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Abstract

We present a model of endogenous formation of R&D agreements among firms in which also the timing of R&D investments is made endogenous. The purpose is to bridge two usually separate streams of literature, the endogenous formation of R&D alliances and the endogenous timing literature. This allows to consider the formation of R&D agreements over time. It is shown that, when both R&D spillovers and investment costs are sufficiently low, firms may find difficult to maintain a stable agreement due to the strong incentive to invest noncooperatively as leaders. In such a case, the stability of an R&D agreement requires that the joint investment occurs at the initial stage, thus avoiding any delay. When instead spillovers are sufficiently high, cooperation in R&D constitutes a profitable option, although firms also possess an incentive to sequence their investment over time. Finally, when spillovers are asymmetric and the knowledge mainly leaks from the leader to the follower, to invest as follower becomes extremely profitable, making R&D alliances hard to sustain unless firms strategically delay their joint investment in R&D.

Keywords: R&D Investment, Spillovers, Endogenous Timing, R&D Alliances, Endogenous Research Cartels.

JEL Classification Numbers: C72, D43, L11, L13, O30.

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1 Introduction

A long-standing theme in the industrial organization literature has been the explanation of the incentives for firms to form R&D alliances and the analysis of the effects of cooperation on innovation and social welfare. A clear understanding of this phenomenon is indeed crucial to guide technology and industrial policies. In this article we approach this issue by focussing on the role of strategic timing in shaping the incentive for firms to engage in R&D cooperative agreements (or R&D cartels).

As is well known, a research agreement is an alliance between firms in order to coordinate their research and development activities in a joint project, and to share, to some extent, the knowledge obtained from this common effort. Therefore, the creation of such research agreements allows the firms not only to coordinate their research efforts but also to improve information-sharing. Many reasons may induce firms to form research cartels. First, innovation is expensive, and the possibility of cost sharing and avoidance of duplication can strongly diminish the expenses to each member. Second, the risk for a firm that its own innovation programmes will not produce valuable results is reduced, since a research agreement has greater possibilities of diversification and each member can share risks with the other members. Third, the members of a research alliance can acquire a greater competitive advantage than nonmembers, which implies that there can be a concrete hazard in being left out of such cartels (see on this topic, Baumol 1992; see also Katz and Ordover 1990, Hernan *et al.* 2003 and Alonso and Marin 2004 for empirical studies).

The IO literature has also stressed the role of knowledge flows (or spillovers) for R&D cooperation. On one side, both theoretical and empirical studies emphasize how spillovers may enhance the benefits stemming from R&D cooperation. When spillovers are high enough, internalizing them produces an increase in the aggregate level of R&D, and the elimination of duplication efforts, which clearly leads to a reduction in research expenditures. On the other hand, high spillovers - typical of loose appropriability regimes - also increase the incentive to cheat by partners in research alliances and profit from free-riding, thus threatening the stability of the research cartel (Kesteloot and Veugelers 1995, Cassiman and Veugelers 2002, Belderbos *et al.* 2004).

Let us consider, as an example, the pharmaceutical sector. While this sector has witnessed a plethora of research agreements over the last years, probably also in light of the substantial technical and market uncertainty inherent in pharmaceutical R&D, only about one third of all alliances between pharmaceutical and biotechnology firms are formed at the initial development stage of the new drug (see Rogers *et al.* 2005 and *Recombinant Capital* web site).[1] Even though the management literature stresses mainly the role of market uncertainty and risk aversion by firms in committing capital to highly uncertain developmental projects, we argue that there could be an interplay between the choice of signing a research agreement *at a given time* and the different strategies to appropriate innovation rents, such as patenting, exploitation of first mover advantage, internalization of information flows, and so on. In this paper we emphasize the strategic use of the timing of R&D investment made by the participants to an R&D collaboration. In particular, a research agreement can strategically be anticipated or postponed to prevent some of its participants from unilaterally exploting a first or second mover advantage in the noncooperative scenario.

With the exception of a few papers, very scant attention is paid to strategic timing issues in both theoretical and empirical studies. among the others, Duso *et al* (2010), analyze the drivers of alliance dynamics across heterogeneous industrial sectors, and observe that firms may prefer to wait and enter a research coalition at a subsequent moment of time, since, in each period of time they weight the benefits against the costs of being a research cartel member. This study finds that, on average, four firms enter a research joint venture (RJV) yearly, while the average entry decreases with the age of these RJVs. In the theoretical literature, a number of papers, departing from d'Aspremont and Jacquemin's (1988) pioneering work, have analyzed the effects of research alliances in models with endogenous R&D (see, among others, Katz and Ordover 1990, Kamien *et al.* 1992, Suzumura 1992, Petit and Tolwinski, 1997, 1999). However, in these models, the creation of research agreements is exogenously assumed.

More recently, the endogenous coalition formation literature has attempted to endogenize the formation of R&D cartels by applying noncooperative models of coalition formation (see Bloch 2003 and 2004 and Yi and Shin 2000). Here a crucial aspect to assess the stability of a given structure of agreements among firms is the sign of the externalities of R&D investments which, in turn, depend on the level of spillovers. For sufficiently high spillovers, forming a research cartel reduces the underinvestment in R&D, since the externalities due

[1]These stylized facts are consistent with what emerges from the R&D Insight database employed by Danzon *et al.* 2005, insofar as they assess the propensity to strategically delay some of the agreements over time.

to the public-good nature of R&D investments are internalized. Thus, alliances of firms can invest more than small groups, and this, in turn, may trigger some firms to leave the coalition and free ride on the existing cartels. Moreover, different R&D alliance formation rules may yield different outcomes in terms of stability of cooperation (see, for instance, Yi & Shin 2000). In some cases, the whole industry alliance of firms investing in R&D can be stable, especially if there are no synergies and, after breaking the agreement, all firms end up investing as singletons (see, for instance, Yi 2003, Bloch 2003 and Marini 2008 for surveys). However, the stability of alliances is no longer guaranteed if firms are assumed to decide endogenously their timing of investment.

The endogenous-timing approach was firstly introduced by Hamilton and Slutsky's (1990) within a duopoly game. In their *extensive game with observable delay*, the authors describe a two stage setup in which, at a preplay stage, two players (duopolists) decide independently whether to move early or late in the basic game (e.g., a duopoly quantity game). If both players announce the same timing, that is (*early*, *early*) or (*late*, *late*), the basic game is played simultaneously. If the players' time-announcements differ, the basic game is played sequentially, with the order of moves as announced by the players. It is shown that the two leader-follower configurations (with either order of play) constitute pure subgame perfect equilibria of the extended game only if at least one player's payoff as follower weakly dominates her corresponding payoff in the simultaneous game. When, conversely, the payoff of a follower is lower than in the simultaneous case, the only pure strategy subgame perfect Nash equilibrium prescribes that both players play simultaneously.

A few recent papers have introduced the possibility for firms to sequence their R&D activities in a model *à la* d'Aspremont & Jacquemin (1988) with asymmetric spillovers. While some of these works assume a given *exogenous* timing for the investment game (Goel 1990, Crampes and Langinier 2003, Halmenschlager 2004, Atallah 2005, De Bondt 2007) some others endogenize the timing of investment (Amir *et al.* 2000, Tesoriere 2008) by adopting a framework *à la* Hamilton and Slutsky (1990). The degree of technological spillovers is shown to be crucial for these games to possess strategic substitutes vs. strategic complements attributes and, thus, to give rise to simultaneous vs. sequential endogenous timing R&D equilibria (see Amir *et al.*, 2000). Nevertheless, these models, comparing sequential versus simultaneous move games, do not consider explicitly the possibility for firms to form research agreements.

Our purpose in this paper is to bridge these two otherwise separate streams of literature, the noncooperative formation of R&D agreements and the endogenous timing approach, with the aim to study the formation of research alliances when the timing of R&D investments is endogenous. This allows for a far more complete picture of R&D agreements, by considering the possible formation of these agreements over time. It is shown that, when both R&D spillovers and investment costs are sufficiently low, firms may find difficult to maintain a stable agreement due to the strong incentive to invest noncooperatively as leaders. In such a case, the stability of an R&D agreement requires that the joint investment occurs at the initial stage, thus avoiding any delay. When instead spillovers are sufficiently high, cooperation in R&D constitutes a profitable option, although firms also possess an incentive to sequence their investment over time. Finally, when spillovers are asymmetric and the knowledge mainly leaks from the leader to the follower, to invest as follower becomes extremely profitable, making R&D alliances hard to sustain unless firms strategically delay their joint investment in R&D. Some of these results can provide an explanation to various stylized facts, such as the tendency of firms to strategically anticipate or postpone their R&D agreements as due to different levels of their R&D investment costs and spillover rates.

This paper is organized as follows. Section 2 lays out the notation and introduces the setup adopted in the paper. Section 3 and 4 apply this setup by building a model *à la* d'Aspremont & Jacquemin (1988) with symmetric and asymmetric R&D spillovers and present the main results. Section 5 concludes.

2 The Setup

The typical modelling approach to R&D collaboration among firms usually assumes that, at a first stage, firms can form an R&D alliance with their competitors and, at a second stage, the formed alliance decides cooperatively its joint level of investment in R&D. At a third and final stage, every firm sets noncooperatively its strategic market variable, typically quantity or price, to compete oligopolistically with all other firms. Our aim is to introduce a variant of this setup assuming that at the first stage each firm decides not only whether to form or not an R&D agreement, but also the timing of its investment in R&D. More specifically, both the R&D agreement formation process and the timing of the investment are made endogenous. Introducing endogenous timing basically determines at which stage of the game a single firm or an R&D cartel will play its investment in R&D. This feature of the model aims to capture the complementarity between the timing of firm R&D investments and the formation of a research cartels. We here focus our analysis on the two-firm case.

2.1 R&D Alliances & Timing Formation Game

We assume that, at a pre-play stage, denoted with t_0, each firm i ($i = 1, 2$) sends simultaneously a message to its rival announcing both its intention to form irrevocably an R&D alliance or stay as singleton as well as its intention to commit to a specific timing for its R&D investment. Every firm message set M_i can be denoted as:

$$M_i = [(\{i,j\}, t_1), (\{i,j\}, t_2), (\{i\}, t_1), (\{i\}, t_2)] \quad i = 1, 2 \text{ and } j \neq i. \tag{1}$$

The message space M contains 16 different message profiles $\mathbf{m} \in M_1 \times M_2$, which, in turn, can induce the following set of nonempty R&D timing-partitions $P(\mathbf{m})$,

$$\mathcal{P} = [(\{1,2\}^{t_1}), (\{1,2\}^{t_2}), (\{1\}^{t_1}, \{2\}^{t_1}), (\{1\}^{t_2}, \{2\}^{t_2}), (\{1\}^{t_1}, \{2\}^{t_2}), (\{1\}^{t_2}, \{2\}^{t_1})].$$

Differently from the Hamilton and Slutsky's (1990) endogenous timing game applied to the R&D investment game (see, for instance, Amir *et al.* 2000), here the two firms may also form a research cartel either at period t_1 or t_2.[2] In our model the temporal choice of an R&D cartel is purely strategic and is made to prevent the rival firm to exploit noncooperatively a first or second mover advantage. We will assume that, in order to be formed, a research alliance with a given timing of investment in R&D requires the *unanimity* of firms decisions. If firms send messages indicating both the same R&D alliance and the same investment timing, then they will sign a binding agreement to invest at the prescribed time. Otherwise, if one firm disagrees, either on the alliance or on the timing of investment, both firms will play as singletons the R&D investment game, with the timing indicated by their own messages. Formally, for $i, j = 1, 2$ and $j \neq i$

$$\begin{cases} P(\mathbf{m}) = \{1,2\}^{\tau} \text{ if } m_i = m_j = (\{i,j\}, \tau) \text{ and} \\ P(\mathbf{m}) = (\{i\}^{\tau_i}, \{j\}^{\tau_j}) \text{ if } m_i \neq m_j. \end{cases}$$

Note that this R&D agreement formation rule reflects an exclusive membership rule, i.e. one in which the consensus of all members is required to complete the agreement.[3]

[2] Note that by allowing the two firms to cooperate across time, one playing cooperatively at time t_1 and the other at timet_2, does not alter the basic results of the analysis.

[3] For a discussion on which coalition formation rule may be more appropriate according to the specific context, see the discussion contained in Hart & Kurz (1983), Yi (2003) and Ray (2007).

2.2 The Investment Game

Once a message profile has been sent and a timing-partition, denoted $P(\mathbf{m}) \in \mathcal{P}$, has been induced, each firm will optimally choose its cooperative or noncooperative investment level according to the timing prescribed by $P(\mathbf{m})$. At this stage, as well as at the following stages, it is assumed that a firm cannot manipulate its level of investment to convince the rival to renege the timing-partition decided at t_0.

As in d'Aspremont & Jacquemin (1988), each firm i, with $i = 1, 2$, is assumed to set a finite level of investment $x_i \in X_i \subset R_+$ affecting its profit via its production cost $c_i(x_1, x_2)$ which, in turn, influences the market competition between individual firms. Denoting with $q_i \in [0, \infty)$ the market competition variable (here quantity), a firm profit function can be denoted as $\pi_i(\mathbf{q}(\mathbf{x}))$, where $\mathbf{q}(\mathbf{x}) = (q_1(x_1, x_2), q_2(x_1, x_2))$.

In a research agreement $\{1, 2\}^{\tau}$ firms will therefore set cooperatively their level of investment at stage $\tau = t_1$ or t_2, i.e.

$$\mathbf{x}^{c^{\tau}} = \left(x_1^{c^{\tau}}, x_2^{c^{\tau}}\right) \tag{2}$$

such that, for every $i, j = 1, 2$ and $j \neq i$

$$x_i^{c^{\tau}} = \arg\max_{x_i} \sum_{i=1,2} \pi_i\left(\mathbf{q}\left(x_i, x_j^{c^{\tau}}\right)\right),$$

given the profile of quantities optimally chosen at the market stage.

