eta_{ab}e^\phi$, $\chi_a = \Gamma_a\zeta$, (3.2) and to make us the identity of bidimensionals gamma matrices $\Gamma_a\Gamma^b\Gamma^a = 0$, the action become

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau\left(\eta^{ab}\partial_a X^\mu\partial_b X^\nu - i\overline{\psi}^\mu\Gamma^a\partial_a\psi^\nu\right)\eta_{\mu\nu}, \quad (3.3)$$ the action of D scalar fields and D fermionic fields free. We have two preserved currents, the energy-momentum tensor, obtained of the change of metric on the world-sheet

$$T_{ab} = -\frac{1}{\alpha'}\left(\partial_a X^\mu\partial_b X_\mu + \frac{i}{4}\overline{\psi}^\mu\left(\Gamma_a\partial_b + \Gamma_b\partial_a\right)\psi_\mu\right) + \frac{1}{2\alpha'}\eta_{ab}\left(\partial^c X^\mu\partial_c X_\mu + \frac{1}{2}\overline{\psi}^\mu\Gamma\partial\psi_\mu\right) = 0, \quad (3.4)$$

and the supercurrent, obtained by varying the gravitino,

$$J^a = \frac{1}{2\alpha'}\Gamma^b\Gamma^a\psi^\mu\partial_b X_\mu = 0. \quad (3.5)$$

The equations (3.4) and (3.5) are constraints of superstring theory. These are called "super-Virasoro" constraints.

3.1. Superstring Interactions.

a)Type IIA superstring.

The action of type IIA superstring is:

$$S_{IIA} = S_{NS} + S_R + S_{CS}, \quad (3.6)$$

$$S_{NS} = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2} e^{-2\Phi}\left(R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}|H_3|^2\right), \quad (3.7)$$

$$S_R = -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_2|^2 + |\tilde{F}_4|^2\right), \quad (3.8)$$

$$S_{CS} = -\frac{1}{4\kappa_{10}^2}\int B_2 \wedge F_4 \wedge F_4. \quad (3.9)$$

We have regrouped terms according to whether the fields are in the NS-NS or R-R sector of the string theory; the Chern-Simons action contains both.
[The NS-NS (Neveu-Schwarz) states in type I and type II superstring theories, are the bosonic closed string states whose left- and right-moving parts are bosonic. The R-R (Ramond-Ramond) states in type I and type II superstring theories, are the bosonic closed string states whose left- and right-moving parts are fermionic. The Chern-Simons term, is a term in the action which involves p-form potentials as well as field strengths. Such a term is gauge-invariant as a consequence of the Bianchi identity and/or the modification of the p-form gauge transformation].

b)Type IIB superstring.

The action of type IIB superstring is:

$$S_{IIB} = S_{NS} + S_R + S_{CS}, \quad (3.10)$$

$$S_{NS} = \frac{1}{2k_{10}^2}\int d^{10}x(-G)^{1/2} e^{-2\Phi}\left(R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}|H_3|^2\right), \quad (3.11)$$

$$S_R = -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right), \quad (3.12)$$

$$S_{CS} = -\frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3, \quad (3.13)$$

where $\tilde{F}_3 = F_3 - C_O \wedge H_3$, (3.14) and $\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$. (3.15)

c)Type I superstring.

The action of type I superstring is:

$$S_I = S_C + S_O \quad (3.16)$$

$$S_C = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}[e^{-2\Phi}\left(R + 4\partial_\mu \Phi \partial^\mu \Phi\right) - \frac{1}{2}|\tilde{F}_3|^2], \quad (3.17)$$

$$S_O = -\frac{1}{2g_{10}^2}\int d^{10}x(-G)^{1/2} e^{-\Phi} Tr_v\left(|F_2|^2\right) \quad (3.18)$$

The open string potential and field strength are written as matrix-valued forms A_1 and F_2, which are in the vector representation as indicated by the subscript on the trace. Here $\tilde{F}_3 = dC_2 - \frac{\kappa_{10}^2}{g_{10}^2}\omega_3$, (3.19) and ω_3 is the Chern-Simons 3-form $\omega_3 = Tr_v\left(A_1 \wedge dA_1 - \frac{2i}{3} A_1 \wedge A_1 \wedge A_1 \right)$. (3.20)

Under an a ordinary gauge transformation $\delta A_1 = d\lambda - i[A_1, \lambda]$, the Chern-Simons form transforms as $\delta\omega_3 = dTr_v(\lambda dA_1)$. (3.21) Thus it must be that $\delta C_2 = \frac{\kappa_{10}^2}{g_{10}^2} Tr_v(\lambda dA_1)$. (3.22)

Hence the equation (3.17) becomes:

$$S_C = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left[\ e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi\right) - \frac{1}{2}\left| dC_2 - \frac{\kappa_{10}^2}{g_{10}^2} Tr_v\left(A_1 \wedge dA_1 - \frac{2i}{3} A_1 \wedge A_1 \wedge A_1 \right)\right|^2\ \right]$$

(3.23)

d)Heterotic strings.

The heterotic strings have the same supersymmetry as the type I string and so we expect the same action. However, in the absence of open strings or R-R fields the dilaton dependence should be $e^{-2\Phi}$ throughout:

$$S_{het} = \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2} e^{-2\Phi}\left[\ R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}\left|\tilde{H}_3\right|^2 - \frac{\kappa_{10}^2}{g_{10}^2} Tr_v\left(\left|F_2\right|^2\right)\ \right]. \quad (3.24)$$

Here

$\tilde{H}_3 = dB_2 - \frac{\kappa_{10}^2}{g_{10}^2}\omega_3$, $\delta B_2 = \frac{\kappa_{10}^2}{g_{10}^2} Tr_v(\lambda dA_1)$ (3.25) are the same as in the type I string, with the form renamed to reflect the fact that it is from the NS sector.

4.<u>D-brane actions.</u>

The coupling of a D-brane to NS-NS closed string fields is the same Dirac-Born-Infeld action as in the bosonic string,

$$S_{Dp} = -\mu_p \int d^{p+1}\xi Tr\left\{ e^{-\Phi}\left[-\det\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)\right]^{1/2}\ \right\}, \quad (4.1)$$

where G_{ab} and B_{ab} are the components of the spacetime NS-NS fields parallel to the brane and F_{ab} is the gauge field living on the brane.

The gravitational coupling is $\kappa^2 = \frac{1}{2}(2\pi)^7 g^2 \alpha'^4$. (4.2) Expanding the action (4.1) gives the coupling of the Yang-Mills theory on the Dp-brane,

$$g_{Dp}^2 = \frac{1}{(2\pi\alpha')^2 \tau_p} = (2\pi)^{p-2} g \alpha'^{(p-3)/2}. \quad (4.3)$$

The Born-Infeld form for the gauge action applies by T-duality to the type I theory, is:

$S = -\frac{1}{(2\pi\alpha')^2 g_{YM}^2}\int d^{10}x Tr\left\{\left[-\det\left(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)\right]^{1/2}\ \right\}$, (4.4) where for the relations (4.2) and (4.3) we have the type I relation $\frac{g_{YM}^2}{\kappa} = 2(2\pi)^{7/2}\alpha'$ (type I) (4.5)

Another low energy action with many applications is that for a Dp-brane and Dp'-brane. There are three kinds of light strings: p-p, p-p', and p'-p', with ends on the respective D-branes. We will consider explicitly the case p=5 and p'=9. The massless content of the 5-9 spectrum amounts to half of a hypermultiplet. The other half comes from strings of opposite orientation, 9-5. The action is fully determined by supersymmetry and the charges; we write the bosonic part:

$$S=-\frac{1}{4g_{D9}^{2}}\int d^{10}xF_{MN}F^{MN}-\frac{1}{4g_{D5}^{2}}\int d^{6}xF'_{MN}F'^{MN}-\int d^{6}x\left[D_{\mu}\chi^{t}D^{\mu}\chi+\frac{g_{D5}^{2}}{2}\sum_{A=1}^{3}\left(\chi_{i}^{t}\sigma_{ij}^{A}\chi_{j}\right)^{2}\right]. \quad (4.6)$$

The integrals run respectively over the 9-brane and the 5-brane, with $M=0,...,9,\quad \mu=0,...,5,$ and $m=6,...,9.$ The covariant derivative is $D_{\mu}=\partial_{\mu}+iA_{\mu}-iA'_{\mu}$ with A_{μ} and A'_{μ} the 9-brane and 5-brane gauge fields. The field χ_{i} is a doublet describing the hypermultiplet scalars. The 5-9 strings have one endpoint on each D-brane so χ carries charges +1 and -1 under the respective symmetries. The gauge couplings g_{Dp} were given in equation (4.3).

