Oxygen Excess Ratio Model Predictive Control for a PEM Fuel Cell System

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***Abstract*—Oxygen excess ratio control is probably the most important task in the control of a Fuel Cell System (FCS), since it affects the Proton Exchange Membrane (PEM) lifetime; in literature, many publications paid attention to the Static Feedforward (SF) control technique, acting on compressor speed. In this paper the results will show how the SF and other classical PI control techniques are inadequate to control the oxygen excess ratio when a realistic model for the AC drive+compressor subsystem is used. Another control input, the return manifold throttle opening, will be used together with the Model Predictive Control (MPC) in order to obtain a better dynamic response for the oxygen excess ratio waveform, without any constraints on the load current.**

***Index Terms*—fuel cell system, oxygen excess ratio control, model predictive control, PMSM**

I. Introduction

NOWADAYS, the energy production and management is an important problem; fuel cells are a competitive solution to this problem, so it is obvious that the fuel cells technology have had a notable increment in this past years. The control of the FCS is very difficult, since it is made up of many interacting subsystem; normally, up to five subsystem may be present: the hydrogen supply subsystem, the oxygen supply subsystem, the cooling system, the humidifier and finally the electrical power management system. Nevertheless, the most part of fuel cell problem remains still unsolved: oxygen starvation, full load drop stack voltage and high time constant to load increment.

A. Literature Overview

The model used in this paper is based on that developed in [1]; other FCS model have been developed in [2], [3]. Only the models developed in [1], [2] present a oxygen supply subsystem control. The FCS model developed in [1] was also used in [4]–[6] to study new control methodologies; in [4] the oxygen supply control system acts only on the compressor motor input. Two control methodologies are presented: dynamic feedforward and static feedforward with a Linear Quadratic Regulator (LQR) feedback.

Both [6] and [5] present a predictive control for the FCS in order to prevent oxygen starvation. The FCS implemented has a hybrid configuration, with a battery/supercapacitor integration; the control system is then able to regulate also the stack current. Predictive controller also implements the compressor and the supercapacitor constraints, to prevent the FCS works in dangerous or instable conditions.

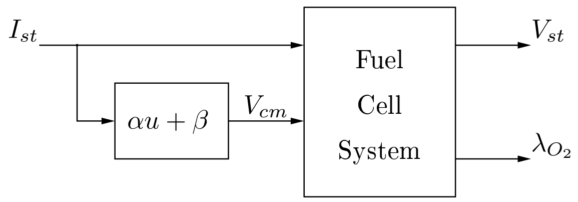

Figure 1. Static feedforward control scheme for the FCS [1]

It should be noted that the compressor motor model implemented in [1] is the static model of a DC machine, i.e.:

$$I_a = \frac{V - k_V \omega_{cp}}{R} \tag{1}$$

$$t_e = k_t I_a \tag{2}$$

$$t_e - t_l = \frac{1}{J}\frac{d\omega_{cp}}{dt} \tag{3}$$

where I_a is the stator current, V the stator applied voltage, ω_{cp} the rotation speed of the compressor, t_e the electromagnetic torque, t_l the compressor torque and finally J the inertia momentum of the whole DC machine + compressor system; the control of the DC machine is a voltage control, acting on the stator voltage V; the lack of the stator inductance is as unrealistic as the consequent immediate current response. Moreover, the compressor motor has a high rotational speed, over $krpm$; at such rotational speed is impossible to use a DC machine, cause the mechanical strenght of the commutator and the overvoltage produced by the commutation process.

