Block Notes of Math

Goldbach, Twin Primes and Polignac

Equivalent RH

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Abstract

In this work the authors will explain and prove three equivalent RH, which are obtained linking G(N)/N with Li and g(N)/N with Li. Moreover the authors will show a new step function for G(N)/N and, through the GRH, a generalization for Polignac.

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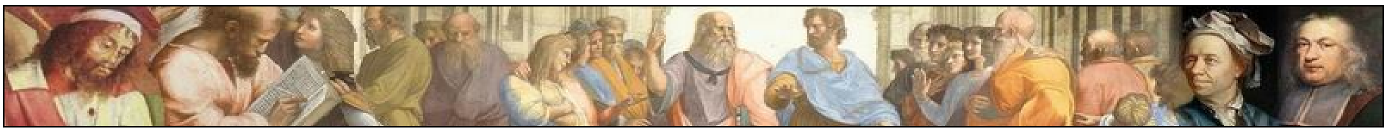

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Equivalent RH with G(x), g(x), P(x,d)

In [25] we have showed a closed form that links G(N) and π(N) or g(N) and π(N).

We says that:

$$\left|\frac{G(x)}{x} - \int_2^x \frac{dt}{t(\ln t)^2}\right| = O(x^{\frac{1}{2}+\varepsilon})$$

This is an equivalent RH (**R. Turco, M. Colonnese, ERATOSTENE group**)".

Can it be showed? Yes.

Several works [see also ERATOSTENE group] showed that:

$$G(N) \approx c\frac{N}{(\ln N)^2} \quad (5)$$

We don't think at the constant, then the PNT in simple form is:

$$\pi(N) \approx \frac{N}{\ln N}, \quad N \to \infty$$

From (5) and from PNT is: $\frac{G(N)}{N} \approx \frac{1}{(\ln N)^2}$, $\frac{\pi(N)}{N\ln N} \approx \frac{1}{(\ln N)^2}$

Then the idea is:

$$\left|\frac{G(N)}{N} - \frac{\pi(N)}{N\ln N}\right| < KC(N),\ K=1,\ C(N)=\frac{1}{\ln N} \quad (6)$$

Introducing now the big O function, the previous expression becomes:

$$\left|\frac{G(N)}{N} - \frac{\pi(N)}{N\ln N}\right| = O((\ln N)^{-1}) \qquad (7)$$

and if instead of π(N) we introduce Li then it is:

$$\left|\frac{G(x)}{x} - \int_2^x \frac{dt}{t(\ln t)^2}\right| = O((\ln x)^{-1}) + O(x^{\frac{1}{2}+\varepsilon}) = O(x^{-\varepsilon}) + O(x^{\frac{1}{2}+\varepsilon}) \approx O(x^{\frac{1}{2}+\varepsilon}) \qquad (8)$$

The (8) is an expression in closed form and links the number of solutions to Goldbach G to Li and the (8) is an equivalent of RH. Then we have a function and its inverse, that can be go back to G(N) and vice versa with (6). We can do some calculation with the excel and set us also a rule that will automatically check the inequality ABS(A-B) < C1(N). We have "YES" if the rule is verified.