5. Mathematics concerning the parallelism between Palumbo's model and string theory.

We take now the relationship of Palumbo's model: $F=\int_{0}^{\infty}F_{i}dF_{i}$ (5.1) Here F denotes the initial energy present at the Big Bang explosion (the explosion of initial black hole) and F_{i} all the partial waves belonging to F (12)-(13). For the parallelism found between Palumbo's model and string theory, F is the oscillation mode of a bosonic string having mass equal to zero (graviton) and F_{i} are the oscillation modes of supersymmetric strings. Then, we have from the equations (d) and (3.1):

$$F=\int_{0}^{\infty}F_{i}dF_{i}\quad -> \quad -\frac{1}{4\pi\alpha'}\int_{M}d\tau d\sigma\sqrt{-\gamma}\gamma^{ab}\partial_{a}X^{\mu}\partial_{b}X_{\mu}=\int_{0}^{\infty}-\frac{1}{4\pi\alpha'}\int d\sigma d\tau\sqrt{-\gamma}$$

$$\left[\gamma^{ab}\partial_{a}X^{\mu}\partial_{b}X^{\nu}-i\overline{\psi}^{\mu}\Gamma^{a}\nabla_{a}\psi^{\nu}-i\overline{\chi}_{a}\Gamma^{b}\Gamma^{a}\psi^{\mu}\left(\partial_{b}X^{\nu}-\frac{i}{4}\overline{X}_{b}\psi^{\nu}\right)\right]\eta_{\mu\nu}\quad (5.2)$$

This equation for the equations (1.15) and (3.24) can be defined also

$$-\frac{1}{8\kappa^{2}}\int d^{26}X\left(\partial_{\mu}h_{\nu\lambda}\partial^{\hat{\mu}}h^{\hat{\nu}\hat{\lambda}}-\frac{1}{2}\partial_{\mu}h_{\nu}^{\hat{\nu}}\partial^{\hat{\mu}}h_{\lambda}^{\hat{\lambda}}+\frac{2}{3}\partial_{\mu}\tilde{\Phi}\partial^{\hat{\mu}}\tilde{\Phi}\right)=\int_{0}^{\infty}\frac{1}{2\kappa_{10}^{2}}\int d^{10}x(-G)^{1/2}e^{-2\Phi}$$

$$\left[R+4\partial_{\mu}\Phi\partial^{\mu}\Phi-\frac{1}{2}\left|\tilde{H}_{3}\right|^{2}-\frac{\kappa_{10}^{2}}{g_{10}^{2}}Tr_{v}\left(\left|F_{2}\right|^{2}\right)\right]\quad (5.3)$$

and for the equation (2.12) of Scherck-Schwarz theory, we have:

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\phi)-\frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi\right]=\int_{0}^{\infty}\frac{1}{2\kappa_{10}^{2}}\int d^{10}x(-G)^{1/2}e^{-2\Phi}$$

$$\left[R+4\partial_{\mu}\Phi\partial^{\mu}\Phi-\frac{1}{2}\left|\tilde{H}_{3}\right|^{2}-\frac{k_{10}^{2}}{g_{10}^{2}}Tr_{v}\left(\left|F_{2}\right|^{2}\right)\right]$$

(5.4), where the sign minus indicates the expansion force: i.e. the Einstein cosmological constant.

With regard the D-branes the equation that is related to F_{i}, thus to the supersymmetric action is (see eq.4.4)

$$S=-\frac{1}{(2\pi\alpha')^{2}g_{YM}^{2}}\int d^{10}xTr\left\{\left[-\det\left(\eta_{\mu\nu}+2\pi\alpha' F_{\mu\nu}\right)\right]^{1/2}\right\},$$

while the equation related to F, thus to the bosonic action is (see eq.4.1)

$$S_{D25}=-\mu_{25}\int d^{26}\xi Tr\left\{e^{-\Phi}\left[-\det\left(G_{ab}+B_{ab}+2\pi\alpha' F_{ab}\right)\right]^{1/2}\right\}.$$

Thus, for parallelism Palumbo's model -> string theory, we have:

$$-\mu_{25}\int d^{26}\xi Tr\left\{e^{-\Phi}\left[-\det\left(G_{ab}+B_{ab}+2\pi\alpha' F_{ab}\right)\right]^{1/2}\right\}=\int_{0}^{\infty}-\frac{1}{(2\pi\alpha')^{2}g_{YM}^{2}}\int d^{10}xTr\left\{\left[-\det\left(\eta_{\mu\nu}+2\pi\alpha' F_{\mu\nu}\right)\right]^{1/2}\right\}$$

(5.5)

6.<u>Further correlations between Palumbo's model and D-brane actions.</u>

a) First order action for type IIB Dirichlet 3-brane.

Now we want to discuss a new first order action for type IIB Dirichlet 3-brane (3). Its form is inspired by the superfield equations of motions obtained from the generalized action principle. The action involves auxiliary symmetric spin tensor fields.
The original D-brane action is of Dirac-Born-Infeld (DBI) type:
$I_{DBI} = -\int_{M_0} d^{p+1}\xi\sqrt{-\det(g_{mn}+F_{mn})}$, (6.1) where $g_{mn} = \partial_m X^{\underline{m}} \eta_{\underline{mn}} \partial_m X^{\underline{n}}$ is the induced metric, and the field F_{mn} are the components of the 2-form field strength $F = dA = d\xi^m \wedge d\xi^n F_{nm}$, $A = d\xi^m A_m$. (6.2)
The moving frame action for the bosonic Dp-brane in flat D=10 space-time reads
$S_D = \int_{M^{p+1}} \left(L_{p+1}^0 + L_{p+1}^1\right)$ (6.3) $L_{p+1}^0 = \frac{1}{(p+1)!} E^{a_0} \wedge ... \wedge E^{a_p} \varepsilon_{a_0...a_p} \sqrt{-\det(\eta_{ab}+F_{ab})}$ (6.4)
$L_{p+1}^1 = Q_{p-1} \wedge \left[dA - \frac{1}{2} E^b \wedge E^a F_{ab} \right]$ (6.5) In (6.4) Q_{p-1} is the Lagrange Multiplier $(p-1)-$form,
$E^a = dX^{\underline{m}}(\xi) u_{\underline{m}}^a(\xi)$ (6.6) is the pull-back of the $(p-1)$ components of a target space vielbein form $(\underline{m} = 0,...,9; \underline{a} = 0,...,9; a = 0,..., p; i = 1,...,9-p)$
$E^{\underline{a}} = dX^{\underline{m}} u_{\underline{m}}^{\underline{a}} = \left(E^a, E^i\right)$ (6.7) which is related to the holonomic vielbein $dX^{\underline{m}}$ by a Lorentz rotation parametrized by the matrix $\left(u_{\underline{m}}^{\underline{a}}\right) = \left(u_{\underline{m}}^a, u_{\underline{m}}^i\right) \in SO(1,9) \Leftrightarrow u_{\underline{m}}^{\underline{a}} \eta^{\underline{mn}} u_{\underline{n}}^{\underline{b}} = \eta^{\underline{ab}}$ (6.8)
The moving frame action for strings and for type I p-branes can be written as
$S_I^{'} = \int_{M^{p+1}} \frac{1}{(p+1)!} E^{a_0} \wedge ... \wedge E^{a_p} \varepsilon_{a_0...a_p}$. (6.9) However, its original form was the first order with respect to the X variable
$S_I^{'} = \int_{M^{p+1}} \left(\frac{1}{p!} E^{a_0} \wedge e^{a_1} \wedge ... \wedge e^{a_p} \varepsilon_{a_0...a_p} - \frac{1}{(p+1)!} e^{a_0} \wedge e^{a_1} \wedge ... \wedge e^{a_p} \varepsilon_{a_0...a_p} \right)$ (6.10) where $e^a = d\xi^m e_m^a(\xi)$ is the (auxiliary) world volume vielbein 1-form field. For the case of a type IIB 3-brane, an adequate choice of the matrix m is given by $m_a^b = \delta_a^b + \frac{1}{z(F)\bar{z}(F)} Tr\left(\tilde{\sigma}_a F \sigma^b \bar{F}\right)$. (6.11) In the trace of (6.11) F, $\bar{F}$ are spinor representations for the selfdual and the anti-selfdual part of the tensor F_{ab}
$F_{\alpha\beta} = F_{\beta\alpha} = \frac{i}{4} F^{ab}\left(\sigma_a \tilde{\sigma}_b\right)_{\alpha\beta}$, $\bar{F}_{\dot{\alpha}\dot{\beta}} = \bar{F}_{\dot{\beta}\dot{\alpha}} = -\frac{i}{4} F^{ab}\left(\tilde{\sigma}_a \sigma_b\right)_{\dot{\beta}\dot{\alpha}}$, (6.12)
$F_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \equiv F_a^b \sigma_{\alpha\dot{\alpha}}^a \tilde{\sigma}_b^{\dot{\beta}\beta} = 2\delta_\alpha^\beta \bar{F}_{\dot{\alpha}}^{\dot{\beta}} + 2\delta_{\dot{\alpha}}^{\dot{\beta}} F_\alpha^\beta$, $* F_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} = 2\delta_\alpha^\beta \bar{F}_{\dot{\alpha}}^{\dot{\beta}} - 2\delta_{\dot{\alpha}}^{\dot{\beta}} F_\alpha^\beta$, (6.13)
and the scalar factors z and its complex conjugate $\bar{z}$ are expressed through the F_{ab} tensor as
$z = \frac{1}{2}\left(1 + \frac{i}{8}\varepsilon^{abcd} F_{ab} F_{cd} + \sqrt{-\det(\eta + F)}\right)$, (6.14)
The action for the D3-brane can be represented as $S_{D3}^{'} = \int_{M^{1+3}} \left(L_4^{0'} + L_4^1\right)$, (6.15) where
$L_4^{0'} = \det(m)\sqrt{-\det(\eta_{ab}+F_{ab})}\left(\frac{1}{3*3!} E^{a'} m^{-1}{}_{a'}^{a} \wedge e^b \wedge e^c \wedge e^d \varepsilon_{abcd} - \frac{1}{3*4!} e^a \wedge e^b \wedge e^c \wedge e^d \varepsilon_{abcd} \right)$
(6.16) can be obtained from (6.10) if one replaces e^a by $e^b m_b^a$, includes an overall multiplier $\sqrt{-\det(\eta + F)}$ and uses the identities

$$\varepsilon_{abcd} m_{a'}^{a} m_{b'}^{b} m_{c'}^{c} m_{d'}^{d} = \varepsilon_{a'b'c'd'} \det(m), \qquad \varepsilon_{abcd} m_{b'}^{b} m_{c'}^{c} m_{d'}^{d} = \det(m) m^{-1}{}_{a}^{a'} \varepsilon_{a'b'c'd'},$$

The second term in the functional (6.15) $L_4^1 = Q_2 \wedge \left[e^{-\frac{\phi}{2}} \left(dA - B_{(2)} \right) - \frac{1}{2} E^b \wedge E^a F_{ab} \right]$, (6.17) corresponds to (6.5), but with the dependence on dilaton $\phi = \phi(X(\xi))$ and NS-NS 2-form background field $B_2 = \frac{1}{2} dX^{\underline{m}} \wedge dX^{\underline{n}} B_{\underline{nm}}(X)$ restored.