For these reasons, the DC motor model has been replaced by a Interior Mounted Permanent Magnet Synchronous Motor (IM-PMSM) model with vector control. This motor is more suitable in high–speed application, thanks to the absence of the brushes and the displacement of the magnets that, being

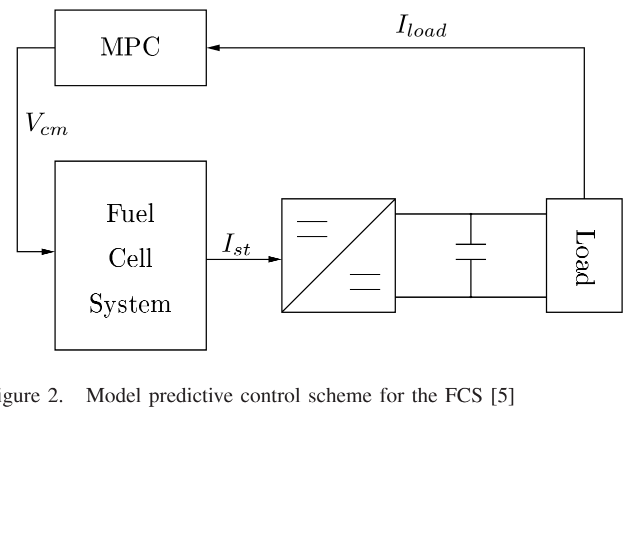

Figure 2. Model predictive control scheme for the FCS [5]

caged in the rotor, permit higher rotational speed without any danger of detachment.

B. Control Problem Formulation

By considering the dynamic of the compressor electrical motor, the prevention of the oxygen starvation become a leading task, since the motor requires a unnegligible time to increase the rotational speed after the speed setting has been modified. This obvious delay cannot be reduced with a SF control, for the reason that the SF can be regarded as a function between the load current and the compressor motor speed reference $\omega_{cp,ref} = f(I_{load})$; for each value of load current, the static feedforward sets a value of speed reference, therefore the transient time, needed by the PMSM to reach the new rotational speed value, depends only on the PMSM drive itself.

Another control input is necessary in order to prevent the oxygen starvation. As the current is drawn from the fuel cell, the oxygen is depleted in the electrochemical reaction; if the compressor drive is inact to quickly replenish the oxygen consumed, the inlet air mass flow can be increased acting on the return manifold throttle opening. In this way, the idraulic resistance seen by the compressor decreases and, with the same rotating speed, the compressor is able to inflate a greater quantity of oxygen.

The control system acts on two control input: the compressor rotational speed reference $\omega_{cp,ref}$ and the return manifold throttle opening reference $A_{t,ref}$. Many constraints must be included in the controller, because the compressor must not work in surge or in choke condition, i.e. in instable conditions, neither the oxygen excess ratio must decrease under a certain limit. For these reasons, a predictive controller is the optimal choice, because:

- it can handle intrinsically multivariable systems;
- it can take into account many linear constraints;
- the controlled plant can work near the constraints, improving dynamic performance without lose stability.

Taking in account the inadequacy of the SF control, a PI controller has been chosen as comparative control technique for the predictive controller. The PI controller acts only on the compressor motor voltage and its parameters has been calculated starting from the linear analysis of the transfer function between the oxygen excess ratio λ_{O_2} and the compressor motor voltage V_{cm}. The O_2 excess ratio is then considered as a measurable variable, for the PI controller; this assumption is necessary in order to simplify the control problem formulation and does not affect the validity of the results, as the PI controller is used just for the comparison. The main problem in realizing the PI controller is, on the contrary respect to the MPC, the impossibility to implement any linear constraint; for this reason, in order to ensure stability to the system, the PI controller can not have a quick response, i.e. a high bandwidth.

II. Modelling and Control of PMSM

The mathematical model of the PMSM in the rotor reference frame can be written as:

$$u_{sd} = R_s i_{sd} + L_{sd}\frac{di_{sd}}{dt} - L_{sq}\omega_r i_{sq} \quad (4)$$

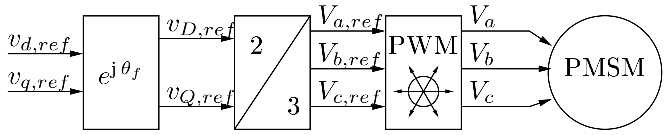

Figure 3. Vector control scheme of the presented IM-PMSM

$$u_{sq} = R_s i_{sq} + L_{sq}\frac{di_{sq}}{dt} + L_{sd}\omega_r i_{sq} + \omega_r \Psi_f \quad (5)$$

where u_{sd} and u_{sq} are the direct and quadrature–axis component of the stator voltage, i_{sd} and i_{sq} are the direct and quadrature–axis component of the stator current, L_{sd} and L_{sq} are the direct and quadrature–axis inductance, Ψ_f is the excitation fluk linkage and, finally, ω_r is the rotational speed measured in electrical radians.