If the firms play simultaneously as singletons at time $\tau = t_1$ or t_2, the appropriate equilibrium concept will be the Nash equilibrium $x^{\tau *}$ of the simultaneous investment game played at stage τ, i.e.

$$\mathbf{x}^{\tau *} = (x_1^{\tau *}, x_2^{\tau *}) \tag{3}$$

such that, for every $i = 1, 2$ and $j \neq i$

$$x_i^{\tau *} = \arg\max_{x_i} \pi_i\left(\mathbf{q}\left(x_i, x_j^{\tau *}\right)\right).$$

Finally, if the firms play sequentially, the relevant equilibrium will be a Stackelberg (subgame perfect Nash) equilibrium, i.e. the profile

$$\mathbf{x}^{\sigma *} = \left(x_i^{\sigma *}, x_j^{\sigma *}\right) \tag{4}$$

such that, for the leader (henceforth firm i)

$$x_i^{\sigma *} = \arg\max_{x_i} \pi_i \left(\mathbf{q} \left(x_i, g_j \left(x_i \right) \right) \right),$$

and for the follower (firm j) $x_j^{\sigma *} = g_j \left(x_i^{\sigma *} \right)$, where $g_j : X_i \to X_j$ is the best-reply mapping:

$$g_j \left(x_i \right) = \arg\max_{x_j} \pi_j \left(\mathbf{q} \left(x_i, x_j \right) \right).$$

Note that for the investment game to be well-defined, all equilibria in (2), (3) and (4) must exist and be unique.

2.3 The Market Game

Once the two firms have either formed a research cartel or chosen their R&D investment as singletons at t_1 or t_2, they will set their market variable at the last stage of the game (denoted with t_3). We assume competition in quantities and a unique Cournot equilibrium among firms, given the equilibrium level of investment $\mathbf{x}^{c^\tau}$, or $\mathbf{x}^{\tau *}$ or $\mathbf{x}^{\sigma *}$ decided at stages t_1, t_2 or both. In particular, the Cournot quantity profile is simply the vector

$$\mathbf{q}^* = (q_1^*, q_2^*)$$

such that, for every firm $i = 1, 2$ and $j \neq i$,

$$q_i^* = \arg\max_{q_i} \ \pi_i(q_i, q_j^*).$$

2.4 Stable R&D Agreements

Given the equilibrium quantities decided by firms at stage t_3, and given the level of investment decided simultaneously or sequentially at stages t_1 and/or t_2 either by the research cartel or by individual firms, firms payoffs can, with a slight abuse of notation, be denoted as $\pi_i(\mathbf{q}^* \left(\mathbf{x}^*(P(\mathbf{m})) \right)$, where $\mathbf{q}^* \left(\mathbf{x}^*(P(\mathbf{m}) \right)$ indicates the equilibrium quantity profile when an investment profile, as defined by (2), or by (3) or finally by (4) is chosen by the firms in a given partition $P(\mathbf{m})$ induced by the message profile $\mathbf{m}$ sent at stage t_0.

Henceforth we make explicit a concept of equilibrium for the message game played at stage t_0. For this purpose, we introduce two different equilibrium concepts. The first is a standard Nash equilibrium of the R&D partition-timing game. The second introduces a social stability requirement, implying that a structure $P(\mathbf{m})$ is stable if and only if the message profile $\mathbf{m}$ is a *strong Nash equilibrium*, i.e., cannot be improved upon by an alternative message announced by a single firm or by the two firms together. This concept is useful to refine over the set of outcome generated by our model. Formally, when a given timing-partition $P \in \mathcal{P}$ is *Nash stable,* the profile $\boldsymbol{\theta}^* = (\mathbf{m}^*, \mathbf{x}^*, \mathbf{q}^*)$ is a subgame perfect Nash equilibrium (SPNE) of the entire game. When, in addition, the message profile $\mathbf{m}$ played at t_0 is also strong Nash, $\boldsymbol{\theta}^*$ IS again a SPNE, with the additional property to be Pareto-optimal for the firms.

Definition 1 *(Nash stability) A feasible R&D timing-partition $P \in \mathcal{P}$ is Nash stable if $P = P(\mathbf{m}^*)$, for some $\mathbf{m}^*$ with the following property:*

$$\pi_i \left(\mathbf{q}^* \left(\mathbf{x}^*(P(\mathbf{m}^*))\right) \geq \pi_i(\mathbf{q}^*(\mathbf{x}^*(P(m_i^{'}, m_j^*)))\right.$$

for every $m_i^{'} \in M_i$ and every firm $i = 1, 2$ with $j \neq i$.

Definition 2 *(Strong Nash stability) A feasible R&D timing-partition $P \in \mathcal{P}$ is strongly stable if $P = P(\widehat{\mathbf{m}})$, for some $\widehat{\mathbf{m}}$ with the following property: there not exists an alternative message profile $\mathbf{m}' \in M_1 \times M_2$ such that*

$$\pi_i(\mathbf{q}^*(\mathbf{x}^*(P(\mathbf{m}'))) \geq \pi_i(\mathbf{q}^*(\mathbf{x}^*(P(\widehat{\mathbf{m}})))$$

for all $i = 1, 2$ and

$$\pi_h(\mathbf{q}^*(\mathbf{x}^*(P(\mathbf{m}'))) > \pi_h(\mathbf{q}^*(\mathbf{x}^*(P(\widehat{\mathbf{m}})))$$

for at least one $h = 1, 2$.

A strong stable Nash equilibrium is at once a Nash equilibrium and a Pareto-optimal message profile.

3 A Duopoly Model with Symmetric Spillovers

We are now ready to apply our framework to the d'Aspremont & Jacquemin's (1988) model. We therefore consider a symmetric duopoly with firms producing a homogeneous good. Along these lines, we assume a linear inverse market demand function

$$P(Q) \ = \max\left\{0,\ a - bQ\right\},$$

with $Q = \sum_{i=1}^{2} q_i$ and a linear cost function for each firm i decreasing in own R&D investment and in a fraction of the rival's effort,

$$c_i(x_i, x_j) = (c - x_i - \beta x_j) \tag{5}$$

for $j \neq i$, and $c \geq x_i - \beta x_j$. In this setup, learning resulting from investment in R&D characterizes the production process, implying that marginal and unit costs decrease as the investment in R&D increases. We allow for the possibility of imperfect appropriability (i.e. for the existence of a technological spillovers between the firms), by introducing a spillover parameter $\beta \in [0, 1]$. Obviously the case of no spillovers ($\beta = 0$) may only arise in a situation of complete intellectual protection. More frequently, however, involuntary information leaks occur due to reverse engineering, industrial espionage or by hiring away employees of an innovative firm. The cases of partial to full spillovers can be modelled by setting $0 < \beta \leq 1$. Here, the parameter β in (5) is assumed to be identical for all firms. However, in Section 4, this parameter, though exogenously given, will differ as due to the cooperative versus non-cooperative nature and to the timing properties of the R&D investment game.

Moreover, we assume a simple quadratic cost function for the investment in R&D given by

$$I_i(x_i) = \gamma \frac{x_i^2}{2},$$

with $\gamma > 0$. This guarantees decreasing returns to R&D expenditure (see e.g. Cheng 1984 and d'Aspremont and Jacquemin 1988). As a result, under Cournot competition in the product market, and setting for simplicity $b = 1$, the last stage profit function for each firm $i = 1, 2$ can be obtained as a function of (x_i, x_j):

$$\pi_i \left(\mathbf{q}^* \left(x_i, x_j\right)\right) = \frac{\left(a - c + \left(2 - \beta\right) x_i + \left(2\beta - 1\right) x_j\right)^2}{9} - \frac{\gamma}{2} x_i^2. \tag{6}$$

3.1 Main Assumptions

Some assumptions are now introduced to ensure the existence and uniqueness of all stages equilibria as well as to simplify the comparative statics.

A.1 *Quantity stage constraint*: $(a/c) > 2$.

A.2 *Profit concavity and best-reply contraction*: $\gamma > 4/3$.

A.3 *Boundaries on R&D efforts*: for every firm, $X_i = [0, c]$. Moreover, for $\beta < 1/2$, we assume $\gamma > \frac{a(2-\beta)(\beta+1)}{4.5c}$ and for $\beta > 1/2$, $\gamma > \frac{a(\beta+1)^2}{4.5c}$.

As explained in detail in the Appendix, assumption A.1 simply ensures that the last stage Cournot equilibrium is unique and interior, with associated positive profits.

Assumption A.2 guarantees both the strict concavity of every firm non-cooperative payoff (6) in its own investment x_i (guaranteed for $\gamma > \frac{8}{9}$) as well as a contraction property on every firm best-replies $g_i(x_j)$, which requires that $\gamma > \frac{4}{3}$.

Assumption A.3 ensures a compact R&D investment set for every firm and imposes some Inada-type conditions to obtain interior investment equilibria in all non-cooperative (simultaneous or sequential) and cooperative R&D games (see also Amir *et al.* 2000, Amir *et al.* 2011a, Tesoriere 2008 and Stepanova and Tesoriere 2011).[4]

Note that by assumption A.2 every firm payoff is strictly concave in its own investment choice and thus best-replies are single-valued and continuous. Investment sets are compact by A.3 and therefore a Nash equilibrium exists by Brower fixed-point theorem. The contraction property implied by A.2 ensures uniqueness of the Nash equilibrium $\mathbf{x}^{\tau *}$. The existence of a Stackelberg equilibrium $\mathbf{x}^{\sigma *}$ - a subgame perfect Nash equilibrium (SPNE) of the sequential R&D game - is guaranteed by both firms continuous payoffs and continuous best-replies, thus implying that a firm as leader faces a continuous maximization problem over a closed set. Then, by the Weierstrass theorem, such a SPNE equilibrium exists. Its uniqueness is guaranteed here by A.3 and by the fact that firms best-replies are single-valued and monotone. Relatively to the cooperative investment level, the strict concavity of every firm profit, under the additional constraint that the two firms select the same collusive investment, implies that also the joint R&D cartel profit is strictly concave. Hence, this will be maximized by a unique investment profile $\mathbf{x}$.

In the next section we characterize all stable R&D agreements with endogenous timing reached by the two firms. As in Hamilton and Slutsky (1990) the firms timing decision is taken conditional on the subgame equilibrium profile induced by the resulting timing structure. So, solving the game amounts to comparing the different basic games associated to all possible scenarios. After a complete analysis of the symmetric case, we extend the setup to the case of asymmetric spillovers. This can offer a broader view on a recent stream

[4] For a detailed description of the consequences occurring to the simultaneous investment game when these boundaries are violated, see, for instance, Amir *et al* 2011b.

of literature concerning endogenous timing under asymmetric spillovers (see, e.g., Amir *et al*, 2000, De Bondt and Vandekerckhove 2008, Tesoriere 2008).

3.2 Cooperative R&D

The R&D cartel made of the two firms investing cooperatively in R&D is assumed to maximize the sum of firms profits, i.e.

$$(7) \qquad \sum_{i=1}^{2} \pi_i \left(\mathbf{q}^* \left(\mathbf{x}\left(\{1,2\}^\tau\right)\right)\right) = \sum_{i=1}^{2} \left\{ \frac{1}{9} \left[a - c + (2-\beta)\, x_i + (2\beta - 1)\, x_j\right]^2 - \gamma \frac{x_i^2}{2} \right\}.$$

where $\mathbf{x} = (x_i, x_j)$ is any arbitrary profile of R&D investment carried out simultaneously by the two firms either at $\tau = t_1$ or at $\tau = t_2$, for $i = 1, 2$ and $j \neq i$. Following most of the literature, we will assume henceforth that the level of investment that maximizes (7) is equal for every firm, i.e., is such that $x_i^{c^\tau} = x_j^{c^\tau}$.[5]

By (7) a firm cooperative investment can be easily obtained as

$$(8) \qquad x_i^{c^\tau}\left(\{1,2\}^\tau\right) = \frac{2(a-c)(1+\beta)}{9\gamma - 2(1+\beta)^2}$$

with an associated equilibrium profit for each firm

$$(9) \qquad \pi_i^C \left(\mathbf{q}^* \left(\mathbf{x}^{c^\tau}\left(\{1,2\}^\tau\right)\right)\right) = \frac{\gamma(a-c)^2}{9\gamma - 2(1+\beta)^2}.$$

3.3 Noncooperative Simultaneous R&D

Differentiating (6) and exploiting the symmetry of firms payoffs, the noncooperative level of investment is obtained as

$$(10) \qquad x_i^{\tau *}\left(\{1\}^\tau, \{2\}^\tau\right) = \frac{2(a-c)(2-\beta)}{9\gamma - 2(2-\beta)(1+\beta)}$$

for $\tau = 1, 2$, with associated a profit given by:

$$\pi_i^N \left(\mathbf{q}^* \left(\mathbf{x}^{\tau *}\left(\{1\}^\tau, \{2\}^\tau\right)\right)\right) = \frac{\gamma(a-c)^2(9\gamma - 2(\beta-2)^2)}{(9\gamma - 2(2-\beta)(1+\beta))^2}.$$

[5] As shown by Salant and Shaffer (1998,1999), for certain values of the parameters, the joint profit maximization may easily imply unequal R&D investments for the two firms.