The superfield (embedding) equations for type II super-D3-brane include the symmetric spin tensor (super)fields $h_{\alpha\beta}$, $\bar{h}_{\dot{\alpha}\dot{\beta}}$. They are expressed in terms of anti-selfdual and selfdual components $F_{\alpha\beta}$ and $\bar{F}_{\dot{\alpha}\dot{\beta}}$ of the antisymmetric tensor F_{ab} by $h_{\alpha\beta} = \frac{1}{z(F)} F^{\alpha\beta}$, $\bar{h}_{\dot{\alpha}\dot{\beta}} = \frac{1}{\bar{z}(F)} \bar{F}_{\dot{\alpha}\dot{\beta}}$, (6.18) where the functions z, $\bar{z}$ are defined in eq.(6.14). The expression (6.11) for the matrix m simplifies when written in the terms of these spin tensors

$$m_a^b = \delta_a^b + \frac{1}{2} Sp\left(\tilde{\sigma}_a h \sigma^b \bar{h} \right), \Leftrightarrow m_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \equiv m_a^b \sigma_{\alpha\dot{\alpha}}^a \tilde{\sigma}_b^{\beta\dot{\beta}} = 2\left(\delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}} + h_\alpha^\beta \bar{h}_{\dot{\alpha}}^{\dot{\beta}} \right). \quad (6.19)$$

From eqs.(6.14) we have $z + \bar{z} = 1 + \sqrt{-\det(\eta + F)}$, $z - \bar{z} = \frac{i}{8} \varepsilon^{abcd} F_{ab} F_{cd}$ (6.20) whereas from eqs.(6.12) and (6.18) we obtain

$\frac{i}{4} \varepsilon^{abcd} F_{ab} F_{cd} \equiv \frac{1}{2} F_{ab} * F^{ba} = z^2 h^2 - \bar{z}^2 \bar{h}^2$, $\frac{1}{2} F_{ab} F^{ba} = -z^2 h^2 - \bar{z}^2 \bar{h}^2$, (6.21) with the abbreviations $h^2 \equiv h_{\alpha\beta} h^{\alpha\beta}$, $\bar{h}^2 \equiv \bar{h}_{\dot{\alpha}\dot{\beta}} \bar{h}^{\dot{\alpha}\dot{\beta}}$. (6.22) These relations and the identity $-\det(\eta + F) \equiv 1 - \frac{1}{2} F_{ab} F^{ba} - \left(\frac{1}{8} \varepsilon^{abcd} F_{ab} F_{cd} \right)^2$ can be used to write the product of z with $\bar{z}$ (6.14) as

$$4z\bar{z} = \left(1 + \sqrt{-\det(\eta + F)} \right)^2 + \left(\frac{1}{8} \varepsilon^{abcd} F_{ab} F_{cd} \right)^2 = 2(z + \bar{z}) + z^2 h^2 + \bar{z}^2 \bar{h}^2. \quad (6.23)$$

From the second eq.(6.20) together with (6.21) and replacing the first eq.(6.20) by (6.23) we arrive at

$$z - \bar{z} = \frac{1}{2} \left(\bar{z}^2 \bar{h}^2 - z^2 h^2 \right), \quad z + \bar{z} = 2z\bar{z} - \frac{1}{2} \left(\bar{z}^2 \bar{h}^2 + z^2 h^2 \right)., \quad (6.24)$$

The sum and the difference of these equations are homogeneous in z and $\bar{z}$, respectively. Thus for nonvanishing z we can extract a system of linear equations for z and $\bar{z}$

$1 = \bar{z} - \frac{1}{2} h^2 z$, $1 = z - \frac{1}{2} \bar{h}^2 \bar{z}$, (6.25) with the solution $z = \frac{1 + \frac{1}{2} \bar{h}^2}{1 - \frac{1}{4} h^2 \bar{h}^2}$, $\bar{z} = \frac{1 + \frac{1}{2} h^2}{1 - \frac{1}{4} h^2 \bar{h}^2}$. (6.26)

Substituting (6.26) into (6.18) we obtain the expression for the selfdual and anti-selfdual parts (6.12) of F_{ab} and, hence, for the whole tensor F_{ab} from (6.13). The expression for the DBI-like square root can be obtained directly from eqs.(6.26) and (6.20):

$$\sqrt{-\det(\eta + F)} = \frac{\left(1 + \frac{h^2}{2} \right) \left(1 + \frac{\bar{h}^2}{2} \right)}{1 - \frac{h^2 \bar{h}^2}{4}} \quad (6.27)$$

The inverse matrix m^{-1} and the determinant $\det(m)$ are $m^{-1}{}_a^b = \left(\delta_a^b - \frac{1}{2} Sp\left(\tilde{\sigma}_a h \sigma^b \bar{h} \right) \right) \frac{1}{\sqrt{\det(m)}}$, (6.28)

$$\det(m) = \left(1 - \frac{h^2\bar{h}^2}{4}\right)^2 . \ (6.29)$$

Eq.(6.28) can be obtained directly from the spinor representation for the matrix m (6.19) while the simplest way to obtain (6.29) is to use a special gauge, where only two of the components of the tensor F_{ab} $F_{01} = f_+$, $F_{34} = f_-$ (6.30) are nonvanishing. Substituting (6.27), (6.28), (6.29), into (6.15), (6.16), (6.17) we arrive at $S'_{D3} = \int_{M^{1+3}} \left(L_4^{0'} + L_4^{1'}\right)$, (6.31)

$$L_4^{0'} = \frac{\left(1+\frac{h^2}{2}\right)\left(1+\frac{\bar{h}^2}{2}\right)}{3*3!}\left[E^{a'}\left(\delta_{a'}^{a} - \frac{1}{2}\tilde{\sigma}_{a'}^{\dot{\beta}\beta} h_\beta^\alpha \sigma_{\alpha\dot{\alpha}}^a \bar{h}_{\dot{\beta}}^{\dot{\alpha}}\right)\wedge e^a \wedge e^c \wedge e^d \varepsilon_{abcd} - \frac{1}{2}\left(1-\frac{h^2\bar{h}^2}{4}\right)e^a \wedge e^b \wedge e^c \wedge e^d \varepsilon_{abcd}\right],$$

(6.32)

$$L_4^{1'} = Q_2' \wedge \left[\left(1-\frac{h^2\bar{h}^2}{4}\right)\left(dA - B_{(2)}\right) - \frac{1}{4}E^{\alpha\dot{\alpha}} \wedge E_{\beta\dot{\beta}}\left(\delta_\alpha^\beta\left(1+\frac{h^2}{2}\right)\bar{h}_{\dot{\alpha}}^{\dot{\beta}} + \delta_{\dot{\alpha}}^{\dot{\beta}}\left(1+\frac{\bar{h}^2}{2}\right)h_\alpha^\beta\right)\right], (6.33)$$

where, in addiction to (6.22) the bispinor representation for the vielbein indices $E^{\alpha\dot{\alpha}} = E^\alpha \tilde{\sigma}_a^{\alpha\dot{\alpha}}$ are used. We recall that $h_{\alpha\beta} = h_{\beta\alpha}$, $\bar{h}_{\dot{\alpha}\dot{\beta}} = \bar{h}_{\dot{\beta}\dot{\alpha}}$ are auxiliary symmetric spin tensor fields replacing F_{ab}. The functional (6.31), (6.32), (6.33) is the result for the first order action for the type IIB D3-brane.

b) A supersymmetric action functional describing the interaction of the fundamental superstring with the D=10, type IIB Dirichlet super-9-brane (4).

The geometric action for the super-D9-brane in flat D=10, type IIB superspace is $S = \int_{M^{10}} L_{10} = \int_{M^{10}} \left(L_{10}^0 + L_{10}^1 + L_{10}^{WZ}\right)$ (6.34), where $L_{10}^0 = \Pi^{\wedge 10}|\eta + F|$ (6.35) with

$$|\eta + F| \equiv \sqrt{-\det\left(\eta_{\underline{mn}} + F_{\underline{mn}}\right)},\quad \Pi^{\wedge 10} \equiv \frac{1}{(10)!}\varepsilon_{\underline{m}_1 \ldots \underline{m}_{10}} \Pi^{\underline{m}_1} \wedge \ldots \wedge \Pi^{\underline{m}_{10}},\ (6.36)$$

$$L_{10}^1 = Q_8 \wedge \left(dA - B_2 - \frac{1}{2}\Pi^{\underline{m}} \wedge \Pi^{\underline{n}} F_{\underline{nm}}\right). \ (6.37)$$

$A = dx^m A_m(x)$ is the gauge field inherent to the Dirichlet branes, B_2 represents the NS-NS gauge field with flat superspace value $B_2 = i\Pi^{\underline{m}} \wedge \left(d\Theta^1\sigma_{\underline{m}}\Theta^1 - d\Theta^2\sigma_{\underline{m}}\Theta^2\right) + d\Theta^1\sigma^{\underline{m}}\Theta^1 \wedge d\Theta^2\sigma_{\underline{m}}\Theta^2$ (6.38) and field strength $H_3 = dB_2 = i\Pi^{\underline{m}} \wedge \left(d\Theta^1\sigma_{\underline{m}} \wedge d\Theta^1 - d\Theta^2\sigma_{\underline{m}} \wedge d\Theta^2\right)$. (6.39) The Wess-Zumino Lagrangian form is the same as the one appearing in the standard formulation

$$L_{10}^{WZ} = e^{F_2} \wedge C\big|_{10},\quad C = \oplus_{n=0}^5 C_{2n},\quad e^{F_2} = \oplus_{n=0}^5 \frac{1}{n!}F_2^{\wedge n},\ (6.40)$$

where the formal sum of the RR superforms $C = C_0 + C_2 + \ldots$ and of the external powers of the 2-form $F_2 \equiv dA - B_2$ (6.41) is used and $\big|_{10}$ means the restriction to the 10-superform input.