The electromagnetic torque produced can be calculated as:

$$t_e = \frac{3}{2}p\left[i_q\Psi_f + (L_{sd} - L_{sq})\, i_{sd} i_{sq}\right] \quad (6)$$

being p the number of pole–pairs. It should be noted that, in an interior mounted PMSM, the saliency torque term $(L_{sd} - L_{sq})\, i_{sd} i_{sq}$ is antagonist for the reason that in an anisotropic PMSM L_{sd} is smaller than L_{sq}; a $i_{sd} = 0$ control is therefore more suitable, since no saliency torque is more present. Through (4) and (5) it can be deduced the possibility to control separately the flux current i_{sd} and the torque current i_{sq} acting on the respective voltage term. Although a coupling term is present in both the equations, the separate control can be obtained thanks a feedforward action, whose task is to cancel out the coupling terms.

Speed vector control has been obtained with classical PI controller, whose parameter have been calculated from analysis of the Bode diagrams for the flux and the torque transfer function. Resulting speed PI controller has a bandwidth equal to 112 Hz and a phase margin equal to 83 degrees. Detailed description of the motor drive model and its control can be found in [7].

III. Model Predictive Control for the FCS

As presented in Section I, the predictive control is the optimal control strategy for the FCS. Infact, the presence of two different control input for the FCS do not require the study of two different controller, because the predictive controller can handle intrisically as many control input as required. Besides, a PI controller cannot handle a linear constraints on its controlled variable without incurring in the *wind–up* problem, and cannot handle multiple linear constraints, or linear combination of constraints on multiple variables, unless looking to *if...then* logical switch. Finally, add a new controlled variable, or a new control input, is much easier with a predictive controller, because only few constraints must be added to the control problem formulation.

A. Basics on the Model Predictive Control

In order to realize the predictive control for a plant, the FCS particularly, a linear model is needed; in general, the linear model must be in the discrete–time form, without direct feedthrough between input and output, i.e. with a $\mathbf{D}$ matrix equal to zero:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}_y\mathbf{x}(k) \\ \mathbf{z}(k) = \mathbf{C}_z\mathbf{x}(k) \end{cases} \tag{7}$$

where $\mathbf{x}$ is the n–dimensional state vector, $\mathbf{u}$ is the l–dimensional input vector, $\mathbf{y}$ is the m_y–dimensional vector of measured output and $\mathbf{z}$ is the m_z–dimensional vector of controlled variables. This model can be obtained from linearizing the nonlinear plant around an equilibrium operating point, analyzing the effects of small perturbations on inputs and states.

A *reference trajectory*, $r(k)$, must be imposd for every controlled variable, plus a *weight*, $Q_{i,j}$, to penalize the deviation of the controlled variable from the desidered values. Potentially, constraints can be setted for the controlled variables. Weights $R_{i,j}$ are also imposed on the input variables[1] to penalize their variations or their absolute values.

The optimal input vector $\mathbf{x}$ for every time instant k, neglecting the constraints, is obtained minimizing a *cost function*, which in the basic form can be written as:

$$V(k) = \sum_{i=H_w}^{H_p} \|\hat{\mathbf{z}}(k+i|k) - \mathbf{r}(k+i|k)\|^2_{Q(i)} + \sum_{i=0}^{H_u-1} \|\Delta\hat{\mathbf{u}}(k+i|k)\|^2_{R(i)} \tag{8}$$

where H_p is the *prediction horizon*, H_u the *control horizon*, $\mathbf{Q}$ and $\mathbf{R}$ matrices composed of the $Q_{i,j}$ and $R_{i,j}$ weights.

The predictive controller can, for each time instant k, compute the optimal variation input vector $\boldsymbol{\Delta}\hat{\mathbf{u}}(k|k)$ that, added to the previous input vector $\mathbf{u}(k-1)$, minimize the cost function 8.