3.4 Sequential R&D Investment Game

Using again (6) we can easily obtain the best-reply of the j-th firm playing as follower the investment game:

$$g_j(x_i) = \frac{2(2-\beta)(a-c-(1-2\beta)x_i)}{9\gamma - 2(\beta-2)^2}. \tag{11}$$

Therefore the leader and the follower equilibrium investment levels are given by

$$x_i^{\sigma*}\left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2\left(2-\beta\right)\left(a-c\right)\left(3\gamma+2\beta^2-2\right)\left(6\beta+3\gamma-2\beta^2-4\right)}{\Delta}$$

$$x_j^{\sigma*}\left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2\left(2-\beta\right)\left(a-c\right)\Gamma}{\Delta}$$

where

$$\Gamma = \left(26\beta\gamma - 20\gamma - 12\beta - 4\beta^2 + 12\beta^3 + 9\gamma^2 - 4\beta^4 - 8\beta^2\gamma + 8\right)$$

and

$$\begin{aligned}\Delta = {} & 160\gamma - 216\gamma^2 + 81\gamma^3 + 32\beta^5 - 8\beta^6 - \beta^4\left(20\gamma+16\right) + \beta^3\left(64\gamma-64\right) \\ & + \beta\left(216\gamma^2 - 224\gamma + 32\right) + \beta^2\left(24\gamma - 54\gamma^2 + 56\right) - 32\end{aligned}$$

with associated equilibrium profits given by

$$\pi_i^L\left(\mathbf{q}^*\left(\mathbf{x}^{\sigma*}\left(\{i\}^{t_1}, \{j\}^{t_2}\right)\right)\right) = \frac{\left(a-c\right)^2\gamma\left(6\beta+3\gamma-2\beta^2-4\right)^2}{\Delta}$$

$$\pi_j^F\left(\mathbf{q}^*\left(\mathbf{x}^{\sigma*}\left(\{i\}^{t_1}, \{j\}^{t_2}\right)\right)\right) = \frac{(a-c)^2\gamma(9\gamma+8\beta-2\beta^2-8)\Gamma^2}{\Delta^2}$$

Comparing R&D equilibrium investment levels under assumptions A.1-A.3, we can state the following:

Proposition 1 *(i) When firms R&D investments are strategic substitutes ($\beta < \frac{1}{2}$) there exists a $\beta^*(\gamma)$ and a $\overline{\gamma}$ such that, for $\beta < \beta^*(\gamma)$ and $\gamma < \overline{\gamma}$,*

$$x_i^{\sigma*} > x_i^{\tau*} > x_i^{c^{\tau}} > x_j^{\sigma*}.$$

(ii) When firms R&D investments are strategic substitutes ($\beta < \frac{1}{2}$) and $\beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$

$$x_i^{\sigma*} > x_i^{\tau*} > x_j^{\sigma*} \geq x_i^{c^\tau}.$$

(iii) When firms R&D investments are strategic complements ($\beta > \frac{1}{2}$),

$$x_i^{c^\tau} > x_i^{\sigma*} > x_j^{\sigma*} > x_i^{\tau*}$$

for $i = 1, 2$ and $j \neq i$.

Proof. See the Appendix. ■

The above proposition provides a full ranking of firms equilibrium investment levels, as it combines the well-known results by d'Aspremont and Jacquemin (1988), who compare cooperative and simultaneous non-cooperative R&D levels, with Amir *et al.* (2000) analysis, focussing on sequential vs. simultaneous non-cooperative outcomes. In particular, the former study proved that, under high (low) spillovers, i.e. with $\beta > \frac{1}{2}$ ($\beta < \frac{1}{2}$) the cooperative investment level is higher (lower) than the simultaneous Nash investment level, that is $x_i^{c^\tau} > x_i^{\tau*}$ ($x_i^{c^\tau} < x_i^{\tau*}$). This finding, if combined with Amir's *et al.* (2000) results, implies that $x_i^{\sigma*} > x_i^{\tau*} > x_i^{c^\tau}$ ($x_i^{c^\tau} > x_i^{\tau*}$ and $x_i^{\sigma*} > x_j^{\sigma*} > x_i^{\tau*}$). Proposition 1 completes this ranking by also including the cooperative investment levels. It can be noticed (see (11)) that the level of spillover is crucial to determine the slope of the follower's best-reply in the investment game. That is, when the spillover rate is very low (case (i)), the follower's best-reply is extremely steep (and negatively sloped) and this player strongly contracts its equilibrium investment, which is thus even lower than that resulting under a cooperative agreement. A firm investing noncooperatively as leader at stage t_1 can therefore profitably expand its investment, and this may occur in particular when the unit cost of investment in R&D (i.e. γ) is very low and the investor is unlikely to be imitated (low β). Under such circumstances, being a leader can be more profitable than participating to an R&D agreement. When, instead, the spillover rates start to increase, the cooperative investment overcomes that of the follower, although the leader's investment remains very high. Finally, for $\beta > 1/2$, cooperation implies the efficient and highest level of R&D investment, regardless of the level of investment costs.

In what follows, we perform some comparisons of the firms payoffs obtained in the different investment games.[6] First, notice that, by the efficiency of profile $\mathbf{x}^{c^\tau}$, we already know

[6] We recall that in Amir's *et al.* (2000) paper, the following ranking is established for simultaneous and sequential payoffs in the symmetric case: $\pi_i^L(\mathbf{x}^{\sigma*}) > \pi_i^N(\mathbf{x}^{\tau*}) > \pi_j^F(\mathbf{x}^{\sigma*})$ for $\beta < \frac{1}{2}$ and $\pi_j^F(\mathbf{x}^{\sigma*}) > \pi_i^L(\mathbf{x}^{\sigma*}) > \pi_i^N(\mathbf{x}^{\tau*})$ for $\beta > \frac{1}{2}$, where L, N and F denote the leader/Nash simultaneous/follower roles, respectively, in the different R&D investment games.

that $\pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right)$. Moreover, the following lemma proves that for $\beta < \frac{1}{2}$ ($\beta > \frac{1}{2}$) a follower (leader) payoff can never be greater than that of a firm in a cooperative agreement.[7]

Lemma 1 *Under high (low) spillovers $\beta > \frac{1}{2}$ ($\beta < \frac{1}{2}$) the profit of each firm in an R&D agreement is always higher than the profit of a leader (follower), namely, $\pi_i^C > \pi_i^L$ ($\pi_i^C > \pi_j^F$).*

Proof. See the Appendix. ■

The following two propositions complete the full ranking of firm payoffs in all different scenarios and for all levels of spillover rates.

Proposition 2 *When firms R&D investments are strategic substitutes ($\beta < \frac{1}{2}$): (i) there exists a $\beta^*(\gamma)$ and a $\overline{\gamma}$ such that, for $\beta < \beta^*(\gamma)$ and $\gamma < \overline{\gamma}$, the profit obtained by a firm playing as leader in a sequential investment game is higher than that obtained in a cooperative R&D agreement, and the following ranking arises*

$$\pi_i^L > \pi_i^C > \pi_i^N > \pi_j^F.$$

(ii) When, instead $\beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$ or both, the following ranking arises:

$$\pi_i^C \geq \pi_i^L > \pi_i^N > \pi_j^F.$$

Proof. See the Appendix. ■

Figure 1 and 2 illustrate the effect of β on the investment levels and on payoffs, respectively. When firm investments are strategic substitutes ($\beta < 1/2$) there exists a narrow range of the spillover rate (between 0 and $\beta^*(\gamma)$) for which being leader, and thus expanding the investment, turns out to be extremely profitable. This occurs only when the cost to invest in R&D is extremely low ($\gamma < \overline{\gamma}$).

[FIGURE 1 AND 2 APPROXIMATELY HERE]

[7]For ease of notation, in what follows π_i^C istand for $\pi_i^C\left(\mathbf{x}^{c^\tau}\right)$. We will use the same notational shortcut in all noncooperative simultaneous and sequential payoffs at the different investment subgames.

The proposition below completes our findings on firms' profitability under different arrangements assuming that R&D investments are strategic complements:

Proposition 3 *When firms investments are strategic complements ($\beta > \frac{1}{2}$) the profit obtained by a firm in a cooperative R&D agreement is always higher than the profit obtained by a firm investing as follower in the sequential investment game, and the following ranking arises*

$$\pi_i^C > \pi_j^F > \pi_i^L > \pi_i^N.$$

Proof. See the Appendix. ■

As it can be observed in figures 1 and 2, for $\beta > 1/2$, the highest level of investment is selected by the research cartel. Under the sequential game the follower free-rides on the leader's investment and gains a higher profit.

Finally, the next two propositions characterize all Nash and strong Nash stable timing-partitions according to Definitions 1 and 2.

Proposition 4 *(Nash stability) (i) When the spillover rate is such that $\beta < \beta^*(\gamma)$, and $\gamma < \overline{\gamma}$, the Nash stable timing-partitions are given by*

$$\mathcal{P}(\mathbf{m}^*) = [(\{1,2\}^{t_1}), (\{1\}^{t_1}, \{2\}^{t_1})].$$

(ii) When $1/2 > \beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$ or both, the Nash stable timing-partitions are instead given by

$$\mathcal{P}(\mathbf{m}^*) = [(\{1,2\}^{t_1}), (\{1,2\}^{t_2}), (\{1\}^{t_1}, \{2\}^{t_1})].$$

(iii) Finally, for $\beta \in (1/2, 1]$, the Nash stable timing-partitions are given by

$$\mathcal{P}(\mathbf{m}^*) = [(\{1,2\}^{t_1}), (\{1,2\}^{t_2}), (\{1\}^{t_1}, \{2\}^{t_2}), (\{1\}^{t_2}, \{2\}^{t_1})].$$

Proof. See the Appendix. ■

It is obvious that, if we require the strong stability of timing-partitions, by the symmetry of firms all noncooperative partitions in which firms invest simultaneously *à la* Nash are Pareto-dominated by the cooperative allainces. Forming a cooperative research agreement to coordinate costly investments in R&D is clearly more profitable than playing the symmetric investment game *à la* Nash. If, however, $\beta < \beta^*(\gamma)$, we have proven that being leader in the investment game yields a higher profit than playing cooperatively, and therefore the only timing-partition that remains strongly stable is the alliance investing at time t_1. Thus, a cooperative agreement, to be stable, requires that firms anticipate strategically their joint investments.

Proposition 5 *(Strong stability) (i) when the spillover rate $\beta < \beta^*(\gamma)$ and $\gamma < \overline{\gamma}$, the only strong Nash stable R&D timing-partition is*

$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_1}\right)\right].$$

(ii) - (iii) When $1 \geq \beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$ or both, the strong Nash stable R&D timing-partitions are

$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right)\right].$$

Proof. See the Appendix. ■

Our results depart from those obtained in the previous literature. In particular, in our setup, differently from Amir *et al.* (2000), firms can form a strategic alliance to invest cooperatively in R&D, and this alliances may be part of a SPNE of the whole game. Moreover, our model suggests that in forming alliance firms have to consider carefully the effect of timing. If firms procrastinate their cooperative investment, they may risk a defection by a partner breaking the alliance to invest as leader. To avoid this problem, firms have to anticipate strategically their joint investment in R&D. As illustrated in detail, this happens only when investing in R&D is not very costly and spillovers are very low. For higher spillovers, to discipline the stability of a research cartel might be easier and time-constraints for the investment less relevant. Our model also shows that, without requiring Pareto-optimality, even the noncooperative simultaneous (sequential) configurations are stable under low (high) spillovers, i.e with $\beta < 1/2$ ($\beta > 1/2$), as already established in Amir *et al.* (2000). To give an intuition, in a scenario characterized by strategic substitutes, the choice to form a R&D alliance at a certain time might be also motivated by the need to avoid to play as follower and singleton. Besides, when both spillovers and unit investment costs are very low, a deliberate strategy meant to deter the exploitation of the first mover advantage (i.e. a possible strategy to appropriate innovation rents) might be in place. In a regime of high appropriability - and thus of low outgoing spillovers - the probability for firms to cooperate is enhanced (as shown by Cassiman and Veugelers 2002, Belderbos *et al.* 2004), and the only problem potentially affecting stability may stem from the willingness to move first. This is not true any longer for higher spillovers and higher investment costs. In this case, we argue that the two forces behind spillovers may push to collaborate at time 1, when the need to internalize high incoming spillovers prevails, or at time 2, to avoid free-riding and potential defections by partners, typical of low appropriability scenarios.

3.5 An Extension to n-symmetric Firms

Extending our model to n-symmetric firms would allow to check the stability of more complex alliances between firms coordinating their investment in R&D. However, including more than two firms into a model with endogenous timing makes the model itself highly unmanageable. Only intuitive conclusions can be drawn employing our previous analysis and some well-known existing results. A first observation concerns the whole industry R&D agreement (or the grand coalition of firms) investing at stage t_2, i.e., using the above notation, the timing-partition $P = \left(\{N\}^{t_2}\right)$ formed when at stage t_0 all firms $i = 1, 2, .., n$ send the message $m_i = (\{N\}, t_2)$. This partition can be strongly stable if every individual firm investing as follower at stage t_2 would be better off than any firm participating to an R&D agreement investing at stage t_1 as leader. Thus, any coalition $S \subset N$ of firms that deviates from the grand coalition $\left(\{N\}^{t_2}\right)$ by sending one of these alternative messages, $m_S^{'} = (\{S\}, t_2)$ or $m_S^{''} = (\{S\}, t_1)$, would induce either the simultaneous partition

$$P(m_S^{'}) = (\{S\}^{t_2}, \{j\}_{j \in N \setminus S}^{t_2}), \tag{12}$$

where all firms outside S are singletons or, analogously, the sequential partition

$$P(m_S^{''}) = (\{S\}^{t_1}, \{j\}_{j \in N \setminus S}^{t_2}). \tag{13}$$

However, if firms in coalition S cannot improve upon partition $\left(\{N\}^{t_2}\right)$ by playing as leaders as in (13) they would not improve *a fortiori* by playing simultaneously as in (12). Therefore, if we show that in the partition (13) all firms within the research cartel S (regardless of its size) do not improve upon the cooperative partition $\left(\{N\}^{t_2}\right)$, the stability of the grand coalition agreement is proved as a result. When investment decisions are strategic complements ($\beta > 1/2$), it can be proved that the payoff of a symmetric firm playing as singleton follower against the coalition S playing as leader is always higher than the payoff of every firm in S.[8]

[8] For a formal proof of this statement see Currarini and Marini (2003, 2004).