The equations of motions from the geometric action (6.34)-(6.37) split into the algebraic ones obtained from the variation of auxiliary fields Q_8 and $F_{\underline{mn}}$

$F_2 \equiv dA - B_2 \equiv \frac{1}{2}\Pi^{\underline{m}} \wedge \Pi^{\underline{n}} F_{\underline{nm}}$, (6.42) $Q_8 = \Pi_{\underline{nm}}^{\wedge 8}\sqrt{|\eta + F|}(\eta + F)^{-1\underline{nm}}$ (6.43) and the dynamical ones

$dQ_8 + dL_8^{WZ-D7} = 0$, (6.44) $\Pi_{\underline{m}}^{\wedge 9}\sqrt{|\eta + F|}(\eta + F)^{-1\underline{mn}}\sigma_{\underline{n\mu\nu}} \wedge \left(d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}}h_{\underline{\rho}}^{\underline{\nu}}\right) = 0.$ (6.45)

Turning back to the fermionic equations (6.45), let us note that after decomposition $d\Theta^{2\underline{\nu}} - d\Theta^{1\underline{\rho}}h_{\underline{\rho}}^{\underline{\nu}} = \Pi^{\underline{m}}\Psi_{\underline{m}}^{\underline{\nu}}$, (6.46) where $\Psi_{\underline{m}}^{\underline{\nu}} = \nabla_{\underline{m}}\Theta^{2\underline{\nu}} - \nabla_{\underline{m}}\Theta^{1\underline{\rho}}h_{\underline{\rho}}^{\underline{\nu}}$ (6.47) and $\nabla_{\underline{m}}$ defined by $d = dx^m\partial_m = \Pi^{\underline{m}}\nabla_{\underline{m}}$, one arrives for (6.45) at

$$-i\Pi^{\wedge 10}\sqrt{|\eta+F|}\sigma_{\underline{k\mu\nu}}\Psi_{\underline{m}}^{\underline{\nu}}(\eta+F)^{-1\underline{mn}}=0, \quad \Leftrightarrow \quad \sigma_{\underline{k\mu\nu}}\Psi_{\underline{m}}^{\underline{\nu}}(\eta+F)^{-1\underline{mn}}=0. \ (6.48)$$

In order to represent the kinetic term $\int_{M^{1+1}}\hat{L}_0 \equiv \int_{M^{1+1}}\frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--}$ as an integral over the D9-brane world volume too, we have to introduce of the harmonic fields in the whole 10-dimensional space or in the D9-brane world volume

$u_{\underline{m}}^{\underline{a}}(x) \equiv \left(u_{\underline{m}}^{++}(x), u_{\underline{m}}^{--}(x), u_{\underline{m}}^{i}(x)\right) \in \ SO(1,9)$ and $v_{\underline{\mu}}^{\underline{\alpha}} = \left(v_{\underline{\mu}q}^{I+}, v_{\underline{\mu}\dot{q}}^{I-}\right) \in \ Spin(1,9)$

Such a "lifting" of the harmonics to the super-D9-brane worldvolume creates the fields of an auxiliary ten dimensional $SO(1,9)/(SO(1,1)*SO(8))$ "sigma model". The only restriction for these new fields is that they should coincide with the "stringy" harmonics on the worldsheet:

$$u_{\underline{m}}^{++}(x(\xi))=\hat{u}_{\underline{m}}^{++}(\xi), \quad \hat{u}_{\underline{m}}^{--}(x(\xi))=\hat{u}_{\underline{m}}^{--}(\xi), \quad u_{\underline{m}}^{i}(x(\xi))=\hat{u}_{\underline{m}}^{i}(\xi) \ (6.49)$$

$v_{\underline{\mu}}^{\underline{\alpha}}(x(\xi))=\hat{v}_{\underline{\mu}}^{\underline{\alpha}}(\xi)$:

$$v_{\underline{\mu}q}^{I+}(x(\xi))=\hat{v}_{\underline{\mu}q}^{I+}(\xi), \quad v_{\underline{\mu}\dot{q}}^{I-}(x(\xi))=\hat{v}_{\underline{\mu}\dot{q}}^{I-}(\xi) \ (6.50)$$

In this manner we arrive at the full supersymmetric action describing the coupled system of the open fundamental superstring interacting with the super-D9-brane:

$$S=\int_{M^{10}}\left(L_{10}+J_8\wedge L_{IIB}+dJ_8\wedge A\right)=$$

$$=\int_{M^{10}}\left[\Pi^{\wedge 10}\sqrt{-\det(\eta_{\underline{mn}}+F_{\underline{mn}})}+Q_8\wedge\left(dA-B_2-\frac{1}{2}\Pi^{\underline{m}}\wedge\Pi^{\underline{n}}F_{\underline{mn}}\right)+e^F\wedge C|_{10}\right]$$

$$+\int_{M^{10}}J_8\wedge\left(\frac{1}{2}E^{++}\wedge E^{--}-B_2\right)+\int_{M^{10}}dJ_8\wedge A \quad (6.51)$$

Also the equations (6.31)-(6.33) and (6.51) can be connected at the Palumbo's model, precisely at the F_i, thus at the superstring action. Then, taking the expression obtained from parallelism between Palumbo's model and string theory and putting, in the right hand, type IIB action, from equations (2.12) and (3.10)-(3.13) we have:

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr(G_{\mu\nu}G_{\rho\sigma})f(\phi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left(R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|H_3|^2\right)-\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(|F_1|^2+|\tilde{F}_3|^2+\frac{1}{2}|\tilde{F}_5|^2\right)-$$

$$-\frac{1}{4\kappa_{10}^2}\int C_4\wedge H_3\wedge F_3 \ \Rightarrow$$

$$\Rightarrow\int_{M^{1+3}}\frac{\left(1+\frac{h^2}{2}\right)\left(1+\frac{\bar{h}^2}{2}\right)}{3*3!}\left[E^{a'}\left(\delta_{a'}^{a}-\frac{1}{2}\tilde{\sigma}_{a'}^{\dot{\beta}\beta}h_\beta^\alpha\sigma_{\alpha\dot{\alpha}}^{a}\bar{h}_{\dot{\beta}}^{\dot{\alpha}}\right)\wedge e^a\wedge e^c\wedge e^d\varepsilon_{abcd}+\right.$$

$$-\frac{1}{2}\left(1-\frac{h^2\bar{h}_2}{4}\right)e^a\wedge e^b\wedge e^c\wedge e^d\varepsilon_{abcd}\ ]+Q_2^{'}\wedge\left[\left(1-\frac{h^2\bar{h}^2}{4}\right)(dA-B_{(2)})-\frac{1}{4}E^{\alpha\dot{\alpha}}\wedge E_{\beta\dot{\beta}}\right.$$

$$\left(\delta_\alpha^\beta\left(1+\frac{h^2}{2}\right)\bar{h}_{\dot{\alpha}}^{\dot{\beta}}+\delta_{\dot{\alpha}}^{\dot{\beta}}\left(1+\frac{\bar{h}^2}{2}\right)h_\alpha^\beta\right)\ ] \ (6.52) \text{ and}$$

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr(G_{\mu\nu}G_{\rho\sigma})f(\phi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left(R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|H_3|^2\right)-\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(|F_1|^2+|\tilde{F}_3|^2+\frac{1}{2}|\tilde{F}_5|^2\right)-$$

$$-\frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3 \Rightarrow$$

$$\Rightarrow \int_{M^{10}}\left[\Pi^{\wedge 10}\sqrt{-\det\left(\eta_{\underline{mn}}+F_{\underline{mn}}\right)}+Q_8 \wedge\left(dA-B_2-\frac{1}{2}\Pi^{\underline{m}}\wedge\Pi^{\underline{n}}F_{\underline{mn}}\right)+e^F \wedge C\Big|_{10}\right]+$$

$$+\int_{M^{10}}J_8 \wedge\left(\frac{1}{2}E^{++}\wedge E^{--}-B_2\right)+\int_{M^{10}}dJ_8 \wedge A \quad (6.53)$$

With regard to the geometrical formulation of gravity, concerning the higher spins gauge theory (6), the MacDowell and Mansouri action is:

$$S_{MM}=-\frac{1}{4\kappa^2\Lambda}\int d^4x\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{abcd}\left(R_{\mu\nu}{}^{ab}R_{\rho\sigma}{}^{cd}+4L^{-2}e_\mu^a e_\nu^b R_{\rho\sigma}{}^{cd}+4L^{-4}e_\mu^a e_\nu^b e_\rho^c e_\sigma^d\right). \quad (6.54)$$

In this equation the last two terms represent the Einstein-Hilbert gravitational action with cosmological constant $\Lambda \neq 0$.