In [8] the method for modify the MPC formulation in order to include any linear constraint or inequality on the $\mathbf{x}$, $\hat{\mathbf{z}}$, $\boldsymbol{\Delta}\hat{\mathbf{u}}$ and $\hat{\mathbf{x}}$ vectors is shown. The control problem become a *Quadratic Programming* (QP) problem, in the form:

$$\min \frac{1}{2}\theta^T \Phi \theta + \varphi^T \theta \;\text{ subject to } \Omega\theta \leq \omega \tag{9}$$

In pratice, the MPC, at each time step k, performs the followings steps:

- measurement of measured output $\mathbf{y}(k)$;
- calculation of the required plant input $\mathbf{u}(k)$;
- application of $\mathbf{u}(k)$ to the plant.

B. The Application to the FCS

As first step to realize the predictive controller of the FCS, a linear model for the FCS has been obtained; linearization point has been found setting the following values for the input variables [4]:

$$i_{st} = 190\ A$$
$$\omega_{cp} = 76.70\ krpm$$
$$A_t = 2 \cdot 10^{-3}\ m^2$$

[1]Also called *manipulated variables*.

where i_{st} is the current drawn from the FCS. Tests performed to check the linear model validity shown that the stationary outputs error between the linear and the nonlinear model is below 10% for i_{st} variations of $\pm 100\ A$.

According to [1], state vector is composed of followings variables:

$$\mathbf{x} = [m_{O_2}\ m_{H_2}\ m_{N_2}\ \omega_{cp}\ p_{sm}\ m_{sm}\ m_{w,an}\ p_{rm}\ i_{sd}\ i_{sq}\ \theta_r] \tag{10}$$

where m_{O_2}, m_{H_2} and m_{N_2} are respectively the oxygen, hydrogen and nitrogen mass, ω_{cp} is the compressor rotational speed, p_{sm} is the supply manifold pressure, m_{sm} is the air mass in the supply manifold, $m_{w,an}$ is the water vapor mass in the anode, p_{rm} is the return manifold pressure and θ_r is the rotor mechanical position.

The $\mathbf{y}$ vector of measured outputs is given by:

$$\mathbf{y} = [W_{cp}\ p_{sm}\ v_{st}] \tag{11}$$

where W_{cp} is the inlet air mass flow from the compressor and v_{st} is the stack voltage.

Only oxygen excess ratio has been controlled, therefore controlled outputs vector is:

$$\mathbf{z} = [\lambda_{O_2}] \tag{12}$$

No control has been performed on the electrical power drawn from the FCS.

The manipulated variables vector is composed of the instant values of the compressor motor rotational speed reference and the return manifold throttle opening reference, while the current drawn from the FCS is regarded as a measured disturbance:

$$\mathbf{u} = [\omega_{cp,ref}\ A_{t,ref}] \tag{13}$$
$$\mathbf{w} = [i_{st}] \tag{14}$$

While speed control on compressor PMSM is a closed–loop control, the throttle opening control is an open–loop control, supposing that $A_t = A_{t,ref}$.

Constraints has been set for the input vector, both to ensure stability to the controlled plant and to avoid unrealistic throttle opening variations:

$$\Delta\omega_{cp,ref} \in [-500; 500]\ \ krpm/s$$
$$\Delta A_{t,ref} \in [-40; 40]\ \ cm^2/s$$

Finally, the constraints on the controlled variable has been set to:

$$\Delta\lambda_{O_2} \in [-0.1; 0.1]$$

The prediction horizon H_p has been set to 40 time steps, as in [5], and the control horizon H_c to 10 time steps, to ensure controller stability. The weight matrices $\mathbf{Q}$ and $\mathbf{R}$ have been chosen to insure the minimal variations for the oxygen excess ratio, having the fastest oxygen replacement. The weight Q_1 for the controlled variable is equal to 10 for the whole prediction horizon, while a null value has been given to the weights for the other output variables. The throttle opening variations weight R_1 has been set to 1 for the control horizon, $R_1 = 0$ for the remaining prediction horizon time steps, to limit excessive variations during the first transient time instant.