N	G(N)	A = G(N)/N	π(N)	B = π(N)/N LN(N)	ABS(A-B)	A-B	C1(N)=1/LN(N)	A-B<C1(N)?
4	1	0,25	2	0,36067376	0,11067376	-0,11067376	0,72134752	YES
6	1	0,166666667	3	0,279055313	0,112388647	-0,11238865	0,558110627	YES
8	1	0,125	4	0,240449173	0,115449173	-0,11544917	0,480898347	YES
10	2	0,2	4	0,173717793	0,026282207	0,026282207	0,434294482	YES
12	1	0,083333333	5	0,167679002	0,084345668	-0,08434567	0,402429604	YES
14	2	0,142857143	6	0,162395649	0,019538506	-0,01953851	0,378923182	YES
16	2	0,125	6	0,13525266	0,01025266	-0,01025266	0,36067376	YES
18	2	0,111111111	7	0,134546322	0,023435211	-0,02343521	0,345976256	YES
20	2	0,1	8	0,13352328	0,03352328	-0,03352328	0,333808201	YES
22	3	0,136363636	8	0,117641983	0,018721653	0,018721653	0,323515453	YES
24	3	0,125	9	0,117996743	0,007003257	0,007003257	0,31465798	YES
26	3	0,115384615	9	0,106244196	0,00914042	0,00914042	0,306927676	YES
28	2	0,071428571	9	0,096461238	0,025032666	-0,02503267	0,300101629	YES
30	3	0,1	10	0,098004701	0,001995299	0,001995299	0,294014104	YES
100	6	0,06	25	0,05428681	0,00571319	0,00571319	0,217147241	YES
200	8	0,04	46	0,043410008	0,003410008	-0,00341001	0,188739166	YES
300	21	0,07	62	0,036233266	0,033766734	0,033766734	0,175322254	YES
400	14	0,035	78	0,0325463	0,0024537	0,0024537	0,1669041	YES
500	13	0,026	95	0,030573127	0,004573127	-0,00457313	0,160911192	YES
600	32	0,053333333	102	0,026575249	0,026758084	0,026758084	0,156324996	YES
1000	28	0,028	168	0,024320491	0,003679509	0,003679509	0,144764827	YES
10000	128	0,0128	1229	0,013343698	0,000543698	-0,0005437	0,10857362	YES
100000	754	0,00754	9592	0,008331505	0,000791505	-0,00079151	0,086858896	YES
1000000*	5239	0,005239	78498	0,005681875	0,000442875	-0,00044287	0,072382414	YES
100000000	2593693	0,02593693	50847534	0,027603504	0,001666574	-0,00166657	0,05428681	YES

*with the formula of ERATOSTENE

Table 1 – G(N) and C1(N)

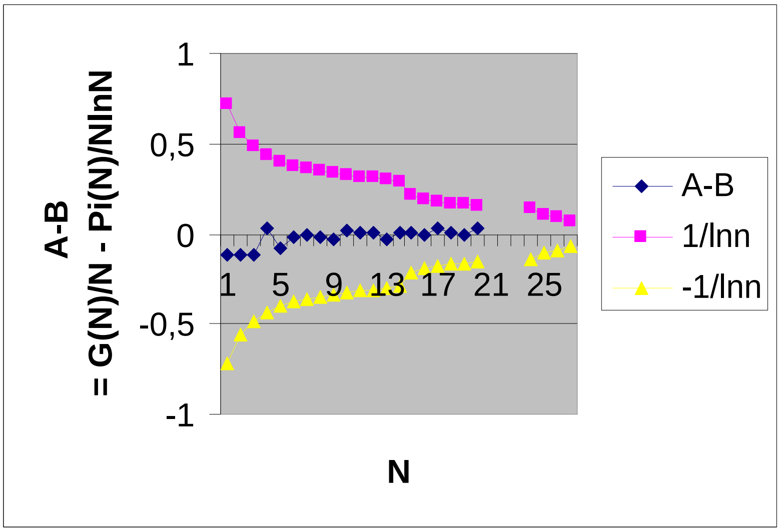


Figure 1 – bounds |1/lnN| of A-B

From (6) is G(N)>0; in fact G(x) make sense for $x \geq 4$. If the integral was equal to zero, we should have a positive area offset by a negative. But G (N) can not be negative, so G(N) $\neq$ 0.

La (6) is almost a "tool that would have liked to Chebyshev", which brings together several concepts of probability (see [25]):

"*The difference in absolute value between the number of solutions Goldbach G(N), compared to the same number N, and the counting of prime numbers up to N compared to 'N-th prime number ($\sim$N ln N), is less than the probability that N is a prime number ($\sim$1/ln N).* "

The term of error 1/ln(N), numerically, *works well*.