With regard to the linear "unfolded" theory, concerning always higher spins gauge theory, we have the following equations:

$(P*Q)(Y)=\frac{1}{(2\pi)^4}\int d^4Ud^4VP(Y+U)Q(Y+V)\exp i\left(U^{\underline{\alpha}}C_{\underline{\alpha\beta}}V^{\underline{\beta}}\right)$, (6.55) where Y,U,V are Majorana spinors of $so(3,2)$ so that $Y_{\underline{\alpha}}=\left(y_\alpha,\bar{y}^{\dot{\alpha}}\right)$, while C is the conjugation charge matrix,

$$C_{\underline{\alpha\beta}}=\begin{pmatrix}\varepsilon_{\alpha\beta} & 0\\ 0 & \varepsilon^{\dot{\alpha}\dot{\beta}}\end{pmatrix}.$$

$P(Y)*Q(Y)=\int\frac{d^4Ud^4V}{(2\pi)^4}e^{i(Y-U)^{\underline{\alpha}}C_{\underline{\alpha\beta}}(Y-V)^{\underline{\beta}}}P(U)Q(V)$, (6.56) that lead to the (6.55) through the variables exchange $U \to U+Y, V \to V+Y$, and

$$(P*Q)(Y,Z)=\frac{1}{(2\pi)^4}\int d^4Ud^4VP(Y+U,Z+U)Q(Y+V,Z-V)\exp i\left(u_\alpha v^\alpha+\bar{u}_{\dot{\alpha}}\bar{v}^{\dot{\alpha}}\right). \quad (6.57)$$

These equations (concerning the higher spins theory, where the cosmological constant $\Lambda \neq 0$) are related with the last term of expression (6.52), thus at the first order action for the type IIB D3-brane, and justifies the sign minus regarding the expansion force in the equation representing the bosonic string action (concerning the graviton, i.e. eq.(2.12)).

7. Correlations between Palumbo's model and some string actions concerning the Pre-Big Bang cosmology.

The string effective action is determined by the usual requirement that the string motion is conformally invariant at the quantum level. The fundamental point is the (non-linear) sigma model describing the coupling of a closed string to external metric $g_{\mu\nu}$, scalar ϕ, and antisymmetric tensor $B_{\mu\nu}$ fields. In the bosonic sector the action reads:

$$S=-\frac{1}{4\pi\alpha'}\int d^2\xi\left[\sqrt{-\gamma}\gamma^{ij}\partial_i x^\mu\partial_j x^\nu g_{\mu\nu}(x)+\varepsilon^{ij}\partial_i x^\mu\partial_j x^\nu B_{\mu\nu}(x)+\frac{\alpha'}{2}\sqrt{-\gamma}R^{(2)}\phi(x)\right]. \quad (7.1)$$

Here $2\pi\alpha'=\lambda_s^2$, $\partial_i \equiv \partial/\partial\xi^i$, and ξ^i are the coordinates spanning the two-dimensional string world-sheet $(i,j=1,2)$, whose induced metric is $\gamma_{ik}(\xi)$. The coordinates $x^\mu=x^\mu(\xi)$ are the fields determining the embedding of the string world-sheet in the external space (target space), ε_{ij} is the two-dimensional Levi-Civita tensor density, and $R^{(2)}(\gamma)$ is the scalar curvature for the world-sheet metric γ.

The (d+1)-dimensional effective action

$S=-\frac{1}{2\lambda_s^{d-1}}\int d^{d+1}x\sqrt{|g|}e^{-\phi}\left[R+(\nabla\phi)^2-\frac{1}{12}H_{\mu\nu\alpha}^2\right]$, (7.2) is the starting point for the formulation of a string-theory-compatible cosmology in the small-curvature and weak-coupling regime, $\alpha' R \ll 1$, $g_s^2 \ll 1$.

The matter action is given by the sum over all components of the string distribution, $S_m = \sum_i S^i_{strings}$, where we use the action (7.1), and we choose the conformally flat gauge for the world-sheet metric (i.e. $\gamma_{ij} = 0,\ R^{(2)} = 0$):

$$S_{string} = -\frac{1}{4\pi\alpha'}\int d^{d+1}x\delta^{d+1}(x - X(\sigma,\tau))d\sigma d\tau \times \left[\left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu\right)g_{\mu\nu}(x) + \left(\dot{X}^\mu X'^\nu - X'^\mu \dot{X}^\nu\right)B_{\mu\nu}(x)\right]$$

(7.3)

Here σ and τ are the world-sheet coordinates, $\dot{X} = dX/d\tau$ and $X' = dX/d\sigma$.
Pre-Big Bang evolution of the Universe can be described in terms of the low-energy, tree-level action of string theory (5). Taking a generic closed superstring theory, this reads

$$S = -\frac{1}{2}\lambda_s^{1-d}\int d^{d+1}x\sqrt{|g|}e^{-\phi}\left[R + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}(dB)^2 - 2\Lambda\right], \text{ (7.4)}$$

where dB is the (three-form) field strength associated with the antisymmetric tensor $B_{\mu\nu}$.
Equation (7.4) receives corrections either when curvatures become $O(\lambda_s^{-2})$, or when the coupling e^ϕ becomes $O(1)$. In general, this is done by defining $g_{\mu\nu} = \tilde{g}_{\mu\nu}e^{\frac{2}{d-1}(\phi-\phi_0)}$. (7.5) Using the Einstein metric $\tilde{g}$, the result for the action (7.4) is simply:

$$S = -\frac{1}{2}\lambda_P^{1-d}\int d^{d+1}x\sqrt{\left|g^{(E)}\right|}\left[R - \frac{1}{d-1}\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}e^{-\frac{4}{d-1}\phi}(dB)^2\right], \text{ (7.6)}$$

where $\lambda_P^{d-1} = e^{\phi_0}\lambda_s^{d-1}$ is the present value of the Planck length, if we take ϕ_0 to coincide with the present (constant) value of the dilaton.
The computation of the spectrum according to the theory of cosmological perturbations requires the choice of the "frame", i.e. of the basic set of fields used to parametrize the action. In the S-frame, and in d+1 dimensions, the gravidilaton effective action takes the form:

$$S = \frac{1}{2\lambda_s^{d-1}}\int d^{d+1}x\sqrt{|g|}e^{-\phi}\left[-R + \omega(\nabla_\mu\phi)^2\right]. \text{ (7.7)}$$

Consider the field redefinition $g \to \tilde{g}$ performed with the help of a new scalar variable ψ: $g_{\mu\nu} = \tilde{g}_{\mu\nu}e^\psi$. (7.8) By expressing the scalar curvature R in terms of $\tilde{g}$, the action (7.7) becomes

$$S = \frac{1}{2\lambda_s^{d-1}}\int d^{d+1}x\sqrt{|\tilde{g}|}e^{-\phi+\frac{(d-1)}{2}\psi}\left[-\tilde{R} + d\tilde{\nabla}^2\psi + \frac{d(d-1)}{4}\left(\tilde{\nabla}_\mu\psi\right)^2 + \omega\left(\tilde{\nabla}_\mu\phi\right)^2\right], \text{ (7.8)}$$

where the tilde denotes geometrical quantities computed with respect to $\tilde{g}$. By setting

$$(\omega = const, \omega > -d/(d-1)):\quad (d-1)\psi = 2\phi,\ \tilde{\phi} = \phi\left[\frac{2d + 2\omega(d-1)}{d-2}\right]^{1/2}, \text{ (7.9)}$$

and neglecting a total derivative, we are led eventually to the E-frame action of general relativity for the new variables $\tilde{g}, \tilde{\phi}$:

$$S = \frac{1}{2\lambda_s^{d-1}}\int d^{d+1}x\sqrt{|\tilde{g}|}\left[-\tilde{R} + \frac{1}{2}\left(\tilde{\nabla}_\mu\tilde{\phi}\right)^2\right]. \text{ (7.10)}$$

The second-order action for tensor perturbations in a cosmological background, can be easily generalized to the S-frame and to the (d+1)-dimensional, Bianchi-I-type metric background, starting from the unperturbed, low energy gravidilaton string effective action (in units $2\lambda_s^{d-1} = 1$):

$$S = -\int d^{d+n+1}x\sqrt{-g}e^{-\phi}\left[R + (\partial_\mu\phi)^2\right], \text{ (7.11)}$$

and considering the transverse, trace-free variable $h_{\mu\nu}$, defined by the first-order perturbation $\delta^{(1)}\phi = 0$, $\delta^{(1)}g_{\mu\nu} = h_{\mu\nu}$, $\delta^{(1)}g^{\mu\nu} = -h^{\mu\nu}$ and $\nabla_\nu h^\nu_\mu = 0 = h^\nu_\mu$, we expand to order h^2 the controvariant components of the metric, $\delta^{(1)}g^{\mu\nu} = -h^{\mu\nu}$, $\delta^{(2)}g^{\mu\nu} = h^{\mu\alpha}h^\nu_\alpha$,

(7.12) the volume density, $\delta^{(1)}\sqrt{-g}=0$, $\delta^{(2)}\sqrt{-g}=-\frac{1}{4}\sqrt{-g}h_{\mu\nu}h^{\mu\nu}$, (7.13) and so on for $\delta^{(1)}R_{\mu\nu}$, $\delta^{(2)}R_{\mu\nu}$. The second-order perturbed action has then contributions from $\sqrt{-g}$, R and $g^{\mu\nu}$:
$\delta^{(2)}S=-\int d^{d+n+1}xe^{-\phi}\left[\delta^{(2)}\left(\sqrt{-g}R\right)+\delta^{(2)}\left(\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)\right]$. (7.14) Integrating by parts, using the unperturbed equations of motion, and neglecting total derivative terms, we obtain the quadratic action:
$\delta^{(2)}S=\frac{1}{4}\int d^{d+n+1}xa^d b^n e^{-\phi}\left(\dot{h}_i^j\dot{h}_j^i+h_i^j\frac{\nabla}{a^2}h_j^i\right)$. (7.15)
The effective action describing the coupling of the dilaton to the fundamental quark and lepton fields building up ordinary macroscopic matter, including all possible loop corrections, can be written in the general form:
$S=\int d^{d+1}x\sqrt{-g}\left[-Z_R(\phi)R-Z_\phi(\phi)(\nabla\phi)^2-V(\phi)+\frac{1}{2}Z_k^i(\phi)(\nabla\psi_i)^2+Z_m^i(\phi)\psi_i^2\right]$, (7.16)
where Z^i are the dilatonic "form factors", to all orders in the loop expansion, and where we have used, for simplicity, a scalar model of fundamental matter fields ψ_i.
The simplest example of quantum-string cosmology model is based on the lowest-order, gravidilaton string effective action:
$S=-\frac{1}{2\lambda_s^{d-1}}\int d^{d+1}x\sqrt{|g|}e^{-\phi}\left[R+\left(\nabla_\mu\phi\right)^2+V\left(\phi,g_{\mu\nu}\right)\right]$, (7.17)
where we have included a (possibly non-local and non-perturbative) dilaton potential V.
Now, for the parallelism between Palumbo's model and string theory, from equation (5.4), (7.4), (7.16), we have:

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\phi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]=$$
$$=\int_0^\infty\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}\left|\tilde{H}_3\right|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_V\left(\left|F_2\right|^2\right)\right]\Rightarrow$$
$$\Rightarrow-\frac{1}{2}\lambda_s^{-9}\int d^{10}x\sqrt{|g|}e^{-\phi}\left[R+g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{12}(dB)^2-2\Lambda\right]\Rightarrow\int d^{10}x\sqrt{-g}\left[-Z_R(\phi)R-Z_\phi(\phi)(\nabla\phi)^2+\right.$$

$\left.-V(\phi)+\frac{1}{2}Z_k^i(\phi)(\nabla\psi_i)^2+Z_m^i(\phi)\psi_i^2\right]$, (7.18) while, from equations (1.12) and (7.17), we have:
$\frac{1}{2\kappa_0^2}\int d^{26}x\sqrt{-G}e^{-2\Phi}\left(R+4\nabla_\mu\Phi\nabla^\mu\Phi\right)\Rightarrow-\frac{1}{2\lambda_s^{d-1}}\int d^{26}x\sqrt{|g|}e^{-\Phi}\left[R+\left(\nabla_\mu\phi\right)^2+V\left(\phi,g_{\mu\nu}\right)\right]$, (7.19) that, for the parallelism above mentioned, can be related also at the equation (7.18).

8. Correlations obtained between some solutions in string theory and Riemann zeta function.

The physicist F.Dyson in 1972 made the crucial observation that the "pair correlation function" of the zeros of the Riemann zeta function is in fact the pair correlation function for the eigenvalues of a matrix taken from a certain family of random Hermitian matrices. What Dyson had spotted was a connection between two apparently unconnected fields of knowledge: quantum physics and number theory. It turned out that physicists looking for ways to characterize the behaviour of atomic particles (the orbital movement of the electrons inside an atom of arbitrary atomic number) had come up with a formula that was very similar to Montgomery's description of the zeros of the Riemann zeta function.
Now, if some equations concerning the Riemann zeta function are related to the orbital movement of the electrons inside an atom of arbitrary atomic number and the particles, in string theory, are the oscillation modes of a fermionic or bosonic string, then it is possible that the Riemann zeta function is related at some equations of string field theory.

In the thesis "Solitonic solutions in string field theory", (7) Prof.V.Puletti has obtained non linear β functions for tachyons of bosonic open strings, in the conjecture of tachyonic profiles slowly variables. Specially, Puletti has found the solutions of conformal points fixed equation of Renormalization Group: $\beta^{T_R} = 0$, solutions that are at finished action (solitons) suitable to describe configurations of unstable branes.
We have obtained interesting connections between some solutions of this thesis and some equations concerning the Riemann zeta function, specifying the Goldston-Montgomery theorem, the mean-value theorems and the Goldbach's problem in short intervals.

a) On some expressions concerning the thesis "Solitonic solutions in string field theory".

The equation of partition function on the disk, that give an effective field theory non local that live on boundary, utilizing the background field method in the disk, is:

$$Z(J) = K\int[dX_\mu]\, e^{-\int_0^{2\pi}\frac{d\tau}{2\pi}\left(\frac{1}{2}X^\mu|i\partial|X^\mu + T(X) - J\cdot X\right)}$$ (8.1) (the partition function of Dirichlet string)

However, the partition function of boundary theory on the disk is generally:

$$Z = K\int[dX_\mu]\, e^{-\left(S_0[X] + \int_0^{2\pi}\frac{d\tau}{2\pi}V[X(\tau)]\right)}.$$ (8.2) Now, from (8.1) we have:

$$Z(k) = K\int[dX_\mu]\, e^{-\int_0^{2\pi}\frac{d\tau}{2\pi}\left(\frac{1}{2}X^\mu|i\partial_\tau|X^\mu + T(X) - ik\cdot\hat{X}\right)},$$ (8.3) that yield at the final expression:

$$Z^{(n)}(k) = K\frac{(-1)^n}{n!}\varepsilon^{-n}\int\prod_{i=1}^{n}dk_i T(k_i)\int_0^{2\pi}\prod_{i=1}^{n}\left(\frac{d\tau_i}{2\pi}\right)e^{-\sum_{i=1}^{n}\frac{k_i^2}{2}G(0) - \sum_{i<j}^{n}k_i k_j G(\tau_i - \tau_j)}\delta\left(k - \sum_{i=1}^{n}k_i\right).$$ (8.4)

Now, computing the partition function at the second order in the expansion in small derivatives of tachyonic field, from (8.4) we obtain:

$$Z_{(2)}^{(n)}(k) = K\frac{(-1)^n}{n!}\varepsilon^{-n}\int\prod_{i=1}^{n}dk_i T(k_i)\int_0^{2\pi}\prod_{i=1}^{n}\frac{d\tau_i}{2\pi}\left\{\frac{1}{2}\sum_{i=1}^{n}\frac{k_i^2}{2}G(0)\frac{1}{2}\sum_{j=1}^{n}\frac{k_j^2}{2}G(0) + \right.$$

$$+\sum_{k=1}^{n}\frac{k_k^2}{2}G(0)\sum_{i<j=1}^{n}k_i k_j G(\tau_i - \tau_j) +$$

$$\left. + \underline{\frac{1}{2}\sum_{i<j=1}^{n}k_i k_j G(\tau_i - \tau_j)\sum_{l<m=1}^{n}k_l k_m G(\tau_l - \tau_m)}\right\}\delta\left(k - \sum_{i=1}^{n}k_i\right),$$ (8.5) and from the term underlined, we obtain:

$$\frac{1}{2}\int_0^{2\pi}\prod_{i=1}^{n}\left(\frac{d\tau_i}{2\pi}\right)\sum_{i<j=1}^{n}k_i k_j G(\tau_i - \tau_j)\sum_{l<m=1}^{n}k_l k_m G(\tau_l - \tau_m).$$ (8.6) When the equals indices are i=l or j=m or j=l or i=m, the integrals in $d\tau$ becomes:

$$\frac{1}{2}\sum_{i<j=1}^{'n}\sum_{l<m=1}^{'n}(k_i k_j)(k_l k_m)\delta_{il}\int_0^{2\pi}\frac{d\tau_i}{2\pi}\frac{d\tau_j}{2\pi}G(\tau_i - \tau_j)\int_0^{2\pi}\frac{d\tau_m}{2\pi}G(\tau_i - \tau_m) =$$

$$= \frac{1}{2}\sum_{i<j=1}^{n}\sum_{l<m=1}^{n}(k_i k_j)(k_l k_m)\delta_{il}\log^2\frac{c}{4}$$ (8.7) =

$$= \int_0^{2\pi}\frac{d\tau_l}{2\pi}\frac{d\tau_m}{2\pi}\frac{d\tau_s}{2\pi}G(\tau_l - \tau_m)G(\tau_s - \tau_m) = \log^2\frac{c}{4}.$$ (8.7a)

From the integral $\int_0^{2\pi}\prod_{i=1}^{n}\left(\frac{d\tau_i}{2\pi}\right)\sum_{k=1}^{n}\frac{k_k^2}{2}G(0)\sum_{i<j=1}^{n}k_i k_j G(\tau_i - \tau_j) = \log\varepsilon\log\frac{c}{4}\sum_{k=1}^{n}k_k^2\log\varepsilon\sum_{i<j=1}^{n}k_i k_j$ and (8.7), we have:

$$\int_0^{2\pi} \frac{d\tau_i}{2\pi}\frac{d\tau_j}{2\pi}G(\tau_i-\tau_j)\Big)\int_0^{2\pi}\frac{d\tau_l}{2\pi}\frac{d\tau_m}{2\pi}\frac{d\tau_p}{2\pi}G(\tau_l-\tau_m)G(\tau_l-\tau_p)\Big)=-\log^3\frac{c}{4}. \quad (8.7b)$$

The fundamental expressions (8.7a) and (8.7b) are related at the equations connected to Riemann zeta function, i.e. the equations concerning the Goldston-Montgomery theorem, the mean-value theorems and the Goldbach's problem in short intervals.

b) <u>On some equations concerning Goldston-Montgomery theorem, mean-value theorems and Goldbach's problem in short intervals.</u>

In the chapter "Goldbach's numbers in short intervals" of Languasco's paper "The Goldbach's conjecture" (11), is described the Goldston-Montgomery theorem.
Assume the Riemann hypothesis. We have the following implications: If $0 < B_1 \le B_2 \le 1$ and $F(X,T) \approx \frac{1}{2\pi} T \log T$ uniformly for $\frac{X^{B_1}}{\log^3 X} \le T \le X^{B_2}\log^3 X$, then

$$\int_1^X (\psi((1+\delta)x)) - \psi(x) - \delta(x)^2 dx \approx \frac{1}{2}\delta X^2 \log\frac{1}{\delta} \quad (8.8),$$ uniformly for $\frac{1}{X^{B_2}} \le \delta \le \frac{1}{X^{B_1}}$. We take the lemma (2) and (3) of this theorem:

<u>Lemma (2)</u>. Let $f(t) \ge 0$ a continuous function defined on $[0,+\infty)$ so that $f(t) << \log^2(t+2)$. If $J(T) = \int_0^T f(t)dt = (1+\varepsilon(T))T\log T$, then: $\int_0^\infty \left(\frac{\sin ku}{u}\right)^2 f(u)du = \left(\frac{\pi}{2} + \varepsilon'(k)\right)k\log\frac{1}{k}$ (8.9), with $|\varepsilon'(k)|$ small for $k \to 0^+$ if $|\varepsilon(T)|$ is uniformly small for $\frac{1}{k\log^2 k} \le T \le \frac{1}{k}\log^2 k$, where for $k=5$, we have $\frac{1}{5\log^2 5} \le T \le \frac{1}{5}\log^2 5$.