Hence, given the efficiency of the grand coalition, it would be impossible for any coalition S to improve by deviating as leader, given that followers would improve even more their payoffs. Similarly, it can be shown that when R&D investments are strategic substitutes ($\beta < 1/2$) a coalition $S \subset N$ made of followers is beaten by individual firms investing as leaders, and therefore the partition $\left(\{N\}^{t_1}\right)$ - made by the grand coalition of firms investing at time t_1- is strong Nash. The strong stability of these two cooperative timing-partitions already observed in our duopoly model thus extends to an analogous endogenous timing game played by n-symmetric firms.

4 A Duopoly Model with Asymmetric Spillovers

Including asymmetric spillovers into the model equals to introducing a higher degree of realism. As is well-known (see e.g. Atallah 2005), asymmetries in knowledge transmission may derive from differences in protection practices, from geographical localization (e.g. Petit *et al.* 2009), from product differentiation (Amir *et al.* 2000), or from sequential moves in the R&D game, as in R&D models with endogenous timing (Tesoriere 2008). Other sources of asymmetry can arise from different technological capabilities, as in Amir and Wooders (1999, 2000), where knowledge may leak only from the more R&D-active firm to the rival, or from a better absorption capacity influencing the outcome of a technological race, as in De Bondt and Henriques (1995).

The spillover asymmetry arising in our model stems instead from the cooperative versus the non-cooperative nature of the R&D game and from the timing of the R&D investment process. The parameter β_i, $(0 \leq \beta_i \leq 1)$ will represent henceforth the *incoming* spillover for firm $i = 1, 2$. Moreover, let β_i^N denote the firm spillover rate under simultaneous noncooperative R&D, β_i^C the spillover rate under R&D cooperation, and β_i^L, β_j^F the spillover rates for the leader and the follower, respectively, in the sequential investment game, with $i, j = 1, 2$, $i \neq j$.

Our assumptions on spillovers asymmetry are based on the following considerations:

(i) When the two firms invest simultaneously and noncooperatively at stage one or two their spillover rate is assumed to be symmetric and lower than or equal to 0.5 (i.e., $\beta_1^N = \beta_2^N \leq 0.5$). The idea is that the competition in R&D and the simultaneity of firm decisions do not allow for a high amount of knowledge transmission.

(ii) When a noncooperative sequential investment in R&D takes place, the spillover rate can be though to be favorable to the firm playing as follower and unfavorable to the firm playing as leader (i.e. $\beta_j^F > \beta_i^L$). In particular we shall set $\beta_j^F > 0.5$ and $\beta_i^L \leq 0.5$. A sequential order of moves in the R&D investment game implies a greater amount of knowledge leaking out from the leader to the follower than *vice versa.* The rationale is that knowledge leaks also through imitation, thus leading to a strong advantage for the firm that is able to observe the first mover innovative outcome. Therefore, benefits from spillovers should be lower for a first mover (see also Tesoriere 2008). Moreover, we assume that sector-specific features determining the intensity of knowledge diffusion[9] affect to the same extent the incoming spillover for the leader in the sequential game (i.e. β_i^L) and the incoming spillovers for both firms in the simultaneous noncooperative game (i.e., $\beta_i^N \quad i = 1, 2$). Therefore we will set $\beta_i^L = \beta_i^N$.

(iii) When the two firms play cooperatively and form a research cartel, they generally also agree to share to some extent the knowledge obtained from their joint R&D effort. It seems realistic to assume that they might agree to fully share their knowledge, and therefore their spillover rates will be symmetric and sufficiently high (i.e. $\beta_1^C = \beta_2^C$ close or equal to one). Moreover, we assume that knowledge leaks occurring mainly through imitation and favouring the follower in a sequential game are less intense if compared with the voluntary exchange of technological knowledge typical of a research agreement. Thus, we maintain that $\beta_i^C > \beta_j^F$, for $i, j = 1, 2$, $i \neq j$.

Taking into account all the above inequalities, our assumptions on the relationship among spillover values can be summarized as follows:

(14) $$1 \geq \beta_i^C > \beta_j^F > \beta_i^L = \beta_i^N \geq 0 \quad i = 1, 2, \, j \neq i$$

with $\beta_i^L = \beta_i^N \leq 0.5$ and $\beta_j^F > 0.5$.

As in the previous section, we introduce here some assumptions needed to ensure the existence and uniqueness of equilibria at all stages (see the Appendix for further details):

[9] Empirical literature aiming at distinguishing between knowledge spillovers that occur within or across different sectors or technological fields leads to conclude that spillovers are technology-specific and, thus, mainly intra-sectoral. As much as about sixty percent of the citations are directed to other patents classified into the same technological field (see, e.g., Jaffe 1985, 1986; Cincera 1997; Malerba *et al* 2007), while the main sources of knowledge are represented by competitors, suppliers and plants belonging to the same business group (Crespi *et al*, 2008).

B.1 (*quantity stage constraint*). As in the case of symmetric spillovers, $a/c > 2$.

B.2 (*Profit concavity and best-reply contraction property*). Again, $\gamma > 4/3$.

B.3 (*Boundaries on R&D efforts*) For every firm $i = 1, 2$, $X_i = [0, c]$ and $\gamma > \frac{2a\left(\beta_i^C+1\right)^2}{9c}$, for $0.5 < \beta_i^C \leq 1$.

4.1 Noncooperative Sequential R&D with Asymmetric Spillovers

Since only in the case of sequential moves at the investment stage our calculations differ from the symmetric case analyzed in the previous sections, we shall deal henceforth extensively with this scenario. Our aim is to investigate whether the asymmetry in the transmission of knowledge between firms is relevant for the endogenous formation of research alliances.

Using an asymmetric-spillover specification, every firm objective function at the market game stage is given by

$$\pi_i = (a - (q_i + q_j))q_i - \left(c - x_i - \beta_j x_j\right) q_i - \gamma \frac{x_i^2}{2}$$

with $i, j = 1, 2$ and $i \neq j$. Solving the game by backward induction, every firm payoff at the investment stage can be obtained as:

$$\pi_i \left(\mathbf{q}^* \left(x_i, x_j\right)\right) = \frac{1}{9} \left[(a - c) + \left(2 - \beta_j\right) x_i + \left(2\beta_i - 1\right) x_j\right]^2 - \gamma \frac{x_i^2}{2}. \tag{15}$$

Differentiating (15) we obtain the best-reply for the follower in the investment game (here player j):

$$g_j(x_i) = \frac{2(2 - \beta_i^L)[(a - c) - (1 - 2\beta_j^F)x_i]}{9\gamma - 2(2 - \beta_i^L)^2}$$

The sequential equilibrium investment levels for the two firms are given by:

$$x_i^{\sigma*}\left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2A\,(a - c)\left(2\left(\beta_i^L\right)^2 - 6\beta_i^L - 3\gamma + 4\right) B}{(\gamma + 2AB^2)}$$

$$x_j^{\sigma*}\left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2\left(2 - \beta_i^L\right)\left((a - c) - \frac{2\left(1 - 2\beta_j^F\right)A(a - c)\left(2\left(\beta_i^L\right)^2 - 6\beta_i^L - 3\gamma + 4\right)B}{2A\left(3\gamma\beta_j^F - 2\beta_i^L - 6\gamma - 4\beta_i^L\beta_j^F + 2\left(\beta_i^L\right)^2\beta_j^F + 4\right)^2 - \gamma}\right)}{9\gamma - 2\left(2 - \beta_i^L\right)^2}$$

where

$$A = \left(9\gamma + 8\beta_i^L - 2\left(\beta_i^L\right)^2 - 8\right)^{-2}$$
$$B = \left(3\gamma\beta_j^F - 2\beta_i^L - 6\gamma - 4\beta_i^L\beta_j^F + 2\left(\beta_i^L\right)^2\beta_j^F + 4\right)$$

Let equation (14) as well as B.1-B.3 hold. By comparing firms R&D equilibrium investment levels under asymmetric spillovers, we can state:

Proposition 6 *There exists a $\tilde{\beta} \in (0, 1/2)$ such that, if $\beta_i^N = \beta_i^L \leq \tilde{\beta}$, then $x_j^{\sigma *} \geq x_i^{c^\tau} > x_i^{\tau *} > x_i^{\sigma *}$. If instead $\beta_i^N = \beta_i^L \geq \tilde{\beta}$, then $x_i^{c^\tau} \geq x_j^{\sigma *} > x_i^{\tau *} > x_i^{\sigma *}$, for $i, j = 1, 2$ and $i \neq j$.*

Proof. See the Appendix. ■

An illustration of this result is shown in Figure 3. To give an intuition, when (infra-sectoral) spillovers are low, the asymmetry between the incoming spillover of the leader (β_i^L) and that of the follower (β_j^F) is pronounced (since β_j^F is always greater than 0.5). Therefore, the leader faces the lowest incentive to invest in R&D since the high outgoing spillover effect overcomes the first-mover advantage effect. On the other hand, the follower takes advantage of a high learning opportunity and of low knowledge leaks. Moreover, in this case, the R&D investment of the follower overcomes that of the cooperative firm, since a competive effect prevails. Conversely, when spillovers become higher, the asymmetry between leader and follower decreases. In this case a free-riding effect may prevail for both players and the cooperative outcome may become convenient, since cooperation between firms succeeds in internalizing knowledge externalities.

Firms profits could be compared only via numerical simulations. In what follows the numerical values assigned to the parameters are as follows: $a = 38$, $c = 18$, $\gamma = 2$. In addition, we assume that in the case of cooperation firms agree to share a high amount of technological knowledge. Thus we assign a constant value $\beta_i^C = 0.8$. Moreover we set the incoming spillover of the follower such that $1 \geq \beta_i^C > \beta_j^F > 0.5$ (for instance $\beta_j^F = 0.6$ as in Figures 3 and 4).

As depicted in figure 4, there exists a value $\hat{\beta} \in (0, 1/2)$, such that the following payoffs ranking emerges:

$$\pi_j^F > \pi_i^C > \pi_i^N > \pi_i^L$$

for $\beta_i^L = \beta_i^N \leq \hat{\beta}$. As a result, in this case the Nash equilibrium timing-partitions are

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_2}\right), \left(\{1\}^{t_2}, \{2\}^{t_2}\right)],$$

while the only strong Nash partition is given by

$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_2}\right)\right].$$

When instead $\beta_1^L = \beta_i^N \geq \hat{\beta}$, the following payoff ranking comes out:

$$\pi_i^C > \pi_j^F > \pi_i^N > \pi_i^L$$

and, thus,

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right), \left(\{1\}^{t_2}, \{2\}^{t_2}\right)],$$
$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right)\right].$$

[FIGURES 3 AND 4 APPROXIMATELY HERE]

These results can be explained by considering that joint cooperative agreements at time t_1 are particularly at risk when there is a strong incentive to be follower in the R&D investment game. As a matter of fact, firms prefer to wait and observe the rival's move rather then trying to reach an agreement. This happens when spillovers are extremely unbalanced (i.e. when $\beta_1^L = \beta_i^N \leq \hat{\beta}$) towards the firms that wait before investing, thus conferring a strong follower-advantage.These results are also in line with some empirical evidence, showing that firms are more likely to choose a follower strategy if they operate in industries with low knowledge leaks. Therefore, with low infra-sectoral spillovers, the only strongly stable R&D timing partition is represented by the research cartel investing at time 2. On the other hand, timing issues become less relevant when spillovers get higher and choosing a first mover strategy may become convenient for the alliance. This is typical of sectors characterized by intensive knowledge exchanges often coupled with high absorptive capacity by the firms (Sofka and Schmidt, 2005).

Our findings complement the few existing results (Amir et al., 2000; Tesoriere, 2008) on endogenous sequencing in R&D investment with asymmetric spillovers. In particular, Tesoriere (2008) considers only the noncooperative case with certain spillovers values ($\beta_1^L = \beta_i^N = 0$ and $\beta_j^F \in (0, 1]$). Under these assumptions he proves that the only timing configuration that is SPNE involves simultaneous noncooperative play at the R&D stage (with zero spillovers). In contrast, in our setup the noncooperative simultaneous configuration may not be the only Nash stable timing-partition and, in addition, it is never strong Nash stable, as firms always prefer to form an R&D cartel than playing (suboptimally) as singletons the investment game.

5 Concluding Remarks

This paper represents a first attempt to bridge two usually distinct streams of the economic literature, the endogenous formation of R&D agreements, and the endogenous timing of R&D investments in a model with spillovers *à la* d'Aspremont and Jacquemin (1988). This is done by introducing a new setup in which firms express both their intention to form or not an alliance as well as the timing of their effort in R&D. Our approach allows to assess the stability of research cartels against deviations occurring over time. We show that the nature of the interaction between the firms in the investment game plays an important role. In particular, under symmetric spillovers and when the level of spillovers is extremely low, both firms want to play the investment game as leaders and, as a result, they may easily end up investing simultaneously either cooperatively or noncooperatively. In this case, any cooperative agreement, to be stable, must contain a commitment to invest at an initial stage. A cooperative agreement of this sort would remain stable against deviations by coalitions of firms even if the number of symmetric firms gets arbitrarily higher than two. When spillovers are higher, our model predicts that both sequential (noncooperative) and simultaneous (cooperative) R&D configurations are stable against individual deviations. However, only cooperative agreements are strongly stable and, in this case, opposite forces, pushing towards either cooperation at the initial stage or to strategic delay of joint R&D investment might be in place. We have argued that this approach, by introducing endogenous timing into the model, may help in explaining some stylized facts, such as the tendency to postpone a portion of agreements in some industries, as the bio-pharmaceutical sector. Finally, when spillovers are asymmetric and favourable to the firm investing as follower, the model shows that an R&D alliance, to be stable, requires the joint investment to be strategically delayed in order to avoid that a firm may break the agreement to exploit the existing "second-mover advantage". This occurs, in particular, when the incoming spillover of the leader is much lower then that of the follower, a scenario typical of low knowledge transmission sectors.