In the table 1 we have also sign A-B without ABS (module) and we can see, in figure 1, A-B fluctuate around the zero. In this work we also indicate that $1/x^{½}$ *works well (See Table 2 and figure 2)* and *the (8) is again true*:

$$\left|\frac{G(x)}{x} - \int_2^x \frac{dt}{t(\ln t)^2}\right| = O(x^{-1/2}) + O(x^{\frac{1}{2}+\varepsilon}) \approx O(x^{\frac{1}{2}+\varepsilon})$$

In this case in the (6) $C2(N)=1/N^{½}$.

N	G(N)	A = G(N)/N	π(N)	B = π(N)/N LN(N)	ABS(A-B)	A-B	C2(N)=1/SQRT(N)	A-B<C2(N)?
4	1	0,25	2	0,36067376	0,11067376	-0,11067376	0,5	YES
6	1	0,166666667	3	0,279055313	0,112388647	-0,11238865	0,40824829	YES
8	1	0,125	4	0,240449173	0,115449173	-0,11544917	0,353553391	YES
10	2	0,2	4	0,173717793	0,026282207	0,026282207	0,316227766	YES
12	1	0,083333333	5	0,167679002	0,084345668	-0,08434567	0,288675135	YES
14	2	0,142857143	6	0,162395649	0,019538506	-0,01953851	0,267261242	YES
16	2	0,125	6	0,13525266	0,01025266	-0,01025266	0,25	YES
18	2	0,111111111	7	0,134546322	0,023435211	-0,02343521	0,23570226	YES
20	2	0,1	8	0,13352328	0,03352328	-0,03352328	0,223606798	YES
22	3	0,136363636	8	0,117641983	0,018721653	0,018721653	0,213200716	YES
24	3	0,125	9	0,117996743	0,007003257	0,007003257	0,204124145	YES
26	3	0,115384615	9	0,106244196	0,00914042	0,00914042	0,196116135	YES
28	2	0,071428571	9	0,096461238	0,025032666	-0,02503267	0,188982237	YES
30	3	0,1	10	0,098004701	0,001995299	0,001995299	0,182574186	YES
100	6	0,06	25	0,05428681	0,00571319	0,00571319	0,1	YES
200	8	0,04	46	0,043410008	0,003410008	-0,00341001	0,070710678	YES
300	21	0,07	62	0,036233266	0,033766734	0,033766734	0,057735027	YES
400	14	0,035	78	0,0325463	0,0024537	0,0024537	0,05	YES
500	13	0,026	95	0,030573127	0,004573127	-0,00457313	0,04472136	YES
600	32	0,053333333	102	0,026575249	0,026758084	0,026758084	0,040824829	YES
1000	28	0,028	168	0,024320491	0,003679509	0,003679509	0,031622777	YES
10000	128	0,0128	1229	0,013343698	0,000543698	-0,0005437	0,01	YES
100000	754	0,00754	9592	0,008331505	0,000791505	-0,00079151	0,003162278	YES
1000000	5239	0,005239	78498	0,005681875	0,000442875	-0,00044287	0,001	YES

Table 2 – G(N) and C2(N)

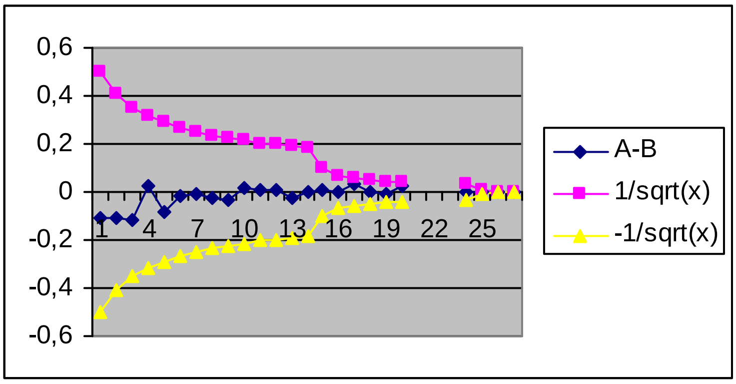


Figure 2 – bounds $|1/x^{1/2}|$ of A-B

In general the term of error is $1/N^{\varepsilon}$ where is $\varepsilon=1/t$ with $t\in R$; but if t is greater than 2 , the bound is too much wide.