IV. Simulation Results

In this section, results of the predictive controller applied to the FCS are shown. Figure shows the oxygen excess ratio waveform when the load current i_{st} varies from 130 to 300 A, with many step variations as shown in Figure 4. This current waveform is quite difficult to realize, but it has not been modified in order to obtain results comparable with [1]. The PI controller used for comparison has a bandwidth equal to 1 Hz and a phase margin equal to 70°. It should be

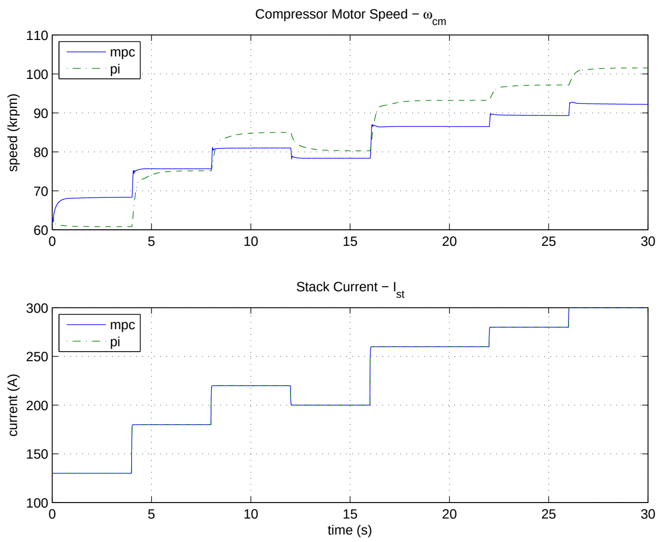

Figure 4. Input stack current of the FCS and corresponding compressor motor speed

noted that the compressor motor speed, in the FCS with the predictive controller, is lower at higher loads; infact, at high load, the return manifold throttle helps the compressor to give the necessary amount of oxygen.

Figure 5 shows how the predictive controller can minimize the danger of oxygen starvation: during few time instants, the compressor motor speed reference is much greater than the stationary value, therefore the PMSM has a quicker response than the PI controlled one. When the compressor motor speed reaches the stationary value, the reference is decreased, to nullify the speed error. In this way, the transient time is minimized and the depleted oxygen can be quckly replenished.

In Figure 6 the action of the predictive controller on the return manifold throttle opening is shown: for each load variation, the throttle opening is increased, or decreased, as necessary. The results of this action is clearly shown in Figure 7: the minimum value of O_2 excess ratio due to a load increment equal to 160 A is equal for the FCS with predictive controller and for the system with a PI controller; neverthless, the PI controller has a slower response, which causes an oxygen starvation for a longer time interval.