<u>Lemma (3).</u>
Let $f(t) \ge 0$ a continuous function defined on $[0,+\infty)$ so that $f(t) << \log^2(t+2)$. If $I(k) = \int_0^\infty \left(\frac{\sin ku}{u}\right)^2 f(u)du = \left(\frac{\pi}{2} + \varepsilon'(k)\right)k\log\frac{1}{k}$ (8.9), then $J(T) = \int_0^T f(t)dt = (1+\varepsilon')T\log T$, (8.10) with $|\varepsilon'|$ small if $|\varepsilon(k)| \le \varepsilon$ uniformly for $\frac{1}{T\log T} \le k \le \frac{1}{T}\log^2 T$, where for $T=5$, we have $\frac{1}{5\log 5} \le k \le \frac{1}{5}\log^2 5$.

The form of the problem of mean-value (8) is that of determining the behaviour of $\frac{1}{T}\int_1^T |\zeta(\sigma+it)|^2 dt$ (8.11) (where ζ is the Riemann zeta function) as $T \to \infty$, for any given value of σ. We have the following theorems: 1) $\int_1^T |\zeta(\sigma+it)|^2 dt < AT\min\left(\log T, \frac{1}{\sigma-\frac{1}{2}}\right)$ (8.12) uniformly for $\frac{1}{2} \le \sigma \le 2$. The particular case $\sigma = \frac{1}{2}$ of this theorem is $\int_1^T \left|\zeta\left(\frac{1}{2}+it\right)\right|^2 dt = 0(T\log T)$ (8.13).

2) As $T \to \infty$ $\int_0^T \left|\zeta\left(\frac{1}{2}+it\right)\right|^2 dt \approx T\log T$ (8.14).

3) Let σ_k be the lower bound of numbers σ such that $\frac{1}{T}\int_1^T \left|\zeta(\sigma+it)\right|^{2k} dt = 0(1)$. (8.15) Then

$\sigma_k \le \max\left(1-\frac{1-\alpha}{1+\mu_k(\alpha)}, \frac{1}{2}, \alpha\right)$ for $0<\alpha<1$.

4) For $0<k\le 2$ $\lim_{T\to\infty}\frac{1}{T}\int_1^T \left|\zeta(\sigma+it)\right|^{2k} dt = \sum_{n=1}^{\infty}\frac{d_k^2(n)}{n^{2\sigma}}$ (8.16) $\left(\sigma>\frac{1}{2}\right)$

Instead of considering integrals of the form $I(T)=\int_0^T \left|\zeta(\sigma+it)\right|^{2k} dt$ (8.17) where T is large, we shall now consider integrals of the form $J(\delta)=\int_0^{\infty}\left|\zeta(\sigma+it)\right|^{2k} e^{-\delta t} dt$ (8.18) where δ is small. The behaviour of these two integrals is very similar. If $J(\delta)=0(1/\delta)$, then $I(T)<e\int_0^T \left|\zeta(\sigma+it)\right|^{2k} e^{-t/T} dt < eJ(1/T)=0(T)$. (8.19) Conversely, if $I(T)=0(T)$, then

$J(\delta)=\int_0^{\infty} I'(t)e^{-\delta t}dt = \left[I(t)e^{-\delta t}\right]_0^{\infty} + \delta\int_0^{\infty} I(t)e^{-\delta t}dt = 0\left(\delta\int_0^{\infty} te^{-\delta t}dt\right) = 0(1/\delta)$. (8.20)

The theorem for integrals is as follows:

If $f(t)\ge 0$ for all t and $\int_0^{\infty} f(t)e^{-\delta t}dt \approx \frac{1}{\delta}$ (8.21) as $\delta\to 0$, then $\int_0^T f(t)dt \approx T$ (8.22) as $T\to\infty$

We first show that, if $P(x)$ is any polynomial, $\int_0^{\infty} f(t)e^{-\delta t}P\left(e^{-\delta t}\right)dt \approx \frac{1}{\delta}\int_0^1 P(x)dx$. It is sufficient to prove this for $P(x)=x^k$. In this case the left-hand side is $\int_0^{\infty} f(t)e^{-(k+1)\delta t}dt \approx \frac{1}{(k+1)\delta} = \frac{1}{\delta}\int_0^1 x^k dx$. Next, we deduce that $\int_0^{\infty} f(t)e^{-\delta t}g\left(e^{-\delta t}\right)dt \approx \frac{1}{\delta}\int_0^1 g(x)dx$ (8.23) if $g(x)$ is continuous, or has a discontinuity of the first kind. For, given ε, we can construct polynomials $p(x), P(x)$, such that $p(x)\le g(x)\le P(x)$ and $\int_0^1 \{g(x)-p(x)\}dx \le \varepsilon$, $\int_0^1 \{P(x)-g(x)\}dx \le \varepsilon$. Then

$\overline{\lim_{\delta\to 0}}\,\delta\int_0^{\infty} f(t)e^{-\delta t}g\left(e^{-\delta t}\right)dt \le \overline{\lim_{\delta\to 0}}\,\delta\int_0^{\infty} f(t)e^{-\delta t}P\left(e^{-\delta t}\right)dt = \int_0^1 P(x)dx < \int_0^1 g(x)dx + \varepsilon$, and making $\varepsilon\to 0$ we obtain $\overline{\lim_{\delta\to 0}}\,\delta\int_0^{\infty} f(t)e^{-\delta t}g\left(e^{-\delta t}\right)dt \le \int_0^1 g(x)dx$. Similarly, arguing with $p(x)$, we obtain

$\underline{\lim_{\delta\to 0}}\,\delta\int_0^{\infty} f(t)e^{-\delta t}g\left(e^{-\delta t}\right)dt \ge \int_0^1 g(x)dx$, and (8.23) follows. Now let $g(x)=0$ $\left(0\le x<e^{-1}\right)$, $=1/x$ $\left(e^{-1}\le x\le 1\right)$. Then $\int_0^{\infty} f(t)e^{-\delta t}g\left(e^{-\delta t}\right)dt = \int_0^{1/\delta} f(t)dt$ and $\int_0^1 g(x)dx = \int_{1/e}^1 \frac{dx}{x} = 1$. Hence $\int_0^{1/\delta} f(t)dt \approx \frac{1}{\delta}$, which is equivalent to (8.22).

If $f(t)\geq 0$ for all t, and, for a given positive m, $\int_0^\infty f(t)e^{-\delta t}dt \approx \frac{1}{\delta}\log^m \frac{1}{\delta}$, (8.24) then

$\int_0^T f(t)dt \approx T\log^m T$. (8.25) The proof is substantially the same. We have

$\int_0^\infty f(t)e^{-(k+1)\delta t}dt \approx \frac{1}{(k+1)\delta}\log^m\left\{\frac{1}{(k+1)\delta}\right\} \approx \frac{1}{(k+1)\delta}\log^m \frac{1}{\delta}$, (8.26) and the argument runs as before, with $\frac{1}{\delta}$ replaced by $\frac{1}{\delta}\log^m \frac{1}{\delta}$. From (8.26) for $m=3, \delta = 4/c$ and $k=\frac{c}{4}-1$, we have:

$\int_0^\infty f(t)e^{-(k+1)\delta t}dt \approx \log^3 \frac{c}{4}$, (8.26a) while, for $m=2, \delta = 4/c$ and $k=\frac{c}{4}-1$, we have:

$\int_0^\infty f(t)e^{-(k+1)\delta t}dt \approx \log^2 \frac{c}{4}$.(8.26b)

In the paper "A note on the exceptional set for Goldbach's problem in short intervals", (9) J.Kaczorowski, A.Perelli and J.Pintz, investigate the size of the exceptional set for Goldbach's problem in short intervals under the assumption of the Generalized Riemann Hypothesis (GRH).

Let $2\leq H\leq N$, $L=\log N$ and $Q=\frac{1}{2}\sqrt{H}$, we have for the major arcs the following equations:

$\sum_{a=1}^{q}{}^{*}\int_{-1/qQ}^{1/qQ}|R(\eta,q,a)|^2 d\eta << \frac{q}{\varphi(q)}\sum_{x(\bmod q)}\int_{-1/qQ}^{1/qQ}|\psi'(2N,\chi,\eta)|^2 d\eta + \frac{\varphi(q)L^4}{qQ}$, (8.27) and, assume GRH, for any $\chi(\bmod q)$ $\int_{-1/qQ}^{1/qQ}|\psi'(2N,\chi,\eta)|^2 d\eta << \frac{NL^2}{qQ}$. (8.28) From (8.27) and (8.28), we get:

$\sum_{a=1}^{q}{}^{*}\int_{-1/qQ}^{1/qQ}|R(\eta,q,a)|^2 d\eta << \frac{NL^2}{Q}$, (8.29) where, for N=5 and H=4, we have

$\sum_{a=1}^{q}{}^{*}\int_{-1/qQ}^{1/qQ}|R(\eta,q,a)|^2 d\eta << 5\log^2 5$. (8.30)

For the minor arcs, we have from (8.29) the following equation:

$\int_{-1/qQ}^{1/qQ}\left|S\left(\frac{a}{q}+\eta\right)\right|^2 d\eta << \frac{1}{\varphi(q)^2}\int_{-1/qQ}^{1/qQ}|T(\eta)|^2 d\eta + \int_{-1/qQ}^{1/qQ}|R(\eta,q,a)|^2 d\eta << \frac{N}{\varphi(q)^2}+\frac{NL^2}{Q} << \frac{N\log^2 P}{P^2}+\frac{NL^2}{Q}$

(8.31), where, for Q=1, P<Q, P=1/2 (P is a function of H and N), we have:

$\frac{1}{\varphi(q)^2}\int_{-1/qQ}^{1/qQ}|T(\eta)|^2 d\eta + \int_{-1/qQ}^{1/qQ}|R(\eta,q,a)|^2 d\eta << 20\log^2 2 + 5\log^2 5$. (8.32)

In the paper "On the exceptional set for the 2k-twin primes problem" of A.Perelli and J.Pintz, (10) we have the following theorem:

Let k be a positive number, $0<\varepsilon<1/2$ and $A>0$ be arbitrary constants and $H=N^{1/2+\varepsilon}$. Then for any $0\leq V\leq N/4$

$\sum_{K=V}^{V+H}\left|\Psi(N,2k)-2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\prod_{\substack{p|k\\p>2}}\left(\frac{p-1}{p-2}\right)(N-2k)\right|^2 <<_{\varepsilon,A} HN^2L^{-A}$. (8.33) We have the following estimates:

$\sum_{K=V}^{V+H}\left|\int_M |S(\alpha)|^2 e(-2k\alpha)d\alpha - 2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\prod_{\substack{p|k\\p>2}}\left(\frac{p-1}{p-2}\right)(N-2k)\ \right|^2 << HN^2L^{-A}$, (8.34) and

$\int_{(\xi-1/H,\xi+1/H)\cap m} |S(\alpha)|^2 d\alpha << NL^{-A-1}$ (8.35) uniformly for $\xi \in [0,1]$. With regard the estimate for the major arcs, we have the following expressions:

$\int_M |S(\alpha)|^2 e(-2k\alpha)d\alpha = N \sum_{q \le L^{B/2}} \frac{\mu(q)^2}{\varphi(q)^2} c_q(-2k) + 0(NL^{-B/2+1})$, (8.36) hence

$$\sum_{k=V}^{V+H} \left| \int_M |S(\alpha)|^2 e(-2k\alpha)d\alpha - 2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\prod_{\substack{p|k \\ p>2}}\left(\frac{p-1}{p-2}\right)(N-2k) \right|^2 << N^2 L^{-B+2} \sum_{k=V}^{V+H} d(2k)^2 + HN^2 L^{-A}$$

(8.37). By Theorem 2 of Brun-Titchmarsh theorem for multiplicative functions, we have that the first term in the right-hand side of (8.37) is $<< HN^2 L^{-A}$ and (8.34) follows. Thus, we have:

$$\sum_{k=V}^{V+H} \left| \int_M |S(\alpha)|^2 e(-2k\alpha)d\alpha - 2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\prod_{\substack{p|k \\ p>2}}\left(\frac{p-1}{p-2}\right)(N-2k) \right|^2 << HN^2 L^{-A} + HN^2 L^{-A}$$, where

for $A = 2, L = \log N, H = N^{1/2+\varepsilon}, \varepsilon = 0.5, N = 5 \rightarrow H = 5$, we have:

$$\sum_{k=V}^{V+H} \left| \int_M |S(\alpha)|^2 e(-2k\alpha)d\alpha - 2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\prod_{\substack{p|k \\ p>2}}\left(\frac{p-1}{p-2}\right)(N-2k) \right|^2 << 250\log^2 5 . \quad (8.38)$$

With regard the estimate for the minor arcs, we have the following expression:

$\int_{\xi_{a/q} \setminus \xi'_{a/q}} \left| S\left(\frac{a}{q}+\eta\right) \right|^2 d\eta << \frac{L^2}{q^2} \int_{L^B/qN}^{1/qQ} |T(\eta)|^2 d\eta + \int_{-1/qQ}^{1/qQ} |R(\eta,q,a)|^2 d\eta$. (8.39) We have that the total contribution of the first integral in the right-hand side of this expression to the quantity in (8.35) is

$<< \sum_{q \le L^{B/2}} \frac{L^2}{q^2} NqL^{-B} + \sum_{L^{B/2} \le q \le Q} \frac{L^2}{q^2} N << NL^{-A-1}$. (8.40) From (8.39)-(8.40) we get

$\int_{(\xi-1/H,\xi+1/H)\cap m} |S(\alpha)|^2 d\alpha << L \sup_{1 \le M \le Q} \sum_{q \in Y_M^{\xi}} \int_{-1/qQ}^{1/qQ} |R(\eta,q,a')|^2 d\eta + NL^{-A-1}$, (8.41) where a' denotes that $a(\bmod q)$ such that $I_{a/q}$ intersects $\left(\xi - \frac{1}{H}, \xi + \frac{1}{H}\right)$. Further, the total contribution to (8.41) of the term $0(N^{1/2})$ in the expression $R(\eta,q,a) = \frac{1}{\varphi(q)} \sum_{\chi} \chi(a)\tau(\overline{\chi})\Psi'(N,\chi,\eta) + 0(N^{1/2})$ is

$<< L \sum_{q \approx M} \frac{N}{qQ} << N^{1/2} L << NL^{-A-1}$. (8.42) Hence, we obtain:

$\int_{(\xi-1/H,\xi+1/H)\cap m} |S(\alpha)|^2 d\alpha << NL^{-A-1} + NL^{-A-1}$, (8.43) where for $A = 2, N = 5$ and $L = \log N$, we have the final expression:

$\int_{(\xi-1/H,\xi+1/H)\cap m} |S(\alpha)|^2 d\alpha << 10\log^3 5$. (8.44)

With regard to the connections between equations of string theory and equations related with the Riemann zeta function, we can see that the eq.(8.7a) is related with lemma(2), lemma(3), (8.26b), (8.30), (8.32) and (8.38);while the eq.(8.7b), is related with (8.26a) and (8.44). For example, we have:

$\int_0^{\infty} f(t) e^{-(k+1)\delta t} dt \Rightarrow \int_0^{2\pi} \frac{d\tau_l}{2\pi} \frac{d\tau_m}{2\pi} \frac{d\tau_s}{2\pi} G(\tau_l - \tau_m) G(\tau_s - \tau_m)$, (8.45) and

$\sum_{a=1}^{q}{}^{*} \int_{-1/qQ}^{1/qQ} |R(\eta,q,a)|^2 d\eta \Rightarrow \frac{c}{4} \left[\int_0^{2\pi} \frac{d\tau_l}{2\pi} \frac{d\tau_m}{2\pi} \frac{d\tau_s}{2\pi} G(\tau_l - \tau_m) G(\tau_s - \tau_m) \right]$. (8.46)

Further, these equations can be related to F (Palumbo's model), therefore at the bosonic string action. Indeed, from (5.2), for example, we have:

$$\int_0^\infty F_i dF_i = F \Rightarrow \int_0^\infty -\frac{1}{4\pi\alpha'}\int d\sigma d\tau\sqrt{-\gamma}\left[\gamma^{ab}\partial_a X^\mu \partial_b X^\nu - i\overline{\psi}^\mu \Gamma^a \nabla_a \psi^\nu - i\overline{\chi}_a \Gamma^b \Gamma^a \psi^\mu \left(\partial_b X^\nu - \frac{i}{4}\overline{X}_b \psi^\nu\right)\right]\eta_{\mu\nu}$$

$$= -\frac{1}{4\pi\alpha'}\int_M d\tau d\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu \Rightarrow \int_0^{2\pi}\frac{d\tau_l}{2\pi}\frac{d\tau_m}{2\pi}\frac{d\tau_s}{2\pi}G(\tau_l - \tau_m)G(\tau_s - \tau_m) \Rightarrow$$

$$\Rightarrow \int_0^\infty f(t)e^{-(k+1)\delta t}dt \;. \quad (8.47)$$

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References

(1) J.Polchinski – "String Theory" Voll.I-II – Cambridge University Press, 1998.
(2) J.Scherk, J.H.Schwarz – "Dual Models for Nonhadrons" – California Institute of Tachnology, April 1974.
(3) I.A.Bandos, W.Kummer – "A Polynomial First Order Action for the Dirichlet 3-brane" – hep-th/9707110 – Phys.Lett.B.
(4) I.Bandos, W.Kummer – Superstring "ending" on super-D9-brane: a supersymmetric action functional for the coupled brane system – hep-th/9906041.
(5) M.Gasperini, G.Veneziano – "The Pre- Big Bang Scenario in String Cosmology" - hep-th/0207130 – CERN-TH/2002-104.
(6) C.Iazeolla – "Sulle interazioni dei campi di gauge di spin arbitrario" – hep-th/0410090.
(7) V.Puletti – "Soluzioni Solitoniche in Teoria di Campo di Stringa" – Tesi di Laurea Specialistica in Fisica – Università degli Studi di Perugia, 2003-2004.
(8) E.C.Titchmarsh – "The Theory of the Riemann Zeta-function" – Oxford Science Publications, 1986.
(9) J.Kaczorowski, A.Perelli, J.Pintz – "A Note on the Exceptional Set for Goldbach's Problem in Short Intervals" – Mh.Math.116, 275-282 (1993).
(10) A.Perelli, J.Pintz – "On the exceptional set for the 2k-twin primes problem" – Compositio Mathematica 82: 355-372, 1992.
(11) A.Languasco – "La Congettura di Goldbach" – Tesi di Dottorato in Matem. – Univ.Genova, 1995.
(12) A.Palumbo – "Dalla Fisica alla Metafisica"- E.DI.S.U. NAPOLI 1 – Luglio 2001.
(13)A.Palumbo – "Origine e Immortalità dell'anima" – Ed.Giuseppe Laterza – Bari 2005.