Another mode to see the same situation is to consider that always $ABS(A-B)/C_1(N) < 1$ (see figure 3).

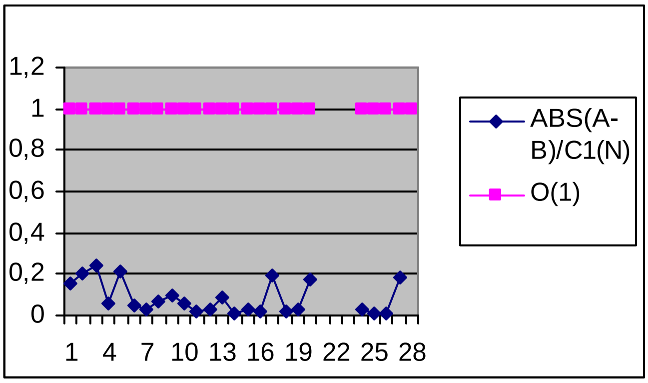


Figure 3 – ABS(A-B)/C1(N)

We can observe that *ln N* in the (6) is about H_n (harmonic number), but we could also introduce the von Mangoldt's function $\Lambda(N)$ or we can calculate G(N)/N from (8) through Li/N. We observe that Li has got an expansion of type:

$$Li(x) = x/\ln x + x/(\ln x)^2 + \ldots + (n-1)! * x/(\ln x)^n + o(x/(\ln x)^n)$$

where n is the number of terms that we consider.

The report found for G (N) is good for calculating g(N) the number of pairs of twin primes:

$$\left|\frac{g(N)}{N} - \frac{\pi(N)}{N\ln N}\right| < KC(N), \quad K=1, \quad C(N)=\frac{1}{\ln N}$$

$$\left|\frac{g(x)}{x} - \int_2^x \frac{dt}{t(\ln t)^2}\right| = O(x^{\frac{1}{2}+\varepsilon}) \quad (9)$$

An immediate verification with some number we can make for a real conviction.