6 Appendix

Proofs of Lemmata and Propositions

Proof of Proposition 1. (i)-(ii) For $\beta \in [0, 1/2)$, the following equation

$$\left(x_j^{\sigma *} - x_i^{c^\tau}\right) = \frac{2(a-c)(2-\beta)\Gamma}{\Delta} - \frac{2(a-c)(1+\beta)}{9\gamma - 2(1+\beta)^2} = 0 \tag{16}$$

can be solved for $\beta^*(\gamma) = \frac{7}{5} - \frac{3}{10}\sqrt{2}\sqrt{5\gamma+2}$, which is strictly positive for $\gamma \leq \overline{\gamma}$, where $\overline{\gamma} = 16/9$. Condition A.3 for $\beta < 1/2$ requires that $\gamma > \frac{a(2-\beta)(\beta+1)}{4.5c}$ and since this constraint reaches its maximum for $\beta = 1/2$, it follows that for $\gamma \in \left[\frac{a}{2c}, \frac{16}{9}\right]$, there exists a $\beta^*(\gamma) \in [0, 1/2)$ for which $\left(x_j^{\sigma *} - x_i^{c^\tau}\right) < 0$. It can be checked that this interval for γ is compatible with a market size-cost ratio $a/c \leq 32/9$. Moreover, by (16) for $1/2 > \beta > \beta^*(\gamma)$ and/or for a $\gamma > 16/9$, $\left(x_j^{\sigma *} - x_i^{c^\tau}\right) > 0$. Combining these facts with Amir's *et al.* (2000) ranking on leader-follower and Nash simultaneous investments, the results follow. (iii) For $\beta \in (1/2, 1]$, by (16), it turns out that $\left(x_j^{\sigma *} - x_i^{c^\tau}\right) < 0$. Moreover it can be easily checked that

$$\text{sign } \left(x_i^{\sigma *} - x_j^{\sigma *}\right) = \text{ sign } 2\gamma\left(2\beta - 1\right)^2 > 0$$

which holds for any β and, thus, also for $\beta \in (1/2, 1]$. Again, combining the above fact with Amir's *et al.* (2000) results, the ranking between R&D investments is proven. ■

Proof of Lemma 1. Suppose by contradiction that for $\beta > \frac{1}{2}$

$$\pi_i^C < \pi_i^L$$

and, following Amir's *et al.* (2000) (see footnote 7),

$$\pi_i^C < \pi_i^L < \pi_j^F.$$

It follows that

$$\sum_{i=1}^{2} \pi_i^C < \pi_i^L + \pi_j^F$$

contradicting the efficiency of profile $x_i^{c^\tau}(\mathbf{q}^*)$. Similarly, for $\beta < \frac{1}{2}$ let

$$\pi_i^C < \pi_j^F$$

and, according to Amir's *et al.* (2000) results,

$$\pi_i^C < \pi_j^F < \pi_i^L.$$

which again implies

$$\sum_{i=1}^{2} \pi_i^C < \pi_i^L + \pi_j^F,$$

which is a contradiction. ■

Proof of Proposition 2. (i) By Lemma 1 and employing Amir's *et al.* (2000) results we know that, under low spillover rates, ($\beta < \frac{1}{2}$), either

$$\pi_i^L > \pi_i^C > \pi_i^N > \pi_j^F. \tag{17}$$

or

$$\pi_i^C > \pi_i^L > \pi_i^N > \pi_j^F. \tag{18}$$

For $\beta \in [0, 1/2)$ the following equation

$$\pi_i^C - \pi_i^L = \frac{\gamma(a-c)^2}{9\gamma - 2(1+\beta)^2} - \frac{\gamma(a-c)^2\left(6\beta + 3\gamma - 2\beta^2 - 4\right)^2}{\Delta} = 0 \tag{19}$$

has only one root $\beta^*(\gamma) = \frac{7}{5} - \frac{3}{10}\sqrt{2}\sqrt{5\gamma + 2}$, requiring that $\gamma < 16/9$ to be positive. Since, by A.3, $\gamma > \frac{a(2-\beta)(\beta+1)}{4.5c}$ and such constraint reaches its maximum for $\beta = 1/2$, we conclude that for $\gamma \in \left[\frac{a}{2c}, \frac{16}{9}\right]$ there exists a $\beta^*(\gamma) \in [0, 1/2)$ ensuring that the inequality $\left(\pi_i^C - \pi_i^L\right) < 0$ holds true. (ii) For $\beta \in [0, 1/2)$, when either $\beta \geq \beta^*$ or $\gamma > \frac{16}{9}$ or both, it can be assessed that

$$\pi_i^C - \pi_i^L = \frac{\gamma(a-c)^2}{9\gamma - 2(1+\beta)^2} - \frac{\gamma(a-c)^2\left(6\beta + 3\gamma - 2\beta^2 - 4\right)^2}{\Delta} \geq 0. \tag{20}$$

The payoffs ranking can therefore be completed using Lemma 1 and Amir's *et al.* (2000) results. ■

Proof of Proposition 3. By Lemma 1 and Amir's *et al.* (2000), we know that under high spillover rates ($\beta > \frac{1}{2}$), either

$$\pi_j^F > \pi_i^C > \pi_i^L > \pi_i^N \tag{21}$$

or

$$\pi_i^C > \pi_j^F > \pi_i^L > \pi_i^N. \tag{22}$$

For $\beta \in [1/2, 1]$ and $\gamma \in \left(\frac{a(\beta+1)^2}{4.5c}, \infty\right)$, the equation

$$\left(\pi_i^C - \pi_j^F\right) = \frac{\gamma(a-c)^2}{9\gamma - 2(1+\beta)^2} - \frac{\gamma(a-c)^2(9\gamma + 8\beta - 2\beta^2 - 8)\left(26\beta\gamma - 20\gamma - 12\beta - 4\beta^2 + 12\beta^3 + 9\gamma^2 - 4\beta^4 - 8\beta^2\gamma + 8\right)^2}{\Delta^2} = 0$$

is solved only for $\beta = 1/2$. It can be checked that for any other spillover rate $1 \geq \beta > 1/2$, the difference $\left(\pi_i^C - \pi_j^F\right)$ is positive and increases monotonically in β. Only for $\gamma \to +\infty$, it occurs that $\left(\pi_i^C - \pi_j^F\right) \to 0$. ■

Proof of Proposition 4. (i) By proposition 2, for $\beta \in [0, \beta^*(\gamma)) < 1/2$ and $\gamma < \overline{\gamma}$, investing as leader at stage t_1 is more profitable for firms than forming a cooperative agreement. As a result, the message $\mathbf{m} = \left(\{1,2\}^{t_2}\right)$ cannot be a Nash equilibrium, since a firm i can profitably deviates with an alternative message $m_i' = (\{i\}, t_1)$ inducing the timing-partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$. Similarly, all sequential timing-partitions $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$ can profitably be objected by the j-th firm who, instead of playing as follower, would rather prefer to invest simultaneously. This is feasible if it sends the message $m_j' = (\{j\}, t_1)$, and induces the timing-partition $\left(\{i\}^{t_1}, \{j\}^{t_1}\right)$. Therefore, we remain with only two partitions $\left(\{1,2\}^{t_1}\right)$ and $\left(\{1\}^{t_1}, \{2\}^{t_1}\right)$ that cannot be profitably objected by any firm. (ii) We know by proposition 2 that, when $\beta \in [\beta^*(\gamma), 1/2)$, the payoff gained in a cooperative agreement is higher than that obtained by a leader (follower or simultaneous) firm, and therefore both cooperative timing-partitions $\left(\{1,2\}^{t_1}\right)$ and $\left(\{1,2\}^{t_2}\right)$ are Nash-stable. Also the simultaneous partition $\left(\{1\}^{t_1}, \{2\}^{t_1}\right)$ cannot be objected by individual deviations. (iii) For $\beta \in (1/2, 1]$, by proposition 3 the payoff gained in a cooperative agreement is the highest obtainable by a firm and, thus, both cooperative timing-partitions $\left(\{1,2\}^{t_1}\right)$ and $\left(\{1,2\}^{t_2}\right)$ are Nash-stable. Also the sequential partitions $\left(\{1\}^{t_1}, \{2\}^{t_2}\right)$ and $\left(\{1\}^{t_2}, \{2\}^{t_1}\right)$ cannot be profitably objected neither by a leader nor by a follower (see proposition 3), and the result follows. ■

Proof of Proposition 5. (i) This result easily follows from proposition 2 and by the fact that all other timing-partitions are Pareto-dominated by a cooperative agreement, with the exception of the sequential partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$. However, since by proposition 2, $\pi_i^N(\mathbf{x}^{\tau *}) > \pi_j^F(\mathbf{x}^{\sigma *})$ for $\beta < 1/2$, the sequential partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$ can profitably be objected by the follower, who prefers to invest simultaneously and, by sending the message $m_j' = (\{j\}, t_1)$ can induce the simultaneous partition $\left(\{i\}^{t_1}, \{j\}^{t_1}\right)$. However, the latter partition can, in turn, be objected by a message $\left(\{1,2\}^{t_1}\right)$ sent by both firms, and therefore, is not Strong Nash stable. Finally, also the partition $\left(\{1,2\}^{t_2}\right)$ can be objected by a firm sending an alternative message $m_i' = (\{i\}, t_1)$, hence inducing the relatively more profitable sequential partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$. (ii) By proposition 2 and 3 it follows that, for $\in [\beta^*(\gamma), 1]$, all sequential and simultaneous noncooperative payoffs are dominated by the cooperative agreements. As a result, the two message profiles $\mathbf{m} = (\{i,j\}, t_1), (\{i,j\}, t_1))$ and $\mathbf{m} =$

$(\{i,j\},t_2),(\{i,j\},t_2))$ are both strongly undominated and the two cooperative partitions $\left(\{1,2\}^{t_1}\right),\left(\{1,2\}^{t_2}\right)$ are both strongly stable. ■

Proof of Proposition 6. Consider first the equilibrium investment levels under the extreme assumptions that $\beta_i^L=0$, $\beta_i^N=0$, $\beta_j^F=1$, $\beta_i^C=1$, $i=1,2$ with $j\neq i$. Thus, we obtain:

$$x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}=\tfrac{2(a-c)(3\gamma-4)^2}{(112\gamma-162\gamma^2+81\gamma^3-32)} \tag{23}$$

and

$$x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}=\tfrac{4(a-c)(9\gamma-8)\gamma}{(112\gamma-162\gamma^2+81\gamma^3-32)}. \tag{24}$$

Moreover, substituting the above values for the spillover parameters into $x_i^{\tau*}$ and $x_i^{c^\tau}$, as derived in Section 3, we have that:

$$x_i^{\tau*}\big|_{\beta_i^N=0}=\tfrac{4(a-c)}{(9\gamma-4)} \tag{25}$$

and

$$x_i^{c^\tau}\big|_{\beta_i^C=1}=\tfrac{4(a-c)}{(9\gamma-8)} \tag{26}$$

for $i=1,2$.

Then,

(a) By simply comparing (25) and (26), we obtain that $x_i^{c^\tau}\big|_{\beta_i^C=1}>x_i^{\tau*}\big|_{\beta_i^N=0}$.

(b) Considering Eqs (24) and (23), it comes out that $(x_i^{\sigma*}-x_j^{\sigma*})\big|_{\beta_i^L=0,\beta_j^F=1}=-9\gamma^2-8\gamma+16<0$ iff $\gamma>-4/9+4\sqrt{10}/9$. This condition is implied by the SOC of firm i competing *a' la Stackelberg* at the R&D investment stage - evaluated at $\beta_i^L=0$, $\beta_j^F=1$ - which requires that $(-112\gamma+162\gamma^2-81\gamma^3+32)<0$.

(c) Also, $x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}>x_i^{c^\tau}\big|_{\beta_i^C=1}$ iff $\gamma>4/3$, and this is implied by assumption B.2.

(d) Finally, $x_i^{\tau*}\big|_{\beta_i^N=0}>x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}$ for $\gamma>4/9+4\sqrt{2}/9$, which, as shown above, is always respected.

Combining all inequalities above, we have

$$x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}>x_i^{c^\tau}\big|_{\beta_i^C=1}>x_i^{\tau*}\big|_{\beta_i^N=0}>x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}.$$

We now examine the ranking of R&D investments when $\beta_i^L = \beta_i^N = 0.5$, still maintaining the assumptions that $\beta_j^F = \beta_i^C = 1$. We obtain:

(i) $x_i^{c^\tau}\big|_{\beta_i^C=1} - x_i^{\tau*}\big|_{\beta_i^N=1/2}) = 3(a-c)(3\gamma+2) > 0$.

(ii) $x_i^{\tau*}\big|_{\beta_i^N=1/2} - x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1} = (162\gamma^3 - 256.5\gamma^2 + 195.75\gamma - 63.875)$. This expression is strictly positive for any $\gamma \geq 0.35$.

(iii) $(x_i^{c^\tau}\big|_{\beta_i^C=1} - x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1}) = (81\gamma^3 - 58.5\gamma^2 + 45\gamma - 18)$, which is strictly positive for $\gamma > 1/2$.

(iv) $(x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1} - x_i^{\tau*}\big|_{\beta_i^{NC}=1/2}) = 9(3\gamma^2 - 3\gamma + 0.5) > 0$ for $\gamma > 1/2$.