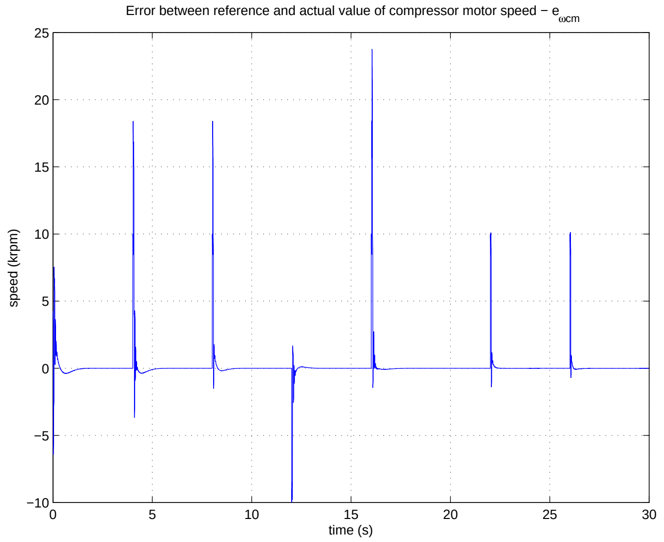

Figure 5. Error between reference and actual value of compressor motor speed

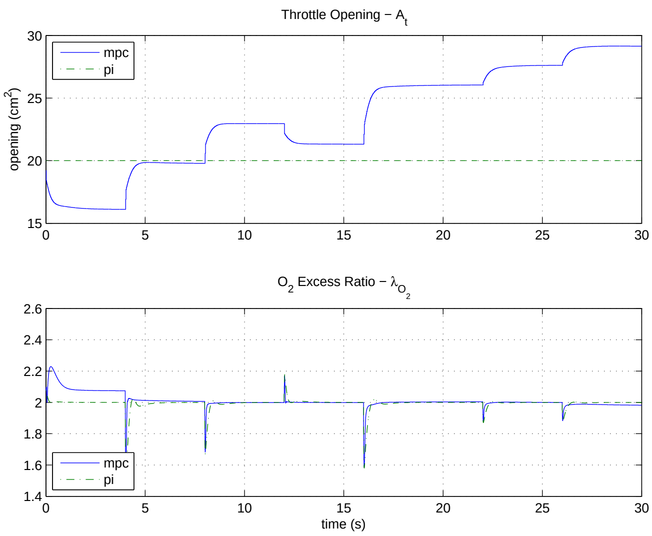

Figure 6. Return manifold throttle opening and O_2 excess ratio

The PI response must be slower than the response of the predictive controller for the reason that the PI can not manage any linear constraints; so, a quicker response causes violations of the surge compressor limit, that entails instability for the whole Fuel Cell System.

V. Conclusions

In this paper an innovative Model Predictive Control based technique for the Fuel Cell Systems has been presented. A comparison with a classical PI controller has been performed; results have been shown that only the predictive controller can manage the linear constraints imposed by the compressor without affecting the whole system performance.

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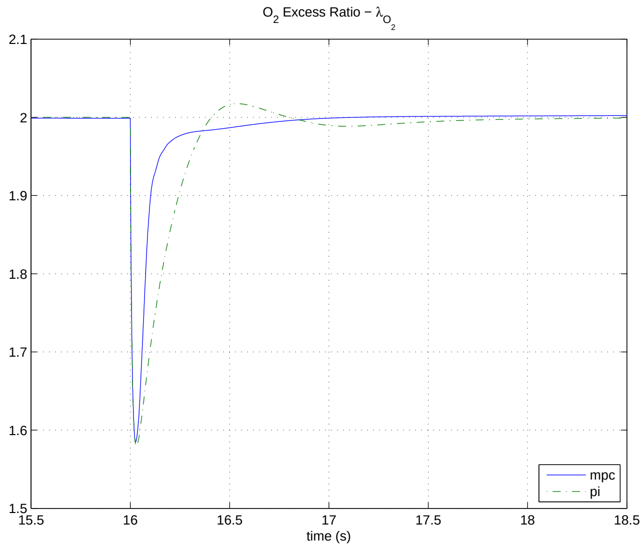

Figure 7. O_2 excess ratio variation at $t = 16\ s$

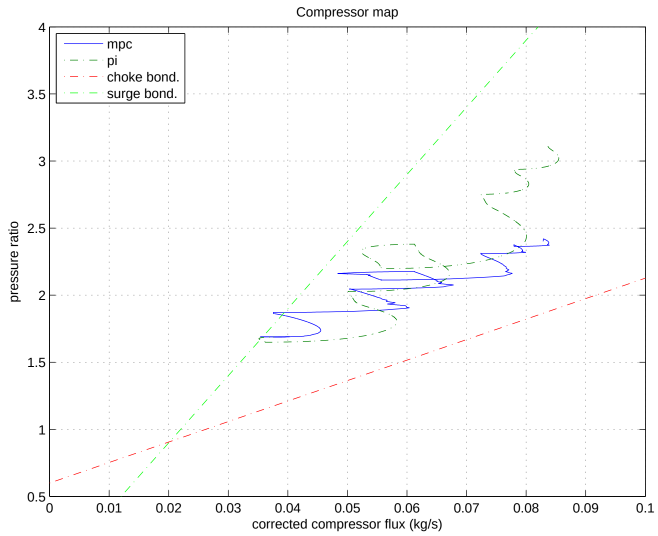

Figure 8. Compressor work trajectory in the compressor map

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