N	g(N)	A=g(N)/N	π(N)	B=π(N)/N*LN(N)	ABS(A-B)	C(N)	ABS(A-B)<C(N)?
3	0	0	2	0,606826151	0,606826151	0,910239227	YES
4	0	0	2	0,36067376	0,36067376	0,72134752	YES
5	1	0,2	3	0,372800961	0,172800961	0,621334935	YES
6	1	0,166666667	3	0,279055313	0,112388647	0,558110627	YES
7	2	0,285714286	4	0,293656196	0,00794191	0,513898342	YES
8	2	0,25	4	0,240449173	0,009550827	0,480898347	YES
9	2	0,222222222	4	0,202275384	0,019946839	0,455119613	YES
10	2	0,2	4	0,173717793	0,026282207	0,434294482	YES
11	2	0,181818182	5	0,189560178	0,007741996	0,417032391	YES
12	2	0,166666667	5	0,167679002	0,001012335	0,402429604	YES
13	3	0,230769231	6	0,179940575	0,050828656	0,389871245	YES
14	3	0,214285714	6	0,162395649	0,051890065	0,378923182	YES
15	3	0,2	6	0,147707749	0,052292251	0,369269373	YES
16	3	0,1875	6	0,13525266	0,05224734	0,36067376	YES
17	3	0,176470588	7	0,145334875	0,031135714	0,352956124	YES
18	3	0,166666667	7	0,134546322	0,032120345	0,345976256	YES
19	4	0,210526316	8	0,142999272	0,067527043	0,339623272	YES
20	3	0,15	8	0,13352328	0,01647672	0,333808201	YES
21	4	0,19047619	8	0,125127139	0,065349052	0,328458739	YES
22	4	0,181818182	8	0,117641983	0,064176199	0,323515453	YES
23	4	0,173913043	9	0,1247983	0,049114743	0,318928989	YES
24	4	0,166666667	9	0,117996743	0,048669924	0,31465798	YES
25	4	0,16	9	0,111840288	0,048159712	0,310667467	YES
26	4	0,153846154	9	0,106244196	0,047601958	0,306927676	YES
27	4	0,148148148	9	0,101137692	0,047010456	0,303413076	YES
28	4	0,142857143	9	0,096461238	0,046395905	0,300101629	YES
29	4	0,137931034	10	0,102404898	0,035526136	0,296974204	YES
30	4	0,133333333	10	0,098004701	0,035328632	0,294014104	YES
100	8	0,08	25	0,05428681	0,02571319	0,217147241	YES
200	15	0,075	46	0,043410008	0,031589992	0,188739166	YES
300	19	0,063333333	62	0,036233266	0,027100067	0,175322254	YES
400	21	0,0525	78	0,0325463	0,0199537	0,1669041	YES
500	24	0,048	95	0,030573127	0,017426873	0,160911192	YES
600	26	0,043333333	102	0,026575249	0,016758084	0,156324996	YES
1000	35	0,035	168	0,024320491	0,010679509	0,144764827	YES

Table 3 – g(N)

So even for g(N), similar considerations apply. Also (9) is an equivalent RH.

A step function and a generalization of Polignac

Now, we introduce the ***von Mangoldt's function*** *(also called lambda function)*:

$$\Lambda(n) = \begin{cases} \log p, & if\ n=p^k,\ \ p\ \ prime,\ k\geq 1 \\ 0, & \text{otherwise} \end{cases} \qquad (10)$$

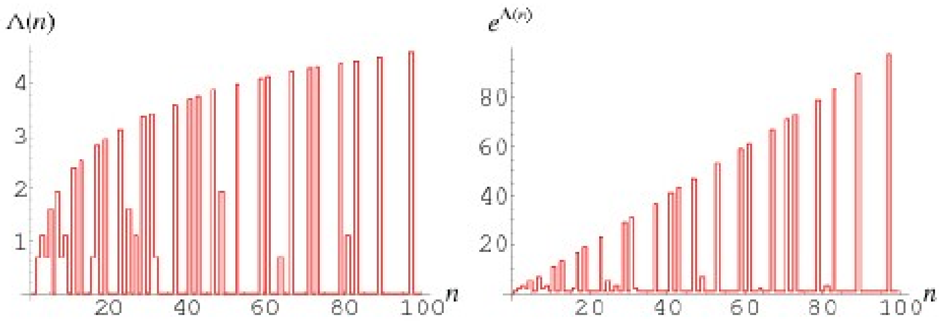


Figure 4 – von Mangoldt's function

The von Mongoldt's function isn't a multiplicative function nor an additive function. Moreover it's:

$$\log n = \sum_{d|n} \Lambda(d) \quad \text{where d | n are divisor of n}$$

<u>Example</u>

n=12

We remember $12=2^2*3$ and that the divisors of 12 are: 1, 2, 3, 4, 6, 12, then:
$\log 12=\Lambda(1)+ \Lambda(2)+ \Lambda(3)+ \Lambda(2^2)+ \Lambda(2*3)+ \Lambda(2^2*3)$