It suffices to take into account the conditions under B.2 as to the feasible values of γ to guarantee that all inequalities sub (i)-(iv) hold. Therefore, for $\beta_i^L = \beta_i^N = 0.5$ and $\beta_j^F = \beta_i^C = 1$, the ranking among equilibrium investments is such that:

$$x_i^{c^\tau}\big|_{\beta_i^C=1} > x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1} > x_i^{\tau*}\big|_{\beta_i^N=1/2} > x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1}.$$

Let now introduce the more general hypotheses that $\beta_i^L = \beta_i^N < 0.5$ and $1 \geq \beta_i^C > \beta_j^F > 0.5$, $i,j = 1,2 \;\; j \neq i$. In what follows, we show that the ranking obtained for $\beta_i^L = \beta_i^N = 0$, $\beta_j^F = \beta_i^C = 1$ and the one obtained for $\beta_i^L = \beta_i^N = 1/2$, $\beta_j^F = \beta_i^C = 1$ are general, i.e. they hold true for all spillover rates assumed.

First, we examine the ranking of R&D investments when $\beta_i^L = \beta_i^N = 0.5$ (and $0.5 < \beta_j^F < \beta_i^C = 1$). It is easy to see that:

(1) $x_i^{c^\tau}\big|_{\beta_i^C=1} - x_i^{\tau*}\big|_{\beta_i^N=1/2} = 3(a-c)(3\gamma+2) > 0$.

(2) Moreover,

$$x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} - x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} = \frac{2(1-2\beta_j^F)(a-c)(\beta_j^F-\gamma-2)}{(2\gamma-1)(8+2\beta_j^{F2}-8\beta_j^F-9\gamma)} < 0$$

due to the SOC for the profit maximization problem when firms compete simultaneously at the investment stage and the constraints hold on γ as stated above.

(3) Also,

$$x_i^{c^\tau}\big|_{\beta_i^C=1} - x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} = \frac{2(a-c)[9\gamma^2+(10\beta_j^{F2}-22\beta_j^F+10)\gamma-16-12\beta_j^{F2}+32\beta_j^F]}{(2\gamma-1)(9\gamma-8)(9\gamma-8-2\beta_j^{F2}+8\beta_j^F)} > 0$$

due to the SOC and the assumed constraints on γ (see B.2).

(4) Then, we obtain that

$$(x_j^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F} - x_i^{\tau*}|_{\beta_i^N=1/2}) = \frac{4(a-c)(2-\beta_j^F)(2\beta_j^F-1)}{3(2\gamma-1)(9\gamma-8-2\beta_j^{F2}+8\beta_j^F)} > 0$$

and finally

$$x_i^{\tau*}|_{\beta_i^N=1/2} - x_i^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F} = \frac{2(a-c)(2+3\gamma-\beta_j^F)(2\beta_j^F-1)}{3(2\gamma-1)(9\gamma-8-2\beta_j^{F2}+8\beta_j^F)} > 0.$$

As a result,

$$x_i^{c^\tau}|_{\beta_i^C=1} > x_j^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F} > x_i^{\tau*}|_{\beta_i^N=1/2} > x_i^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F}.$$

The same ranking holds also for any value of β_i^C such that $0.5 < \beta_j^F < \beta_i^C < 1$. This can be proven considering that

$$(x_i^{c^\tau} - x_j^{\sigma*})|_{\beta_i^L=1/2,\beta_j^F=\beta_i^C=\beta>1/2} = \frac{2(a-c)(1-2\beta)[2\beta^3+2(\gamma-1)\beta^2-(17\gamma-4)\beta+17\gamma-9\gamma^2]}{(2\gamma-1)(9\gamma-8-2\beta+8\beta)}.$$

The above expression is strictly positive since the term in square brackets at the numerator is negative (and decreasing in β), the second term at the denominator is the SOC for simultaneous competition at the investment stage (see B.2), and the third term at the denominator is negative for any $\beta > 0.5$ due to the constraints on γ (see B.2). Now, $\beta_i^C > \beta_j^F > 0.5$ implies that x_i^C increases as well. Thus, *a fortiori*, $x_i^{c^\tau}|_{\beta_i^C} > x_j^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F}$. Therefore:

$$x_i^{c^\tau}|_{\beta_i^C} > x_j^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F} > x_i^{\tau*}|_{\beta_i^N=1/2} > x_i^{\sigma*}|_{\beta_i^L=1/2,\beta_j^F}$$

for any value of β_i^C and β_j^F such that $0.5 < \beta_j^F < \beta_i^C < 1$.

Now we consider the ranking at $\beta_i^L = \beta_i^N = 0$ and we let $0.5 < \beta_j^F < \beta_i^C \leq 1$ Note that

$$x_i^{c^\tau}|_{\beta_i^C=1} - x_j^{\sigma*}|_{\beta_i^L=0,\beta_j^F} = \frac{8(a-c)(3\gamma-4)(2\beta_j^F-1)[(3\beta_j^F-6)\gamma+4]}{(9\gamma-8)(72\gamma^2\beta_j^F+160\gamma-18\beta_j^{F2}\gamma^2-216\gamma^2-32+81\gamma^3-48\beta_j^F\gamma)} = 0$$

iff $\beta_j^F = 1/2$. Now, let $\beta_i^C = 1-\epsilon$, with ϵ sufficiently small. It is easy to see that $\frac{\partial x_i^{c^\tau}}{\partial \beta_i^C} > 0$. Therefore, $x_i^{c^\tau}|_{\beta_i^C} < x_j^{\sigma*}|_{\beta_i^L=0,\beta_j^F=1/2}$. Moreover, letting β_j^F be greater than 1/2, directly implies that $x_i^{c^\tau}|_{\beta_i^C} < x_j^{\sigma*}|_{\beta_i^L=0,\beta_j^F}$, since $x_j^{\sigma*}$ is monotonically increasing in β_j^F.

Finally, $x_i^{\sigma*}|_{\beta_i^L=0,\beta_j^F}$ increases for β_j^F such that $0.5 < \beta_j^F < 1$. We proceed now by contradiction, wondering if the ranking $x_i^{\sigma*}|_{\beta_i^L=0,\beta_j^F} > x_i^{\tau*}|_{\beta_i^N=0}$ could ever be feasible. It is easily found that the inequality $x_i^{\sigma*} > x_i^{\tau*}$ contradicts the above finding, i.e. that

$x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} < x_i^{\tau*}\big|_{\beta_i^N=1/2}$ at $\beta_i^L = \beta_i^N = 0.5$, and $\beta_j^F > 0.5$, combined with the fact that $x_i^{\sigma*}$ is monotonically increasing in β_i^L. As a result,

$$x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F} > x_i^{c^\tau}\big|_{\beta_i^C} > x_i^{\tau*}\big|_{\beta_i^N=0} > x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F}.$$

The fact that both $x_j^{\sigma*}$ and $x_i^{\tau*}$ are monotonically decreasing in $\beta_i^L = \beta_i^N$, and that, conversely, $x_i^{\sigma*}$ is monotonically increasing in β_i^L completes the proof. Figure 3 illustrates this proposition by means of a numerical example. ■

Assumptions under Symmetric Spillovers

A.1 Straightforward manipulations of firms' payoffs at the quantity-stage yield

$$q_i^* = q_j^* = \frac{1}{3}\left[(a-c) + (2-\beta)\,x_i + (2\beta-1)\,x_j\right] \tag{27}$$

and then

$$\pi_i\left(\mathbf{q}^*\left(\mathbf{x}\right)\right) = \frac{1}{9}\left[(a-c) + (2-\beta)\,x_i + (2\beta-1)\,x_j\right]^2 - \frac{\gamma}{2}x_i^2. \tag{28}$$

Since $\beta \in [0,1]$ and $x_i \in [0,c]$, and given that for a firm the worst investment scenario occurs when $x_i^* = 0$, $\beta = 0$ and $x_j^* = c$, by (27) this yields

$$q_i^*\left(x_i = 0, x_j = c, \beta = 0\right) = \frac{1}{3}\left[(a-2c)\right]. \tag{29}$$

This condition implies that for

$$a > 2c$$

a unique interior (positive) Cournot profile of quantities, with associated positive equilibrium profits, always exists.

A.2 It is easily shown that the investment-stage SOCs are respected, for every $i = 1, 2$, for

$$\frac{\partial^2 \pi_i\left(\mathbf{x}\left(\mathbf{q}^*\right)\right)}{\partial x_i^2} = \frac{1}{9}\left(8 + 2\beta^2 - 8\beta - 9\gamma\right) < 0$$

which requires that $\gamma > \frac{2}{9}\left(2-\beta\right)^2$ and then strict-concavity of $\pi_i\left(\mathbf{x}\left(\mathbf{q}^*\right)\right)$ in x_i is guaranteed for $\gamma > \frac{8}{9}$ for any $\beta \in [0,1]$. Firms' best-replies are obtained from (28) and are given by:

$$x_i = g_i\left(x_j\right) = \frac{2\left(2-\beta\right)\left(a-c+(2\beta-1)\,x_j\right)}{\left(9\gamma + 8\beta - 2\beta^2 - 8\right)}.$$

Moreover, since for every firm

$$g_i'(x_j) = -\frac{\partial^2 \pi_i (x_i(\mathbf{q}^*), x_j(\mathbf{q}^*)) / \partial x_i \partial x_j}{\partial^2 \pi_i (x(\mathbf{q}^*)) / \partial x_i^2} = -\frac{2(2\beta-1)(2-\beta)}{\left(8+2\beta^2-8\beta-9\gamma\right)}$$

increasing differences of $\pi_i(x_i, x_j)$ in (x_i, x_j) (and then non decreasing best-replies) are implied by $\beta > \frac{1}{2}$ and decreasing differences (and non increasing best-replies) are implied by $\beta < \frac{1}{2}$, given that

$$\frac{\partial^2 \pi_i (\mathbf{x}(\mathbf{q}^*))}{\partial x_i \partial x_j} = \frac{2}{9}(2\beta-1)(2-\beta).$$

To guarantee that uniqueness of Nash equilibrium $\mathbf{x}^{\tau *}(\mathbf{q}^*)$, a contraction condition would serve the scope. This condition is respected for $g_i'(x_j) < 1$ when the function is increasing and for $g_i'(x_j) > -1$, when the function is decreasing, thus requiring

$$g_i'(x_j) = -\frac{2(2\beta-1)(2-\beta)}{\left(8+2\beta^2-8\beta-9\gamma\right)} < 1 \tag{30}$$

for $\beta > \frac{1}{2}$ and

$$g_i'(x_j) = -\frac{2(2\beta-1)(2-\beta)}{\left(8+2\beta^2-8\beta-9\gamma\right)} > -1 \tag{31}$$

for $\beta < \frac{1}{2}$. Condition (30) implies

$$\left(8+2\beta^2-8\beta-9\gamma\right) > -2(2\beta-1)(2-\beta)$$

which is satisfied for

$$\gamma > \frac{2}{9}(\beta+1)(2-\beta). \tag{32}$$

Since the RHS in (32) is monotonically increasing in β, (32) becomes

$$\gamma > \frac{1}{2}.$$

Condition (31) equals to

$$-2(2\beta-1)(2-\beta) > -\left(8+2\beta^2-8\beta-9\gamma\right), \tag{33}$$

and thus

$$\gamma > \frac{2}{3}(\beta-1)(\beta-2). \tag{34}$$

Since the expression on the RHS of (34) is monotonically increasing in β, we obtain the condition $\gamma > \frac{4}{3}$.

Therefore, for any $\beta \in [0,1]$ the two firms' investment best-replies $g_i(x_j)$ are contractions if $\gamma > \frac{4}{3}$.

A.3 In order to obtain interior values for the equilibrium investment level $\mathbf{x}^*$under symmetric spillovers and in all simultaneous, cooperative and sequential games, some assumptions are in order.

(i) Using the FOC for each firm $i = 1,2$ when playing simultaneously the investment game, we obtain that

$$\frac{\partial \pi_i (\mathbf{x} (\mathbf{q}^*))}{\partial x_i} = \frac{2(2-\beta)}{9} [a - c + (2-\beta) x_i + (2\beta - 1) x_j] - \gamma x_i,$$

which, setting $x_i = 0$, becomes

$$\frac{\partial \pi_i (0, x_j)}{\partial x_i} = \frac{2(2-\beta)}{9} [a - c + (2\beta - 1) x_j] > 0 \tag{35}$$

for every $x_j \in [0, c]$. As a result, to play $x_i = 0$ is never a best-reply for a firm .

(ii) Secondly, when a firm $i = 1,2$ participates to a cooperative R&D agreement, its FOC is

$$\begin{aligned}\frac{\partial \pi_i (0, x_j)}{\partial x_i} + \frac{\partial \pi_j (0, x_j)}{\partial x_i} = \frac{2(2-\beta)}{9} [a - c + (2-\beta) x_i + (2\beta - 1) x_j] - \gamma x_i + \\ + \frac{2}{9} (2\beta - 1) (a - c + (2-\beta) x_j + (2\beta - 1) x_i)\end{aligned}$$

which, evaluated at $\mathbf{x} = (0, x_j)$, becomes

$$\frac{\partial \pi_i (0, x_j)}{\partial x_i} + \frac{\partial \pi_j (0, x_j)}{\partial x_i} = \frac{2}{9} \left((a - c)(1 + \beta) + 10\beta x_j - 4x_j - 4\beta^2 x_j \right) > 0$$

for every $x_j \in [0, c]$. It is thus never rational for a firm in a cooperative agreement to play $x_i = 0$, no matter what the other firm does.