From (10) it is:
log 12= 0 + log 2 + log 3 + log 2 + 0 + 0= log(2*3*2) = log 12

We know $\pi(N)$ as a counting prime function (a step function):

$$\pi(N) = \sum_{p \le x} 1$$

If we introduce the von Mangoldt's function $\Lambda(N)$ then *we propose* a "*step function* $\upsilon(N)$" (see figure 6):

$$\nu(N) = \frac{\pi(N)}{N\ \Lambda(N)}$$

$$\frac{G(N)}{N} \sim \nu(N) \qquad \text{or} \qquad \lim_{x\to\infty} \frac{G(x)}{x} \Big/ \nu(x) = 1$$

For example

π(N)	**Λ(N)**	**υ(N)**	**G(N)/N**
π(10) = 4	Λ(10) = ln 10 = 2,3	υ(10) = 4/(10*2,3)=0,17	G(10)/10 = 0,1
π(30) = 10	Λ(30) = ln 30 = 3,401	υ(30) = 10/(30*3,401)=0,098≈0,1	G(30)/30 = 3/30=0,1
π(100) = 25	Λ(100) = ln 100 = 4,605	υ(100) = 25/(100*4,605)=0,054	G(100)/100 = 6/100=0,06
π(1000) = 168	Λ(1000) = ln 1000 = 6,907	υ(1000) = 168/(1000*6,907)=0,0243	G(1000)/1000 = 28/1000=0,028

The approssimations of the step function v(N) improve when N grows (see figure 5).

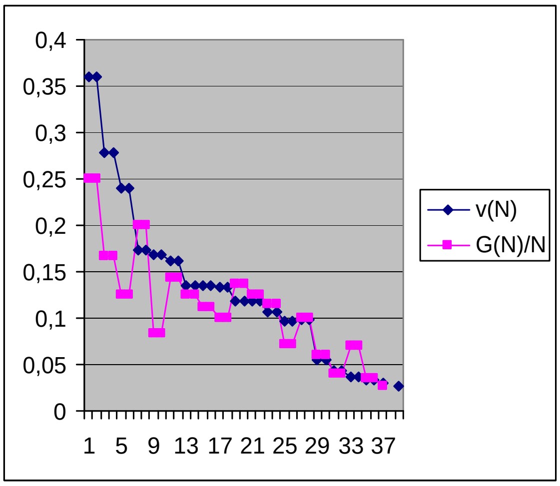


Figure 5 – step function v(N)

Can we generalize this result as *a generalization of Polignac*? Yes. If we call *P(x , d)* the number of primes $\leq$x and which are far d, if we remember the GRH [see 25], since it is:

$$\pi(x,a,d) = \frac{1}{\varphi(d)}\int_2^x \frac{1}{\ln t}dt + O(x^{\frac{1}{2}+\varepsilon}), \quad x \to \infty \quad (11)$$

where a, d are such that gcd(a, d)=1 and φ(d) is the *totient function* of Euler.

Then it is:

$$|\frac{P(x,d)}{x} - \frac{\pi(x,a,d)}{x\ln x}| < kC3(x), k = 1, C3(x) = \frac{1}{\ln x}$$

or (12)

$$|\frac{P(x,d)}{x} - \frac{1}{\varphi(d)}\int_2^x \frac{1}{t(\ln t)^2}dt| = O(x^{\frac{1}{2}+\varepsilon})$$

Example

x=127, a=2,d=9

gcd(a , d) =1

a + d: 11, 20, 29, 38, 47, 56, 65, 74, 83, 92, 101, 110, 119, 128 …

We have underlined the prime numbers above.

φ(9) = 6, in fact 1,2,4,5,7,8 are the numbers without nothing in common with 9,

Li ~ x/ln x

π(127-2,9) ~ [1/φ(9)]*(127/ln 127) ≈ 4,36 about 5.

But this result is also P(x, 9). In fact if we consider a=1,2,4,5,7,8 or the numbers less than 9 and without nothing in common with 9, we have six *arithmetic progressions*:

a=1, a + d:	10, 19, 28, 37, 46, 55, 64, 73, 82, 91, 100, 109, 118, 127 …
a=2, a + d:	11, 20, 29, 38, 47, 56, 65, 74, 83, 92, 101, 110, 119, 128 …
a=4, a + d:	13, 22, 31, 40, 49, 58, 67, 76, 85, 94, 103, 112, 121, 130 …
a=5, a + d:	14, 23, 32, 41, 50, 59, 68, 77, 86, 95, 104, 113, 122, 131 …
a=7, a + d:	16, 25, 34, 43, 52, 61, 70, 79, 88, 97, 106, 115, 124, 133 …
a=8, a + d:	17, 26, 35, 44, 53, 62, 71, 80, 89, 98, 107, 116, 125, 134 …

In all arithmetic progression we have prime numbers. How many are the prime numbers in each arithmetic progression?

About:

$$[1/\varphi(d)] * (x/\ln x).$$

Then the absolute value of difference [P(x, d)/x] - [π(x, a, d)/x ln x] is very little.

In fact for a = 2 is:

P(127,9)/127 = 5/127 = 0,00031

π(127, 2, 9)/(127 ln 127) = 0,007102

|[P(127, 9)/127] - [π(127, 2, 9)/127 ln 127]| = 0,00679 < 1/ln 127 = 0,206433.

We can obtain, with the integral, a value better than x/ln x; in fact (see Appendix) it is:

$$\int_2^x \frac{dt}{t \cdot \ln^2 t} = \frac{1}{\ln 2} - \frac{1}{\ln x}$$

Then it is:

π(127, 2, 9) = 1/6 * (1/ln 2 - 1/ln 127) = 0,20604

π(127, 2, 9) / 127 ln 127 = 0,000334 a similar result of P(x, d)/x

|[P(127, 9)/127] - [π(127, 2, 9)/127 ln 127]| = 0,000024 < 1/ln 127 = 0,206433.

So, also (12) is an equivalent RH.

Appendix

$$\int_2^x \frac{dt}{t\cdot \ln^2 t} = \int_2^x d(\ln x)\cdot\frac{dt}{\ln^2 t} = \left[\ln t\cdot\frac{1}{\ln^2 t}\right]_2^x - \int_2^x \ln t\cdot d\left(\frac{1}{\ln^2 t}\right)dt =$$

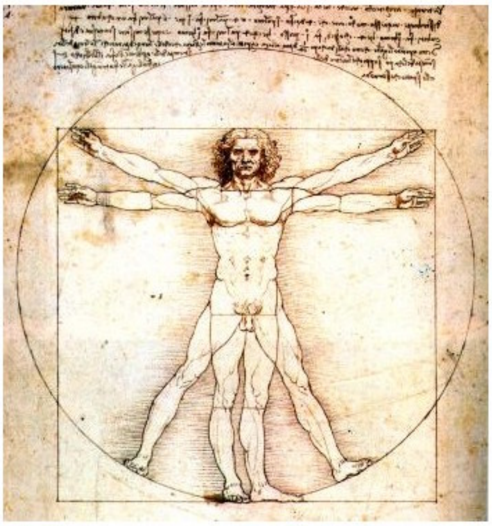
$$\frac{1}{\ln x}-\frac{1}{\ln 2}-\int_2^x \ln t\cdot\left(\frac{-2\ln t}{\ln^4 t}\cdot\frac{1}{t}\right)dt = \frac{1}{\ln x}-\frac{1}{\ln 2}+2\int_2^x\frac{dt}{t\cdot\ln^2 t}$$

so :

$$\int_2^x \frac{dt}{t\cdot \ln^2 t} = \frac{1}{\ln x}-\frac{1}{\ln 2}+2\int_2^x\frac{dt}{t\cdot\ln^2 t}$$

then :

$$\frac{1}{\ln 2}-\frac{1}{\ln x} = \int_2^x\frac{dt}{t\cdot\ln^2 t}$$



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