(iii) Finally, for a firm $i = 1,2$ investing as a leader, the FOC is

$$\frac{\partial \pi_i (x_i, g_j(x_i))}{\partial x_i} = \frac{\partial \pi_i (x_i, g_j(x_i))}{\partial x_i} + \frac{\partial \pi_i (x_i, g_j(x_i))}{\partial x_j} g_j'(x_i) = 0. \tag{36}$$

Notice that for $\beta > \frac{1}{2}$ both $\partial \pi_i(x_i, x_j)/\partial x_j > 0$ and $g_j'(x_i) > 0$ while for $\beta < \frac{1}{2}$, the opposite holds, given that

$$\frac{\partial \pi_i (0, g_j(0))}{\partial x_i} = \frac{(a-c)\left(2\beta^2 + 4 - 6\beta - 3\gamma\right)\left(2 - 2\beta^2 - 3\gamma\right)(2-\beta)}{2\left(8\beta + 9\gamma - 2\beta^2 - 8\right)^2} > 0$$

for $\gamma > \frac{4}{3}$, and expression (35) guarantees that a firm as a leader will always invest a strictly positive amount at a sequential equilibrium. Moreover, since the FOC for a follower is the same as in the simultaneous Nash equilibrium, at the sequential equilibrium both firms will never play the profile $\mathbf{x}^{\sigma*} = (0,0)$. To conclude, we want to be sure that both firms will never play their full cost reduction investment (corner solution), and that instead either their best-replies or their cooperative decisions always lie below their maximum rational level

$$\overline{x}_i = \overline{x}_j = \frac{c}{\beta+1}. \tag{37}$$

(i) and (iii) Under noncooperative behaviour and using (37), this is guaranteed if

$$\frac{\partial \pi_i\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} = \frac{2\left(2-\beta\right)}{9}\left(\left(a-c\right)+\left(2-\beta\right)\frac{c}{\beta+1}+\left(2\beta-1\right)\frac{c}{\beta+1}\right)-\gamma\frac{c}{\beta+1}<0$$

which holds for

$$\gamma > \frac{a\left(2-\beta\right)\left(\beta+1\right)}{4.5c}. \tag{38}$$

As a result, for

$$\gamma > \frac{a\left(2-\beta\right)\left(\beta+1\right)}{4.5c} \Leftrightarrow x_i^{\tau*} = g_i(x_j^{\tau*}) \text{ and } \mathbf{x}^{\sigma*} = (x_i^{\sigma*}, g_j(x_i^{\sigma*})) \tag{39}$$

that is, both simultaneous and sequential investment equilibria are interior and lie below the boundary points. Instead for

$$\gamma \leq \frac{a\left(2-\beta\right)\left(\beta+1\right)}{4.5c} \Leftrightarrow x_i^{\tau*} = \frac{c}{\beta+1} \text{ and } \mathbf{x}^{\sigma*} = \left(\frac{c}{\beta+1}, \frac{c}{\beta+1}\right). \tag{40}$$

(ii) For a firm participating to a cooperative R&D agreement, its FOC evaluated at $\overline{\mathbf{x}} = \left(\frac{c}{\beta+1}, \frac{c}{\beta+1}\right)$ is

$$\begin{aligned}\frac{\partial \pi_i\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} + \frac{\partial \pi_j\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} &= \tfrac{2(2-\beta)}{9}\left[a-c+\left(2-\beta\right)\tfrac{c}{\beta+1}+\left(2\beta-1\right)\tfrac{c}{\beta+1}\right]-\gamma x_i+ \\ &+\frac{2}{9}\left(2\beta-1\right)\left[a-c+\left(2-\beta\right)\tfrac{c}{\beta+1}+\left(2\beta-1\right)\tfrac{c}{\beta+1}\right]<0\end{aligned}$$

which holds if

$$\gamma > \frac{a\left(\beta+1\right)^2}{4.5c}. \tag{41}$$

Notice that for $\beta < \frac{1}{2}$ ($\beta > \frac{1}{2}$) the cooperative constraints on γ is less (more) demanding than the noncooperative constraints. Therefore, the constraint used to avoid full cost reductions for $\beta > \frac{1}{2}$ is (41) while for $\beta < \frac{1}{2}$ we can impose that (38).

Assumptions under Asymmetric Spillovers

B.1 It is easily found that equilibrium quantities as function of R&D investments are given by:

$$q_i^* = \frac{1}{3}\left[(a-c) + \left(2-\beta_j\right)x_i + \left(2\beta_i - 1\right)x_j\right] \tag{42}$$

for $i, j = 1, 2\ \ , i \neq j$. Therefore

$$\pi_i\left(\mathbf{q}^*\left(\mathbf{x}\right)\right) = \frac{1}{9}\left[(a-c) + \left(2-\beta_j\right)x_i + \left(2\beta_i - 1\right)x_j\right]^2 - \frac{\gamma}{2}x_i^2. \tag{43}$$

Substituting in Eq. (42) or in Eq. (43), $x_i^* = 0$, $x_j^* = c$, $\beta_i = 0$ and $0.5 < \beta_j < 1$, we obtain that under asymmetric spillovers the condition $a > 2c$ is again needed to guarantee an interior Cournot profile of equilibrium quantities, and hence the strict positivity of equilibrium profits.

B.2 Given our assumptions on spillovers for the simultaneous noncooperative R&D investment game, that is, $\beta_i^N = \beta_j^N \leq 0.5$, the SOC for the investment game played a' la Cournot does not vary and requires that, for every $i = 1, 2$,

$$\gamma > \frac{2}{9}\left(2 - \beta_i^N\right)^2. \tag{44}$$

Being the RHS of (44) decreasing in β_i^N, we obtain that the most stringent condition on γ is given by $\gamma > \frac{8}{9}$.

Note that this condition also guarantees that the SOC for the maximization problem of an R&D alliance playing the investment game is respected. In this case the SOC is given by

$$\gamma > \frac{2}{9}\left(1 + \beta_i^C\right)^2,$$

and, being increasing in β_i^C - and given our assumptions on β_i^C- the above condition is respected for $\gamma > \frac{8}{9}$.

Moreover, when both firms play simultaneously the investment stage, and given that $\beta_i^N = \beta_j^N \leq 0.5$, best replies are contractions for $\gamma > \frac{4}{3}$.

In addition, to guarantee the uniqueness of the sequential equilibrium, the contraction approach applied to the follower best-reply requires that

$$g_j'(x_i) = \frac{2\left(2\beta_j - 1\right)(2 - \beta_i)}{\left(9\gamma + 8\beta_i - 2\beta_i^2 - 8\right)} < 1 \tag{45}$$

Since $0.5 < \beta_j^F < 1$ and the follower's best-reply is increasing, (45) requires

$$\gamma > \frac{2}{9}(2 - \beta_i)\left(1 + 2\beta_j - \beta_i\right). \tag{46}$$

As a result, since (46) is increasing in β_j and decreasing in β_i, the most stringent constraint on γ becomes $\gamma > \frac{4}{3}$.

B.3 First define as $(\overline{\mathbf{x}}_i, \overline{\mathbf{x}}_j)$ the point at which the boundary lines given by

$$x_i = c - \beta_i x_j$$
$$x_j = c - \beta_j x_i$$

intersect. It is easily found that:

$$\overline{x}_i = \frac{c(1 - \beta_i)}{1 - \beta_i\beta_j}$$
$$\overline{x}_j = \frac{c\left(1 - \beta_j\right)}{1 - \beta_i\beta_j}$$

>From the profit maximization problem for a firm under asymmetric spillovers, we have

$$\frac{\partial \pi_i\left(\mathbf{x}\left(\mathbf{q}^*\right)\right)}{\partial x_i} = \frac{2(2 - \beta)}{9}\left[(a - c) + (2 - \beta)x_i + (2\beta - 1)x_j\right] - \gamma x_i = 0$$

from which the following best-reply is obtained:

$$g_i(x_j) = \frac{2\left(2 - \beta_j\right)\left(2(a - c) + (2\beta_i - 1)x_j\right)}{\left(9\gamma + 8\beta_j - 2\beta_j^2 - 8\right)}$$

In order to show that this best-reply lies underneath the point $(\overline{x}_i, \overline{x}_j)$, it suffices to impose that, when the incoming spillover β_i is greater that 1/2 for at least one firm, $\frac{\partial \pi_i(\overline{\mathbf{x}}_i, \overline{\mathbf{x}}_j)}{\partial x_i} < 0$. If this condition holds true, then the equilibrium R&D investment profile will lie at the interior of the full cost reduction boundary $(\overline{x}_i, \overline{x}_j)$. More specifically,

$$\begin{aligned}
\frac{\partial \pi_i\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} &= \frac{2\left(2 - \beta_j\right)}{9}\left[(a - c) + \left(2 - \beta_j\right)\overline{x}_i + (2\beta_i - 1)\overline{x}_j\right] - \gamma = \\
&= \frac{2\left(2 - \beta_j\right)}{9}\left[(a - c) + \left(2 - \beta_j\right)\frac{c(1 - \beta_i)}{1 - \beta_i\beta_j} + (2\beta_i - 1)\frac{c\left(1 - \beta_j\right)}{1 - \beta_i\beta_j}\right] - \gamma\frac{c(1 - \beta_i)}{1 - \beta_i\beta_j} < 0
\end{aligned}$$

and, since $\left[(a-c)+\left(2-\beta_j\right)\frac{c(1-\beta_i)}{1-\beta_i\beta_j}+(2\beta_i-1)\frac{c\left(1-\beta_j\right)}{1-\beta_i\beta_j}\right]=a$, the above inequality becomes

$$\frac{2a\left(2-\beta_j\right)}{9}-\gamma\frac{c\,(1-\beta_i)}{1-\beta_i\beta_j}<0$$

requiring that:

$$\gamma>\frac{2}{9}\frac{a\left(1-\beta_i\beta_j\right)\left(2-\beta_j\right)}{c\,(1-\beta_i)}. \tag{47}$$

Given that, for a sequential equilibrium, $1>\beta_j^F>0.5$ and $0\leq\beta_i^L\leq 0.5$, the constraint in (47), which is increasing in β_i and decreasing in β_j, boils down into the following condition on γ:

$$\gamma>\frac{a}{2c} \tag{48}$$

which is the most stringent one for firm i. The boundary points required for an interior equilibrium under noncooperative behavior and simultaneous moves were derived in the previous section. By simply substituting for β_i^N, for $i=1,2$, we have

$$\gamma>\frac{2a\left(2-\beta_i^N\right)\left(\beta_i^N+1\right)}{9c}. \tag{49}$$

Moreover, since our assumptions assumptions on spillovers imply that $\beta_i^N\leq 0.5$, and given that (49) is increasing in β_i^N, the (most stringent) condition on γ becomes:

$$\gamma>\frac{a}{2c}. \tag{50}$$

For a firm entering an R&D alliance, the constraint on γ does not vary with respect to the case with symmetric spillovers, but for the assumption $0.5<\beta_i^C\leq 1$. As a result, the condition

$$\gamma>\frac{2a\left(\beta_i^C+1\right)^2}{9c}. \tag{51}$$

boils down into:

$$\gamma>\frac{8a}{9c}. \tag{52}$$

Also in this case the constraint on γ required under cooperation (eq. 52) is the most stringent and thus will be the one to be imposed.

Finally, combining both constraints in B.1 and in B.3 for the sequential investment game, the most demanding condition on γ is $\gamma > 1$. Moreover, in the noncooperative simultaneous investment stage, the same constraint on γ has to be satisfied, whilst, under the cooperative case, it is required that $\gamma > 16/9$.

This is the most stringent condition also employed in the numerical simulations with asymmetric spillovers.

References

[1] Alonso, T. and P.Marín, (2004) "Research Joint Ventures: the Role of Public Research Institutes," *Investigaciones Economicas*, 28, 213-231.

[2] Amir, R., Wooders, J., (1999). "Effects of one-way spillovers on market shares, industry price, welfare and R&D cooperation", *Journal of Economics and Management Strategy*, 8, 223-249.

[3] Amir, R., M. Amir and J. Jin (2000) "Sequencing R&D decisions in a two-period duopoly with spillovers", *Economic Theory*, 15(2), 297-317.

[4] Amir, R., Wooders, J., (2000). "One-Way Spillovers, Endogenous Innovator/Imitator Roles and Research Joint Ventures", *Games and Economic Behavior*, 31, 1-25.

[5] Amir, R. , F. Garcia, C. Halmenschlager, J. Pais (2011a), "R&D as a Prisoner's Dilemma and R&D- avoiding Cartels", *The Manchester School*, 79 , 1, 81–99.

[6] Amir, R., Halmenschlager, C. and J. Jin (2011b) "R&D Induced Industry Polarization and Shake-outs", *International Journal of Industrial Organization*, 29 (4), 386-398.

[7] Atallah, G. (2005) "R&D Cooperation with Asymmetric Spillovers", *Canadian Journal of Economics* 38, 3, 919-936.

[8] Baumol, W. (1992) "Horizontal collusion and innovation", *The Economic Journal*, 102, 129-137.

[9] Belderbos, R., M.Carree, B. Diederen, B. Lokshin and R.Veugelers (2004). "Heterogeneity in R&D Cooperation Strategies", *International Journal of Industrial Organization*, 22, 8-9, 237-1263,

[10] Bloch, F. (2003) "Coalition Formation in Games with Spillovers". in: Carraro C. (eds.) *The endogenous formation of economic coalitions*, Fondazione Eni Enrico Mattei Series on Economics and the Environment, Cheltenham, U.K. and Northampton, Mass.: Elgar.

[11] Bloch, F. (2004) "Group and Network Formation in Industrial Organization", in G. Demange and M.Wooders (eds.) *Group Formation in Economics; Networks, Clubs and Coalitions*, Cambridge University Press: Cambridge U.K..

[12] Cassiman, B. and Veugelers, R. (2002), "R&D Cooperation and Spillovers: Some Empirical Evidence from Belgium", *American Economic Review*, 92, 1169- 1184.

[13] Cheng, L. (1984) "International competition in R&D and technological leadership: An examination of the Posner-Hufbauer hypothesis", *Journal of International Economics*, 17, 15–40.

[14] CBO (Congressional Budget Office, Congress of the United States) (2006), "Research and Development in the Pharmaceutical Industry.", Available at: http://www.cbo.gov/ftpdocs/76xx/doc7615/10-02-DrugR-D.pdf

[15] Crampes, C. and C. Langinier. C. (2003) "Following the Follower in the Innovation Game". *IDEI Working Papers*, n° 216, Institut d'Économie Industrielle, Toulouse.

[16] Currarini, S. and M., Marini, (2003), "A Sequential Approach to the Characteristic Function and the Core in Games with Externalities", Sertel, M., Kara, A.(eds.), *Advances in Economic Design*. Springer Verlag, Berlin.

[17] Currarini, S. and M., Marini, (2004), "A Conjectural Cooperative Equilibrium in Strategic Form Games", in Carraro C. , Fragnelli V. (eds.) *Game Practise and the Environment*. Kluwer Academic Press.

[18] Danzon, P.M., Nicholson S. and Pereira N.S. (2005), "Productivity in pharmaceutical-biotechnology R&D: the role of experience and alliances", *Journal of Health Economics*, 24(2): 317–340.

[19] d'Aspremont, C., A. Jacquemin (1988), "Cooperative and noncooperative R&D in duopoly with spillovers, *American Economic Review*, 78, 1133 - 1137

[20] De Bondt, R. and I. Henriques (1995). "Strategic Investment with Asymmetric Spillovers", *Canadian Journal of Economics*, 28, 3, 656-674.

[21] De Bondt, R. and J. Vandekerckhove (2008), "Asymmetric Spillovers and Investments in R&D of Leaders and Followers", *Economics of Innovation and New Technology*, 17, 417 - 433.

[22] Duso, T. , Pennings, E. and Jo Seldeslachts (2010) "Learning Dynamics in Research Alliances: A Panel Data Analysis", *Research Policy*, 39, 776-789.

[23] Goel, R.A. (1990) "Innovation, Market Structure, and Welfare: A Stackelberg Model". *Quarterly Review of Economics and Business*, 30, 40-53.

[24] Halmenschlager, C. (2004). "R&D-Cooperating Laggards versus a Technological Leader". *Economics of Innovation and New Technology*, 13, 8, 717-732.

[25] Hamilton, J., and S. Slutsky (1990), "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria". *Games and Economic Behavior* 2, 29–46.

[26] Hart S, Kurz M (1983), "Endogenous Formation of Coalitions", *Econometrica* 52, 1047-1064.

[27] Hernan, R., Marin, P.L. and G. Siotis , 2003, "An empirical evaluation of the determinants of Research Joint Venture Formation", *Journal of Industrial Economics*, 51, 75-89.

[28] Kamien, M.I. E. Muller, I. Zang (1992), "Research joint ventures and R&D cartels", *American Economic Review*, 82, 1293 - 1306.

[29] Katz, M.L. and Ordover J.A. (1990), "R&D cooperation and competition", *Brookings Institution*, Washington DC, 137-203.

[30] Kesteloot, K. and Veugelers, R. (1995), "Stable R&D Cooperation with Spillovers ", *Journal of Economics and Management Strategy*, 4 (4), 651 - 672.

[31] Marini, M. A. (2008), "Games of Coalition and Network Formation: a Survey", in Naizmada A. *et al.* (eds), *Networks, Dynamics and Topology*, London & Berlin, Springer-Verlag.

[32] Paul S.M., Mytelka D.S., Dunwiddie C.T., Persinger C.C., Munos B.H., Lindborg S.R. and Schacht A.L. (2010): "How to improve R&D productivity: the pharmaceutical industry's grand challenge ", *Nature Reviews Drug Discovery* 9, 203-214.

[33] Petit, M.L. and Tolwinski, B., (1997), "Technology sharing cartels and industrial structure", *International Journal of Industrial Organization*, 15, 77-101

[34] Petit, M.L. and Tolwinski, B., (1999), "R&D cooperation or competition?", *European Economic Review*, 43, 185-208.

[35] Petit, M. L., Sanna-Randaccio, F. and Sestini, R., (2009): "Asymmetric Knowledge Flows and Localization with Endogenous R&D: A Dynamic Analysis", *Economic Modelling*, 26, 536–547.

[36] Ray, D. (2007), *A Game-Theoretic Perspective on Coalition Formation*. Oxford, U.K.: Oxford University Press.

[37] Recombinant Capital. http://www.recap.com.

[38] Rogers M.J., Ding M. and Maranas C.D., (2005):"Valuation and Design of Pharmaceutical R&D Licensing Deals", *AIChE Journal*, 51, 198–209.

[39] Salant, S. and G. Shaffer (1998), "Optimal Asymmetric Strategies in Research Joint Ventures", *International Journal of Industrial Organization*, 16, 195–208.

[40] Salant, S. and G. Shaffer (1999), "Unequal Treatment of Identical Agents in Cournot Equilibrium", *American Economic Review*, 89, 585-604.

[41] Stepanova, A., A. Tesoriere (2011), "R&D with Spillovers: Monopoly versus Noncooperative and Cooperative Duopoly", *The Manchester School*, 79, 1, 125–144.

[42] Suzumura, K., 1992, "Cooperative and Non-cooperative R&D in an Oligopoly with Spillovers", *American Economic Review*, 82, 1307-1320.

[43] Tesoriere, A.(2008), "Endogenous R&D Symmetry in Linear Duopoly with One-way Spillovers", *Journal of Economic Behavior & Organization*, 66, 213-225.

[44] Yi, S. S. and Shin H. S. (2000), "Endogenous Formation of Research Coalitions with Spillovers", *International Journal of Industrial Organization*, Vol. 18, 229-256

[45] Yi, S. S., (2003), "The Endogenous Formation of Economic Coalitions: The Partition Function Approach", in C. Carraro (ed.) *The endogenous formation of economic coalitions*, Fondazione Eni Enrico Mattei Series on Economics and the Environment, Cheltenham, U.K. and Northampton, Mass.: Elgar.

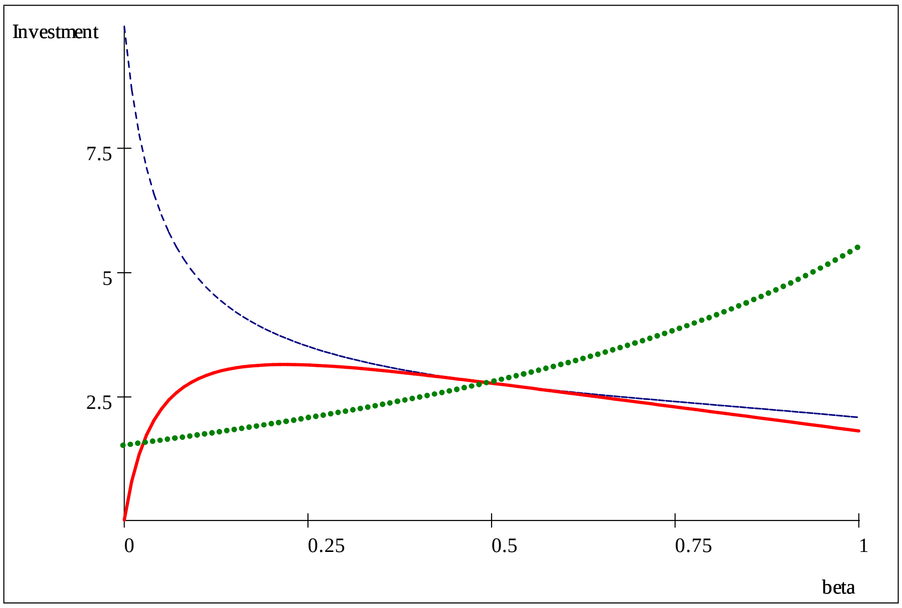

Fig.1 - R&D investment for the leader (dashed line), follower (continuous line) and cooperative firm (dotted line) for $a = 38$, $c = 18$, $\gamma = 1.7$, $\beta \in [0.1]$.

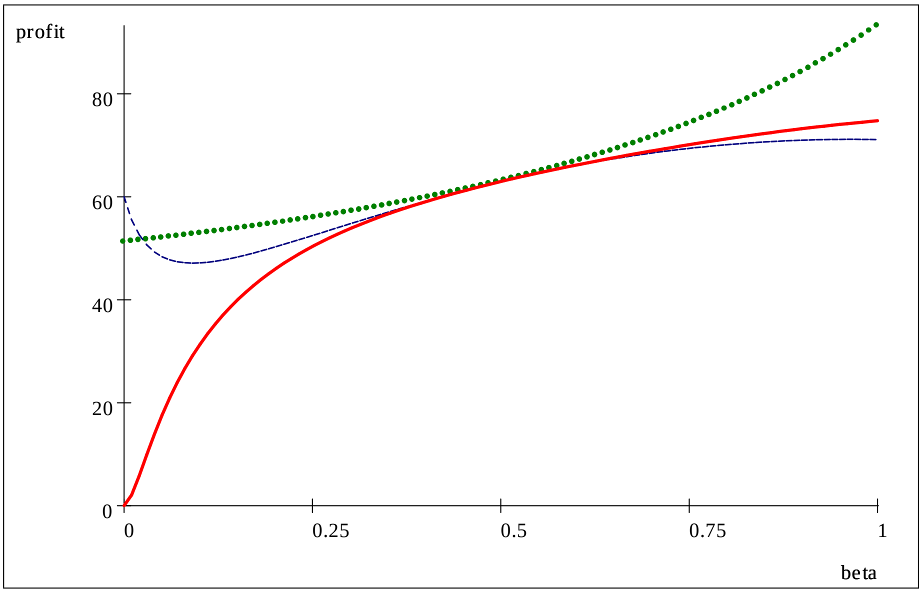

Fig.2 - Payoff for the leader (dashed line), follower (continuous line) and cooperative firm (dotted line) for $a = 38$, $c = 18$, $\gamma = 1.7$, $\beta \in [0.1]$.

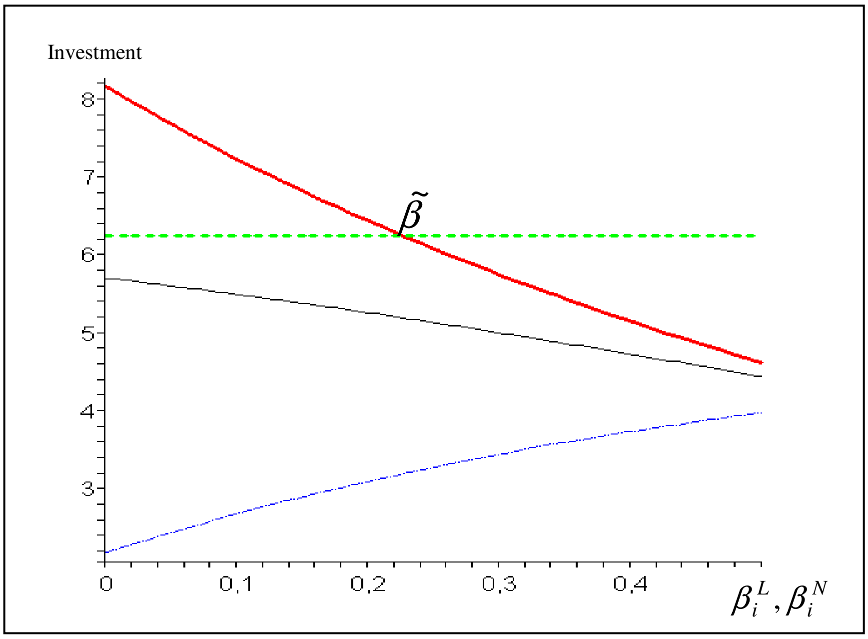

Fig 3: R&D investment for a leader (dashed line), a follower (solid thick line), a cooperative firm (dotted thick line), a non-cooperative firm (solid line) for a=38, c=18, $\gamma = 2$, $\beta_j^F = 0.6$, $\beta_i^C = 0.8$.

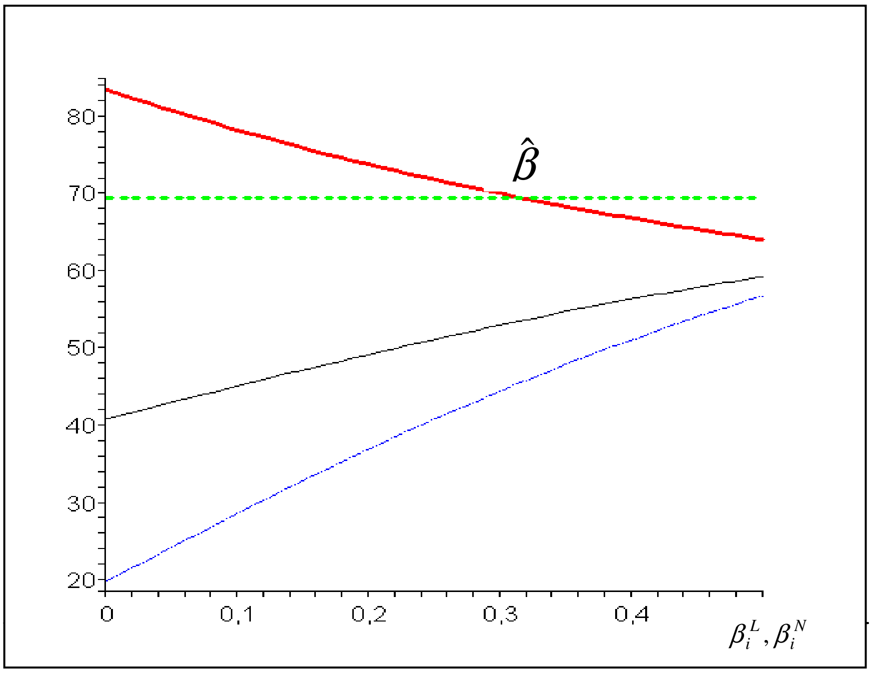

Fig 4: payoffs for a leader (dashed line), a follower (solid thick line), a cooperative firm (dotted thick line), a non-cooperative firm (solid line) for a=38, c=18, $\gamma = 2$, $\beta_j^F = 0.6$, $\beta_i^C = 0.8$.